Non-linear deformations of duality-symmetric theories

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with Paolo Pasti and Mario Tonin arXiv:1205.4243

XVIII European Workshop on String Theory, September 22, 2012
Motivation

- Duality symmetry plays important role in many theor. models of physical interest
- $N=8$ supergravity is invariant under $E_{7(7)}$ \textit{(Cremmer & Julia '79)}

\[ F'^i_{\mu\nu} = (F^A_{\mu\nu}, G^\bar{A}_{\mu\nu}) \quad i = 1, \ldots, 56 \text{ of } E_{7(7)} \text{ and } A, \bar{A} = 1, \ldots, 28 \text{ of } SU(8) \]

On-shell linear (twisted self-) duality:

\[ G^\bar{A}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^\rho\sigma^A \quad \Rightarrow \quad F'^{-i}_{\mu\nu} \equiv F'^i_{\mu\nu} - \frac{1}{2} \Omega^i j \varepsilon_{\mu\nu\rho\sigma} F'^\rho\sigma^j = 0, \quad \Omega^i_k \Omega^k_j = -\delta^i_j \]

- $N=8$ supergravity is perturbatively finite at 3 and 4 loops \textit{(Bern et. al.)}
- Assumption: SUSY + $E_{7(7)}$ may be in charge of the absence of divergences \textit{(Kallosh)}
- $E_{7(7)}$-invariant counterterms can appear at 7 loops $\partial^{2k} F^4, \partial^{2k} R^4$
Motivation

- higher-order deformations $\partial^{2k}F^4$ in the effective action will lead to a non-linear deformation of the twisted self-duality condition

$$L = \frac{1}{4}F^2 + \partial^{2k}F^4 + \cdots,$$

$$\tilde{G} = 2 \frac{\delta L(F)}{\delta F} \Rightarrow F^i_+ = \frac{\delta \Delta(F)}{\delta F^i_+} \neq 0, \quad F^i_- = F^i_- + \Omega^i_j F^j_-, \quad \tilde{G}_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\rho\lambda} G^{\rho\lambda}

F^i_\sim = (F^A, G^{\sim A}) \quad \Delta(F) - \text{duality-invariant counterterm}

Questions to answer:

- how, exactly, possible higher-order terms may deform the effective action and duality relation between ‘electric’ and ‘magnetic’ fields, while keeping duality symmetry?
- check whether this deformation is compatible with supersymmetry

- in this talk we shall mainly concern with the first problem
- brief comments on supersymmetry in conclusion
Two ways of dealing with duality-symmetric theories

I. Lagrangian depends only on ‘electric’ fields $L(F)$ and is not duality-invariant

- **Duality symmetry** manifests itself only on-shell: 
  \[ \tilde{G} = 2 \frac{\delta L(F)}{\delta F} = F + \Delta(F) \]
  in the linear case
  \[ F' = (F, G), \quad \delta F'^i = M^i{}_j F'^j \] - linear duality transform $M^i{}_j \subset Sp(2N)$

- The variation of $L(F)$ under duality transform should satisfy a condition
  \[(Gaillard-Zumino '81, '97; Gibbons-Rasheed '95)\]
  \[ \delta L = \frac{1}{4} \delta (F \tilde{G}) \Rightarrow F\tilde{F} + G\tilde{G} = 0 \]

II. Lagrangian depends on both ‘electric’ and ‘magnetic’ fields $L(F'^i)$.
It is manifestly duality invariant. Duality condition follows from e.o.m.
Subtleties with space-time covariance
Duality-invariant actions

- Space-time invariance is not manifest
  \((Zwanziger '71, Deser & Teitelboim '76, Henneaux & Teitelboim '87, ...)

**Example:** duality-symmetric Maxwell action for \(F^i = dA^i \ (i = 1, 2)\)

\[
L = \frac{1}{8} F_{\mu\nu}^i F^{i\mu\nu} + \frac{1}{4} (F_{0a}^i - \varepsilon^{ij} \tilde{F}_{0a}^j) (F^{0ai} - \varepsilon^{ij} \tilde{F}^{0aj})
\]

\(\mu = (0, a) \ a = 1, 2, 3\)

breaks manifest Lorentz invariance

**Modified Lorentz invariance:** \(\delta A^i_\mu = \delta A^i_\mu + x^a \Lambda^0_a (F_{0\mu}^i - \varepsilon^{ij} \tilde{F}_{0\mu}^j)\)

**Twisted self-duality condition is obtained by integrating the e.o.m.:**

\[
\frac{\delta L}{\delta A^i} = 0 \quad \Rightarrow \quad F_{\mu\nu}^i - \varepsilon^{ij} \tilde{F}_{\mu\nu}^j = 0 \quad \Rightarrow \quad F_{\mu\nu}^1 = \tilde{F}_{\mu\nu}^2
\]
Space-time covariance can be restored by introducing an auxiliary scalar field $a(x)$ (Pasti, D.S. & Tonin '95)

$$L_{\text{nonc}} = \frac{1}{8} F_{\mu \nu}^{i} F^{i \mu \nu} + \frac{1}{4} (F_{0 a}^{i} - \epsilon^{ij} \tilde{F}_{0 a}^{j})(F^{0 ai} - \epsilon^{ij} \tilde{F}^{0 aj}) \quad \mu = (0, a) \quad a = 1, 2, 3$$

$$L_{\text{cov}} = \frac{1}{8} F_{\mu \nu}^{i} F^{i \mu \nu} + \frac{1}{4} \nu^{\mu} (F_{\mu \nu}^{i} - \epsilon^{ij} \tilde{F}_{\mu \nu}^{j})(F^{\nu \lambda i} - \epsilon^{ij} \tilde{F}^{\nu \lambda j}) \nu_{\lambda} (x)$$

Local symmetries:

$$\delta A_{\mu}^{i} = \partial_{\mu} \lambda^{i} (x)$$

$$\delta_{I} A_{\mu}^{i} = \Phi(x) \partial_{\mu} a(x), \quad \delta_{I} a(x) = 0 \quad \rightarrow \quad \nu^{\mu} A_{\mu}^{i} \quad \text{is pure gauge}$$

$$\delta_{II} a(x) = \varphi(x), \quad \delta_{II} A_{\mu}^{i} = \frac{\varphi(x)}{\sqrt{(\partial a)^{2}}} \nu^{\nu} (F_{\mu \nu}^{i} - \epsilon^{ij} \tilde{F}_{\mu \nu}^{j}) \quad \rightarrow \quad \text{gauge fixing}$$

$$\nu_{\mu} (x) = \frac{\partial_{\mu} a(x)}{\sqrt{(\partial a)^{2}}}, \quad \nu_{\mu} \nu^{\mu} = 1$$

$$a(x) = x^{0}, \quad \nu_{\mu} = \delta_{\mu}^{0}$$
Non-linear generalization

Another form of the Lagrangian:

\[ L_{\text{cov}} = \frac{1}{4} \Omega^{ij} (\nu^\mu F'_{\mu i}) (\nu_\lambda \tilde{F}'^{\lambda j}) - \frac{1}{4} (\nu^\mu \tilde{F}'_{\mu i})(\nu_\lambda \tilde{F}'^{\lambda i}), \quad \Omega^2 = -1, \quad i, j = 1, \ldots, 2N \]

pure gauge component \( \nu^\mu A^i_\mu \) enters only the 1st term under the total derivative

\[ \nu^\mu F'_{\mu i} = \frac{1}{2} \varepsilon_{\mu \nu \rho \lambda} \nu^\mu F'^{\rho \lambda i} \]
does not contain \( \nu^\mu A^i_\mu \)

Higher-order Lagrangian:

\[ L = \frac{1}{4} \Omega^{ij} (\nu^\mu F'_{\mu i}) (\nu_\lambda \tilde{F}'^{\lambda j}) - \frac{1}{4} (\nu^\mu \tilde{F}'_{\mu i})(\nu_\lambda \tilde{F}'^{\lambda i}) - \frac{1}{4} \mathcal{L} [i_\nu \tilde{F}, d_\nu \tilde{F}, \phi] \]

where \( (i_\nu \tilde{F}^\rho)_{\nu} = \nu^\mu \tilde{F}'_{\mu \nu} \)

By construction \( L \) is invariant under \( \delta_i A^i_\mu = \Phi(x) \partial_\mu a(x), \quad \delta_i a(x) = 0 \)
Non-linear Lagrangian & local $a(x)$-symmetry

$$L = \frac{1}{4} \Omega^{ij} (\nu^\mu F^{ri}_{\mu \nu}) (\nu_\lambda \widetilde{F}^{\lambda \nu j} - \frac{1}{4} (\nu^\mu \widetilde{F}^{\lambda i}_{\mu \nu})(\nu_\lambda \widetilde{F}^{\lambda \nu i}) - \frac{1}{4} \mathcal{L} [i_v \widetilde{F}^{\lambda}, d_i v \widetilde{F}^{\lambda}, ...]$$

**2nd local symmetry in the linear case:**

$$\delta_H a(x) = \varphi(x), \quad \delta_H A^i_\mu = \frac{\varphi(x)}{\sqrt{(\partial a)^2}} \nu^\nu (F^{ri}_{\mu \nu} - \Omega^{ij} \widetilde{F}^{\lambda j}_{\mu \nu} ) = 0 \text{ on shell}$$

**$A^i$ equation of motion:**

$$\frac{\delta L}{\delta A^i} = d \left( \nu (i_v F^{ri}_{\mu \nu} - \Omega^{ij} i_v \widetilde{F}^{\lambda j}_{\mu \nu} - \Omega^{ij} \frac{\delta L}{\delta (i_v \widetilde{F}^{\lambda j}_{\mu \nu})} ) = 0 \Rightarrow \nu^\nu (F^{ri}_{\mu \nu} - \Omega^{ij} \widetilde{F}^{\lambda j}_{\mu \nu} ) - \Omega^{ij} \frac{\delta L}{\delta (v^\nu F^{ri}_{\mu \nu})} = 0$$

**2nd local symmetry in non-linear case:**

$$\delta_H a(x) = \varphi(x), \quad \delta_H A^i_\mu = \frac{\varphi(x)}{\sqrt{(\partial a)^2}} \left( \nu^\nu (F^{ri}_{\mu \nu}) - \Omega^{ij} \widetilde{F}^{\lambda j}_{\mu \nu} ) - \Omega^{ij} \frac{\delta L}{\delta (v^\nu F^{ri}_{\mu \nu})} \right)$$
Consistency condition on non-linear deformation $\mathcal{L}(F)$

\[ \delta_{\parallel} L = 0 \quad \Rightarrow \quad \Omega^{ij} d \left[ \frac{\nu}{\sqrt{(\partial a)^2}} \left( i_v \tilde{F}^i + \frac{\delta \mathcal{L}}{2\delta(i_v \tilde{F}^i)} \right) \frac{\delta \mathcal{L}}{\delta(i_v \tilde{F}^j)} \right] = 0 \]

The condition on $\mathcal{L}$ ensures the auxiliary nature of the scalar $a(x)$ upon gauge fixing $a(x)$ it ensures non-manifest space-time invariance

**Known examples:**

- Born-Infeld-like form of the M5-brane action
  *(Perry & Schwarz ‘96; Pasti, D.S. and Tonin ‘97)*

- Born-Infeld-like form of the duality-symmetric D3-brane action
  *(Berman ‘97; Nurmagambetov ‘98)*

- New Born-Infeld-like deformations *(Kuzenko et. al, Bossard & Nicolai; Kallosh et. al ‘11)*
\( a(x) \)-independence of twisted self-duality condition

\[ v^\nu (F'^i_{\mu\nu} - \Omega^{ij} F'^j_{\mu\nu}) - \Omega^{ij} \frac{\delta L}{\delta (\nu_j F'_j^{\mu\nu})} = 0 \]

\[ F'^i - \Omega^{ij} * F'^j = v \frac{\delta L}{\delta (i_v F'_i)} - \Omega^{ij} * v \frac{\delta L}{\delta (i_v F'_j)} = \frac{\delta \Delta (F')}{\delta F'^i_+}, \quad F'^i_+ = F'^i + \Omega^{ij} * F'^j \]

should not depend on \( v(x) \sim da(x) \) independently of gauge fixing

This establishes on-shell relation between manifestly duality-symmetric and Gaillard-Zumino approach to the construction of non-linear self-dual theories

**Main issues:** Whether counterterms of \( N=8,4 \) sugra can provide the form of \( \Delta(F) \)? If yes, whether this deformation is consistent with supersymmetry?
Supersymmetry issue

- Counterterms $\partial^{2k} F^4$ that can appear at 7 loops in $N=8$ sugra are supersymmetric and $E_7(7)$-invariant on the mass-shell, i.e.

  modulo linear twisted self-duality $F_-'^i = F'^i - \Omega^i_j F'^j = 0$

  $\Delta_0(F'^i, \phi) = \Delta_0(F'_+^i, \phi) = \Delta_0(F^A, \phi)$

- When included into the effective action, $I_0(F)$ deforms duality condition

  $F_-'^i = \frac{\delta \Delta_0(F', \phi)}{\delta F'_+^i} \neq 0 \quad \Rightarrow \quad \Delta(F'_+^i, F'_-^i, \phi), \quad \Delta(F'_+^i, F'_-^i, \phi)\bigg|_{F'_-^i = 0} = \Delta_0$

  whose form is determined by GZ- Gibbons-Rasheed condition or space-time invariance of the deformed action

  Supersymmetry of $\Delta(F'_+^i, F'_-^i, \phi)$ should be checked

  Standard $N=8$ superspace methods are not applicable. Use component formalism
Supersymmetry of duality-symmetric actions

- **Example:** duality-symmetric $N=1$ Maxwell action \( F^i = dA^i (i = 1,2) \)

\[
L_{N=1} = \frac{1}{8} F^i_{\mu \nu} F^{i \mu \nu} + \frac{1}{4} \nu^\mu (F^i_{\mu \nu} - \varepsilon^{ij} \tilde{F}^j_{\mu \nu})(F^{\nu \lambda i} - \varepsilon^{ij} \tilde{F}^{\nu \lambda j})\nu_\lambda + \frac{i}{2} \overline{\psi} \gamma^\mu \partial_\mu \psi
\]

Susy transformations *(Schwarz & Sen ’93, Pasti, D.S. & Tonin ‘95)*

\[
\delta A^i_\mu = i \overline{\psi} \gamma_\mu \zeta^i, \quad \zeta^i = \varepsilon^{ij} \gamma_5 \zeta^j, \quad \delta_\sigma a(x) = 0
\]

\[
\delta \psi = \frac{1}{8} K^i_{\mu \nu} \gamma^{\mu \nu} \zeta^i, \quad K^i_{\mu \nu} = F^i_{\mu \nu} + \nu_{[\mu} (F^i_{\nu] \rho} - \varepsilon^{ij} \tilde{F}^j_{\nu] \rho})\nu^\rho
\]

On shell \((F^2 = -\tilde{F}^1)\):

\[
\delta A^1_\mu = i \overline{\psi} \gamma_\mu \zeta^1, \quad \delta \psi = \frac{1}{4} F^1_{\mu \nu} \gamma^{\mu \nu} \zeta^1
\]

- **In the non-linear case:**

\[
K^i = F^i - \nu (i \nu F^i - \overline{\delta I(F, \psi)} \overline{\delta F^i_+})
\]
Non-linear duality, supersymmetry and UV

Examples:
- N=1,2,3,4, D=4 Born-Infeld theories (D3-branes) (*known since ’95*)
- Abelian N=(2,0) D=6 self-dual theory on the worldvolume of the M5-brane (‘96)
- BI models (including higher-order derivatives) coupled to N=1,2 D=4 sugra
  (*Kuzenko and McCarthy ‘02, Kuzenko ’12, Kallosh et. all ’12...*)

In most of the known examples non-linear deformation of duality is related to a partial spontaneous breaking of supersymmetry

Issues:
- Whether non-linear deformations are possible for vector fields inside supergravity multiplets, in particular, in N=4,8 supergravities? (for N=2 sugra, *Kallosh et.all 08.2012*)
- Whether this interplay between dualities and supersymmetry may shed light on the UV behavior of N=4,8 supergravities?