Minimal Flavour Violation with two Higgs Doublets

M. N. REBELO

CFTP/IST, UTL LISBOA

CORFU 2012

SM and BSM

Work in collaboration with

F. J. BOTELLA, G. C. BRANCO, MIGUEL NEBOT

IFIC (CSIC, U. VALENCIA) (LISBON) IFIC (CSIC and U. VALENCIA)

Neutral currents have played an important rôle in the construction and experimental tests of unified gauge theories

EPS Prize in 2009 to Gargamelle, CERN

In the Standard Model Flavour Changing Neutral Currents (FCNC) are forbidden at tree level

- in the gauge sector, ie no Z FCNC
- in the scalar sector, ie no HFCNC

Models with two or more Higgs doublets potentially large HFCNC

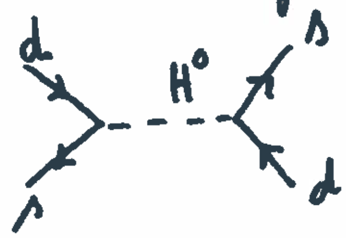
strict limits on FCNC processes!

In the SM, FCNC are generated only at loop level

$\Rightarrow$ very suppressed

$K^0-\bar{K}^0$ mixing
$D^0-\bar{D}^0$ mixing
$B_d^0-\bar{B}_d^0$ mixing
$B_s^0-\bar{B}_s^0$ mixing
rare Kaon decays
rare B-meson decays
CP violation

Most dangerous couplings



K_L-K_S mass diff.
$\Downarrow$
$m_H \gtrsim 1\,\mathrm{TeV}$

CP violation ε_K
$m_H \gtrsim 30\,\mathrm{TeV}$

processes that play a crucial rôle in testing the SM and putting limits in Models for Physics Beyond the SM

Proposed solutions, case of Multi-Higgs models

NFC Weinberg, Glashow (1977)
Paschos (1977)

or

existence of suppression factors in HFCNC
Antaramian, Hall, Rasin (1992)
Hall, Weinberg (1993)
Joshipura, Rindani (1991)

first models of this type with no ad-hoc assumptions suppression by small elements of V_{CKM} : BGL models
Branco, Grimus, Lavoura (1996)

More recently, we have generalized BGL models to larger class of models of "Minimal Flavour Violation" type

About Minimal Flavour Violation

Buras, Gambino, Gorbahn, Jager, Silvestrini (2001)
D'Ambrosio, Giudice, Isidori, Strumia (2002)

leptonic sector
Cirigliano, Grinstein, Isidori, Wise (2005)

$G_F = U(3)^5$ largest symmetry of the gauge sector
flavour violation completely determined by Yukawa couplings

Our framework
- multi-Higgs models
- no Natural Flavour Conservation
- obey above condition (one of the defining ingredients of MFV framework)

"Higgs-mediated FCNC's: Natural Flavour Conservation vs. Minimal Flavour Violation"
Buras, Carlucci, Gori, Isidori, arXiv:1005.5310 (JHEP)
Barbieri, Lodone, Straub, Jones-Perez; Cervero, Gerard; ....

Question : Under what conditions the neutral Higgs couplings are only functions of V_{CKM} ?

The case of two Higgs doublets

Yukawa interactions

$$\mathcal{L}_Y = -\bar{Q}_L^0 \Gamma_1 \Phi_1 d_R^0 - \bar{Q}_L^0 \Gamma_2 \Phi_2 d_R^0 - \bar{Q}_L^0 \Delta_1 \tilde{\Phi}_1 u_R^0 - \bar{Q}_L^0 \Delta_2 \tilde{\Phi}_2 u_R^0 + h.c.$$

$$\tilde{\Phi}_i = -i\tau_2 \phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}}\left(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2\right) \; ; \; M_u = \frac{1}{\sqrt{2}}\left(v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2\right)$$

Diagonalized by

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \mathrm{diag}\,(m_d, m_s, m_b)$$

$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \mathrm{diag}\,(m_u, m_c, m_t)$$

Expansion around the vev's

$$\Phi_j = e^{i\alpha_j} \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix} \qquad j = 1,2$$

We perform the following transformation

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = O \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} ; \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} ; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = O \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$O = \frac{1}{v} \begin{pmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{pmatrix} ; \quad v = \sqrt{v_1^2 + v_2^2} = \left(\sqrt{2}\, G_F\right)^{-1/2} \approx 246 \text{ GeV}$$

O singles out

H^0 with couplings to quarks proportional to mass matrices

G^0 the neutral pseudo-Goldstone boson

G^+ charged pseudo-Goldstone boson

Physical neutral Higgs fields are combinations of H^0, R and I

Yukawa couplings in terms of quark mass eigenstates for H^+, H^0, R, I

$$\mathcal{L}_Y = \cdots \sqrt{2}\,\frac{H^+}{v}\,\bar{u}\left(-V N_d \gamma_R + N_u^\dagger V \gamma_L\right) d + h.c. -$$

$$- \frac{H^0}{v}\left(\bar{u} D_u u + \bar{d} D_d d\right) -$$

$$- \frac{R}{v}\left[\bar{u}\left(N_u \gamma_R + N_u^\dagger \gamma_L\right) u + \bar{d}\left(N_d \gamma_R + N_d^\dagger \gamma_L\right) d\right] +$$

$$+ i\,\frac{I}{v}\left[\bar{u}\left(N_u \gamma_R - N_u^\dagger \gamma_L\right) u - \bar{d}\left(N_d \gamma_R - N_d^\dagger \gamma_L\right) d\right]$$

$$\gamma_L = (1-\gamma_5)/2 \; ; \; \gamma_R = (1+\gamma_5)/2 \qquad V \equiv V_{CKM}$$

Flavour changing neutral currents controlled by:

$$N_d = \frac{1}{\sqrt{2}} U_{d_L}^{\dagger} \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{u_L}^{\dagger} \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{uR}$$

For generic two Higgs doublet models N_u, N_d non-diagonal arbitrary

For definiteness rewrite N_d :

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{d_L}^{\dagger} e^{i\alpha} \Gamma_2 U_{dR}$$

conserves flavour

leads to FCNC

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{dL}^{\dagger} e^{i\alpha} \Gamma_2 U_{dR}$$

We want N_d entirely controlled by V_{CKM} elements (together with ratios of v_1 and v_2 and quark masses)

$V_{CKM} = U_{uL}^{\dagger} U_{dL}$

Obstacles : (i) Dependence on U_{dL} rather than V_{CKM}

(ii) Need to get rid of U_{dR}

Solution to first difficulty :

Flavour symmetry constraining $U_{uL} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$V_{CKM} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ U_{d31} & U_{d32} & U_{d33} \end{pmatrix} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ U_{d31} & U_{d32} & U_{d33} \end{pmatrix}$$

$$(V_{CKM})_{3j} = (U_{dL})_{3j}$$

together with $\Gamma_2 U_{dR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix} \Rightarrow$ only third row of U_{dL} appears in N_d

$$\text{FCNC} \quad \propto \quad U_{d_L}^\dagger\, e^{i\alpha}\, \Gamma_2\, U_{d_R}$$

to get rid of U_{d_R}, choose $\Gamma_2 \propto P M_d$, P projector

$$U_{d_L}^\dagger \Gamma_2 U_{d_R} \propto U_{d_L}^\dagger P M_d U_{d_R} \propto U_{d_L}^\dagger P U_{d_L} D_d$$

for $P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix}$

$$\left(U_{d_L}^\dagger \Gamma_2\right)_{ij} = \left(U_{d_L}^\dagger\right)_{i3} \left(\Gamma_2\right)_{3j} = \left(V_{CKM}^\dagger\right)_{i3} \left(\Gamma_2\right)_{3j}$$

$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) (V_{CKM}^\dagger)_{i3} (V_{CKM})_{3j} (D_d)_{jj}$$

Symmetry BGL

$Q_{L3}^0 \longrightarrow e^{i\alpha} Q_{L3}^0$; $u_{R3}^0 \rightarrow e^{i2\alpha} u_{R3}^0$; $\phi_2 \rightarrow e^{i\alpha} \phi_2$ $\alpha \neq 0, \pi$

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix} ; \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix} ; \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

Both Higgs doublets have non-zero Yukawa couplings in up and down sector

$$N_u = -\frac{v_1}{v_2} \operatorname{diag}(0, 0, m_t) + \frac{v_2}{v_1} \operatorname{diag}(m_u, m_c, 0)$$

Six different models

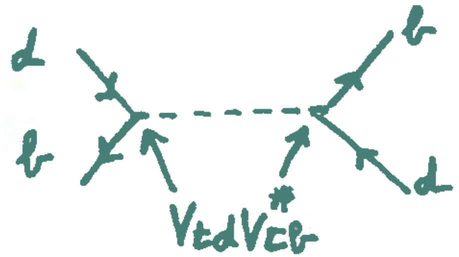
$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \overbrace{(V^\dagger_{CKM})_{i3} (V_{CKM})_{3j}}^{\text{MFV}} (D_d)_{jj}$$

$$N_u = -\frac{v_1}{v_2}\, \mathrm{diag}(0,0,m_t) + \frac{v_2}{v_1}\, \mathrm{diag}(m_u, m_c, 0)$$

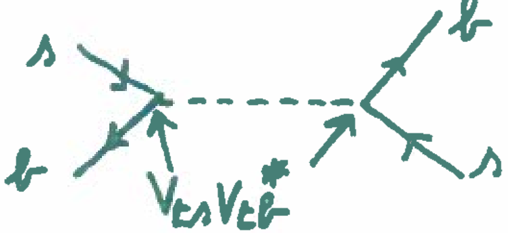
FCNC only in the down sector
suppression by the 3rd row of V_{CKM}

Strong and Natural suppression of the most constrained processes

$\Delta s = 2$ processes
$|V_{td}V^*_{ts}| \sim \lambda^5$ (λ^{10} suppression)
$\sim 10^{-4}$



may contribute significantly to $B_d - \bar{B}_d$ mixing



contribution to $B_s - \bar{B}_s$ mixing

How to find a general expansion of N_d^0, N_u^0 which conforms to the MFV requirements?

$$N_d^0 = U_{dL} N_d U_{dR}^\dagger = \frac{1}{\sqrt{2}}\left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2\right)$$

$$N_u^0 = U_{uL} N_u U_{uR}^\dagger = \frac{1}{\sqrt{2}}\left(v_2 \Delta_1 - v_1 e^{i\alpha} \Delta_2\right)$$

Necessary condition N_d^0, N_u^0 to be of MFV type:

should be functions of M_d, M_u no other flavour dependence

Furthermore, N_d^0, N_u^0 should transform under WB appropriate form

$$Q_L^0 \to W_L Q_L^0 \quad , \quad d_R^0 \to W_R^d d_R^0 \quad ; \quad u_R^0 \to W_R^u u_R^0$$

$$M_d \to W_L^\dagger M_d W_R^d \quad ; \quad M_u \to W_L^\dagger M_u W_R^u$$

$$U_{dL} \to W_L^\dagger U_{dL} \; ; \; U_{uL} \to W_L^\dagger U_{uL} \; ; \; U_{dR} \to W_R^{d\dagger} U_{dR} \; ; \; U_{uR} \to W_R^{u\dagger} U_{uR}$$

$$H_{d,u} \equiv (M_{d,u})(M_{d,u}^\dagger) \quad , \quad H_{d,u} \to W_L^\dagger H_{d,u} W_L$$

N_d^0, N_u^0 transform as M_d, M_u

It is convenient to write H_d, H_u in terms of projection operators

Botella, Nebot, Vives 2004

$$H_d = \sum_i m_{d_i}^2 P_i^{dL} \; ; \quad P_i^{dL} = U_{dL} P_i U_{dL}^\dagger \; ; \quad (P_i)_{jk} = \delta_{ij}\delta_{ik} \qquad u \leftrightarrow d$$

MFV expansion for N_d^0 and N_u^0

$$N_d^0 = \lambda_1 M_d + \lambda_{2i} U_{dL} P_i U_{dL}^\dagger M_d + \lambda_{3i} U_{uL} P_i U_{uL}^\dagger M_d + \ldots$$

$$N_u^0 = \tau_1 M_u + \tau_{2i} U_{uL} P_i U_{uL}^\dagger M_u + \tau_{3i} U_{dL} P_i U_{dL}^\dagger M_u + \ldots$$

In green terms that do not lead to FCNC

In red terms that lead to FCNC

In the quark eigenstate basis

$$N_d = \lambda_1 D_d + \lambda_2 P_i D_d + \lambda_{3i} (V_{CKM})^\dagger P_i V_{CKM} D_d + \ldots$$

$$N_u = \tau_1 D_u + \tau_{2i} P_i D_u + \tau_{3i} V_{CKM} P_i (V_{CKM})^\dagger D_u + \ldots$$

At this stage λ and τ coefficients appear as free parameters, MFV
Need for additional symmetries in order to constrain these coeff.

BGL example again

corresponds to the following truncation of our MFV expansion

$$N_d^0 = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_{uL}^\dagger M_d$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_{uL}^\dagger M_u$$

together with

$$N_d^0 = \frac{v_2}{v_1} M_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) e^{i\alpha} \Gamma_2$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) e^{-i\alpha} \Delta_2$$

implies BGL model fully defined in covariant way under WB transform.

$$\frac{v_2}{\sqrt{2}} e^{i\alpha} \Gamma_2 = U_{uL} P_3 U_{uL}^\dagger M_d \; ; \quad \frac{v_2}{\sqrt{2}} e^{-i\alpha} \Delta_2 = U_{uL} P_3 U_{uL}^\dagger M_u$$

we have

$$U_{uL} P_3 U_{uL}^\dagger \Gamma_2 = \Gamma_2 \; ; \quad U_{uL} P_3 U_{uL}^\dagger \Gamma_1 = 0 \; ; \quad U_{uL} P_3 U_{uL}^\dagger \Delta_2 = \Delta_2$$

$$U_{uL} P_3 U_{uL} \Delta_1 = 0$$

BGL is the only implementation of models where Higgs FCNC are a function of V_{CKM} only (together with v_1, v_2) which are based on an Abelian symmetry obeying the sufficient conditions of having M_u block diagonal together with the existence of a matrix P such that

$$P\Gamma_2 = \Gamma_2 \quad ; \quad P\Gamma_1 = 0$$

Ferreira, Silva arXiv: 1012287

The leptonic sector

Required for completeness

- study of experimental implications
- study of stability under RGE

Models with two Higgs doublets with FCNC

- controlled by V_{CKM} in the quark sector
- controlled by V_{PMNS} in the leptonic sector

Case of Dirac neutrinos, straightforward

Minimal Flavour Violation with Majorana neutrinos

Low energy effective theory and stability

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2} \nu_L^{0T} C^{-1} m_\nu \nu_L^0 + h.c.$$

generated from effective dimension five operator

$$\mathcal{O} = \sum_{i,j=1}^{2} \sum_{\alpha,\beta=e,\mu,\tau} \sum_{a,b,c,d=1}^{2} \left(L^T_{L\alpha a} \kappa^{(ij)}_{\alpha\beta} C^{-1} L_{L\beta c} \right) \left(\varepsilon^{ab} \phi_{ib} \right) \left(\varepsilon^{cd} \phi_{jd} \right)$$

$$\mathcal{L}_{Y_\ell} = - \overline{L^0_L}\, \Pi_1 \phi_1 \ell^0_R - \overline{L^0_L}\, \Pi_2 \phi_2 \ell^0_R + h.c.$$

$$\Pi_1 , \quad \Pi_2 , \quad \kappa^{11}, \quad \kappa^{12}, \quad \kappa^{21}, \quad \kappa^{22} \qquad (\kappa^{(ij)})$$

$$L^0_{Lj} \longrightarrow \exp(i\alpha) L^0_{Lj} , \quad \phi_2 \longrightarrow \exp(i\alpha)\, \phi_2$$

$$\alpha = \pi/2 , \quad Z_4 \text{ symmetry}$$

Imposing this Z_4 symmetry implies: $(j=3)$

$$\kappa^{(12)} = \kappa^{(21)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\kappa^{(11)} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \kappa^{(22)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$\alpha = \pi/2$ ensures $\kappa^{(22)}_{33} \neq 0$

$$\frac{1}{2} m_\nu = \frac{1}{2} v_1^2 \kappa^{(11)} + \frac{1}{2} v_2^2 e^{2i\theta} \kappa^{(22)}$$

$$\Pi_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

Higgs FCNC in the charged sector

Stability: $\kappa^{(12)} = \kappa^{(21)} = 0$

$\kappa^{(11)} P_3^\nu = 0$ $\qquad \kappa^{(22)} P_3^\nu = \kappa^{(22)}$

$P_3^\nu \Pi_1 = 0$ $\qquad P_3^\nu \Pi_2 = \Pi_2$

stable under renormalization

W. Grimus, L. Lavoura 2005

Seesaw framework

$$\mathcal{L}_Y + \text{mass} = -\bar{L}_L^0 \Pi_1 \phi_1 \ell_R^0 - \bar{L}_L^0 \Pi_2 \phi_2 \ell_R^0 -$$
$$- \bar{L}_L^0 \Sigma_1 \tilde{\phi}_1 \nu_R^0 - \bar{L}_L^0 \Sigma_2 \tilde{\phi}_2 \nu_R^0 +$$
$$+ \frac{1}{2} \nu_R^{0T} C^{-1} M_R \nu_R^0 + h.c.$$

$$m_\ell = \frac{1}{\sqrt{2}} \left(v_1 \Pi_1 + v_2 e^{i\theta} \Pi_2 \right) , \quad m_D = \frac{1}{\sqrt{2}} \left(v_1 \Sigma_1 + v_2 e^{-i\theta} \Sigma_2 \right)$$

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \bar{\ell}_L^0 \gamma^\mu \nu_L^0 + h.c$$

$$\mathcal{L}_Y (\text{neutral, lepton}) = - \bar{\ell}_L^0 \frac{1}{v} \left[m_\ell H^0 + N_\ell^0 R + i N_\ell^0 I \right] \ell_R^0 -$$
$$- \bar{\nu}_L^0 \frac{1}{v} \left[m_D H^0 + N_\nu^0 R + i N_\nu^0 I \right] \nu_R^0 + h.c.$$

$$N_\ell^0 = \frac{v_2}{\sqrt{2}} \Pi_1 - \frac{v_1}{\sqrt{2}} e^{i\theta} \Pi_2$$
$$N_\nu^0 = \frac{v_2}{\sqrt{2}} \Sigma_1 - \frac{v_1}{\sqrt{2}} e^{-i\theta} \Sigma_2$$

$$\mathcal{L}_{mass} = -\bar{\ell}_L^0 m_\ell \ell_R^0 + \frac{1}{2}\left(\nu_L^{0T}, (\nu_R^0)^{cT}\right) C^{-1} \mathcal{M}^* \begin{pmatrix} \nu_L^0 \\ (\nu_R^0)^c \end{pmatrix} + hc$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \qquad (\Psi_L)^c \equiv C \gamma_0^T (\Psi_L)^*$$

BGL type example, Z_4 symmetry

$L_{L3}^0 \rightarrow \exp(i\alpha) L_{L3}^0$, $\nu_{R3}^0 \rightarrow \exp(i2\alpha)\nu_{R3}^0$, $\phi_2 \rightarrow \exp(i\alpha)\phi_2$

$\alpha = \frac{\pi}{2}$

$$\Pi_1 \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad \Pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

$$\Sigma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad M_R = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$

New feature m_{ν_i} from $m_{eff} \equiv -m_D \frac{1}{M_R} m_D^T$ $\qquad M_{33} \neq 0$

Three light neutrinos ν_i , plus heavy neutrinos N_j

light - light , light - heavy, heavy - heavy couplings

H^0, R, I couplings

$U^\dagger m_{eff} U^* = d$, $\quad m_D \frac{1}{D} m_D^T = -U d U^T$ (WB M_D diag)

$m_D = i\, U \sqrt{d}\, O^c \sqrt{D}$ Casas and Ibarra , 2001

$$(N_\ell)_{ij} = \frac{v_2}{v_1} (D_\ell)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) (U_\nu^\dagger)_{i3} (U_\nu)_{3j} (D_\ell)_{jj}$$

light - light neutral couplings : diag , d

light - heavy neutral couplings : sensitive to O^c, d, D

heavy - heavy neutral couplings : diag , sensitive to O^c, d, D

H^+ couplings

$$\frac{\sqrt{2}\, H^+}{v} \left(\overline{\nu_L^0}\, N_\ell^0\, \ell_R - \overline{\nu_R^0}\, N_\nu^{0\dagger}\, \ell_L^0 \right) + h.c.$$

Scalar Potential

Z_4 forbids $\phi_1^\dagger\phi_2$, $\phi_1^\dagger\phi_2\,\phi_i^\dagger\phi_i$, $\phi_1^\dagger\phi_2\,\phi_1^\dagger\phi_2$

ungauged accidental continuous symmetry
not a symmetry of full Lagrangean

after spontaneous gauge symmetry breaking $\rightarrow$
$\rightarrow$ pseudo Goldstone boson

solution : soft symmetry breaking $m_{12}\phi_1^\dagger\phi_2 + h.c.$

Conclusions

Multi-Higgs models are very interesting candidates for NP

There are new mechanisms beyond NFC to obtain strong suppression of FCNC as required by experiment

LHC results may bring surprises for the Higgs sector

Models with three Higgs doublets

Yukawa interactions

$$\mathcal{L}_Y = -\overline{Q}_L^0 \Gamma_1 \Phi_1 d_R^0 - \overline{Q}_L^0 \Gamma_2 \Phi_2 d_R^0 - \overline{Q}_L^0 \Gamma_3 \Phi_3 d_R^0 - $$
$$- \overline{Q}_L^0 \Delta_1 \tilde{\Phi}_1 u_R^0 - \overline{Q}_L^0 \Delta_2 \tilde{\Phi}_2 u_R^0 - \overline{Q}_L^0 \Delta_3 \tilde{\Phi}_3 u_R^0 + h.c.$$

$$\tilde{\Phi}_i = -i \tau_2 \Phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} \left(v_1 e^{i\alpha_1} \Gamma_1 + v_2 e^{i\alpha_2} \Gamma_2 + v_3 e^{i\alpha_3} \Gamma_3 \right)$$

$$M_u = \frac{1}{\sqrt{2}} \left(v_1 e^{-i\alpha_1} \Delta_1 + v_2 e^{-i\alpha_2} \Delta_2 + v_3 e^{-i\alpha_3} \Delta_3 \right)$$

after spontaneous symmetry breakdown

$$\Phi_j = e^{i\alpha_j} \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}} (v_j + \rho_j + i\eta_j) \end{pmatrix}$$

We perform the following transformation

$$\begin{pmatrix} H^0 \\ R \\ R' \end{pmatrix} = O \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ I \\ I' \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

$$O = \begin{pmatrix} \frac{v_1}{v} & \frac{v_2}{v} & \frac{v_3}{v} \\ \frac{v_2}{v'} & -\frac{v_1}{v'} & 0 \\ \frac{v_1}{v''} & \frac{v_2}{v''} & \frac{-(v_1^2+v_2^2)/v_3}{v''} \end{pmatrix}, \qquad \begin{aligned} v &= \sqrt{v_1^2+v_2^2+v_3^2} \\ v' &= \sqrt{v_1^2+v_2^2} \\ v'' &= \sqrt{v_1^2+v_2^2+(v_1^2+v_2^2)^2/v_3^2} \end{aligned}$$

O singles out

H^0 with couplings to quarks proportional to mass matrices

G the neutral pseudo-Goldstone boson

$$\mathcal{L}_Y(\text{neutral}) = -\frac{H^0}{v}\left(\bar{d}_L D_d d_R + \bar{u}_L D_u u_R\right) -$$

$$-\bar{d}_L \frac{1}{v'} N_d (R+iI) d_R - \bar{u}_L \frac{1}{v'} N_u (R-iI) u_R -$$

$$-\bar{d}_L \frac{1}{v''} N_d' (R'+iI') d_R - \bar{u}_L \frac{1}{v''} N_u' (R'-iI') u_R + h.c.$$

With

$$N_d = \frac{1}{\sqrt{2}} U_{d_L}^{\dagger} \left(v_2 e^{i\alpha_1} \Gamma_1 - v_1 e^{i\alpha_2} \Gamma_2\right) U_{d_R}$$

$$N_u = \frac{1}{\sqrt{2}} U_{u_L}^{\dagger} \left(v_2 e^{-i\alpha_1} \Delta_1 - v_1 e^{-i\alpha_2} \Delta_2\right) U_{u_R}$$

$$N_d' = \frac{1}{\sqrt{2}} U_{d_L}^{\dagger} \left(v_1 e^{i\alpha_1} \Gamma_1 + v_2 e^{i\alpha_2} \Gamma_2 + x e^{i\alpha_3} \Gamma_3\right) U_{d_R}$$

$$N_u' = \frac{1}{\sqrt{2}} U_{u_L}^{\dagger} \left(v_1 e^{-i\alpha_1} \Delta_1 + v_2 e^{-i\alpha_2} \Delta_2 + x e^{-i\alpha_3} \Delta_3\right) U_{u_R}$$

$$x = -(v_1^2 + v_2^2)/v_3$$

Imposing the following discrete symmetry on the Lagrangian

$$Q^0_{L1} \to \omega\, Q^0_{L1} \,, \quad Q^0_{L2} \to \omega^2 Q^0_{L2} \,, \quad Q^0_{L3} \to \omega^4 Q^0_{L3}$$

$$\Phi_1 \to \omega\, \Phi_1 \,, \quad \Phi_2 \to \omega^2 \Phi_2 \,, \quad \Phi_3 \to \omega^4 \Phi_3$$

$$u^0_{R1} \to \omega^2 u^0_{R1} \,, \quad u^0_{R2} \to \omega^4 u^0_{R2} \,, \quad u^0_{R3} \to \omega^8 u^0_{R3}$$

$$d^0_{Rj} \to d^0_{Rj} \qquad \text{with} \quad \omega = \exp i\pi/4$$

restricts the Yukawa coupling matrices. Following structure

$$\Gamma_1 = \begin{bmatrix} X & X & X \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ X & X & X \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Gamma_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ X & X & X \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} X & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Delta_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & X \end{bmatrix}$$

all three Higgs doublets have non-zero Yukawa couplings both in the up and down sectors

In this case there are Higgs mediated FCNC only down sector

$$(N_d)_{ij} = \frac{v_2}{v_1}(D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(V_{CKM}^\dagger)_{i2}(V_{CKM})_{2j}\,(D)_{jj} -$$
$$- \frac{v_3}{v_1}(V_{CKM}^\dagger)_{i3}\,(V_{CKM})_{3j}\,(D)_{jj}$$

$x = -(v_1^2 + v_2^2)/v_3$

$$(N_d')_{ij} = (D_d)_{jj} - \frac{v_3 - x}{v_3}(V_{CKM}^\dagger)_{i3}\,(V_{CKM})_{3j}\,(D_d)_{jj}$$

N_d includes FCNC terms where the suppression factor in $\Delta S = 2$ transitions is only $(V_{cd}^* V_{cs})^2$, which then requires quite heavy neutral Higgs