Holographic duals of 2+1d QFT's with Minimal SUSY and Massive Fundamental Flavours

Niall T. Macpherson

Swansea University

JHEP06(2012)136 [hep-th/1204.4222]

Talk Plan

- Review the holographic dual of $\mathcal{N}$=1 SYM-CS in 2+1 d.

- Sketch of construction of dual of $\mathcal{N}$=1 SQCD-CS with massive flavours in 2+1 d and implications.

  - Massless flavour, Canoura, Merlatti, Ramallo, leads to IR singularity.

- Generate type-IIA solution with interpolating G_2 structure.

  - NS5, D4, D2

  - A dual to a cascading higgsing $\mathcal{N}$=1 quiver in 2+1 d?

$\mathcal{N}$=1 SYM in 2+1 dimensions

SU(N_c): Gauge fields + Gauginos:

$$S_{\rm SYM} = \int d^3x Tr\left(-\frac{1}{4}F_{\mu\nu}^2 - i\bar{\lambda}\gamma^\mu D_\mu \lambda\right) + \frac{k}{4\pi}\int d^3x Tr\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A + \bar{\lambda}\lambda\right)$$

Witten calculated the index on R$\times$T^2

$k \geq \frac{N_c}{2} \Rightarrow$ SUSY Unbroken

$k = \frac{N_c}{2} \Rightarrow$ Single, Confining Vacuum

Holographic Dual: Maldacena, Nastase 2001: D5 branes wrapping a 3 cycle

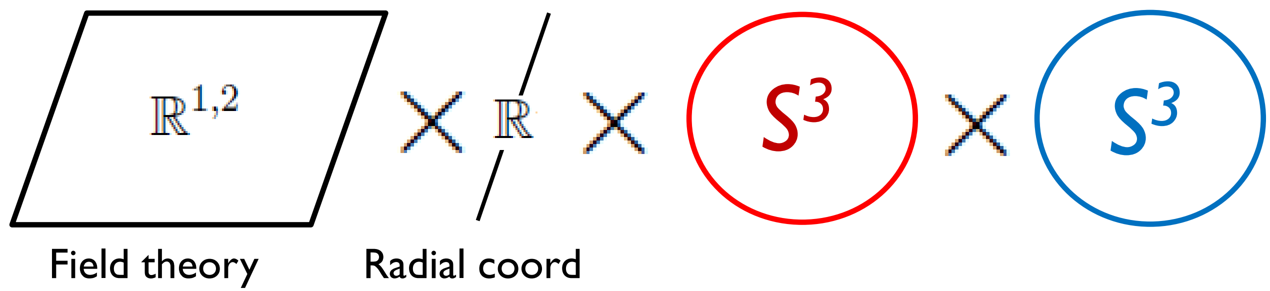
World Volume

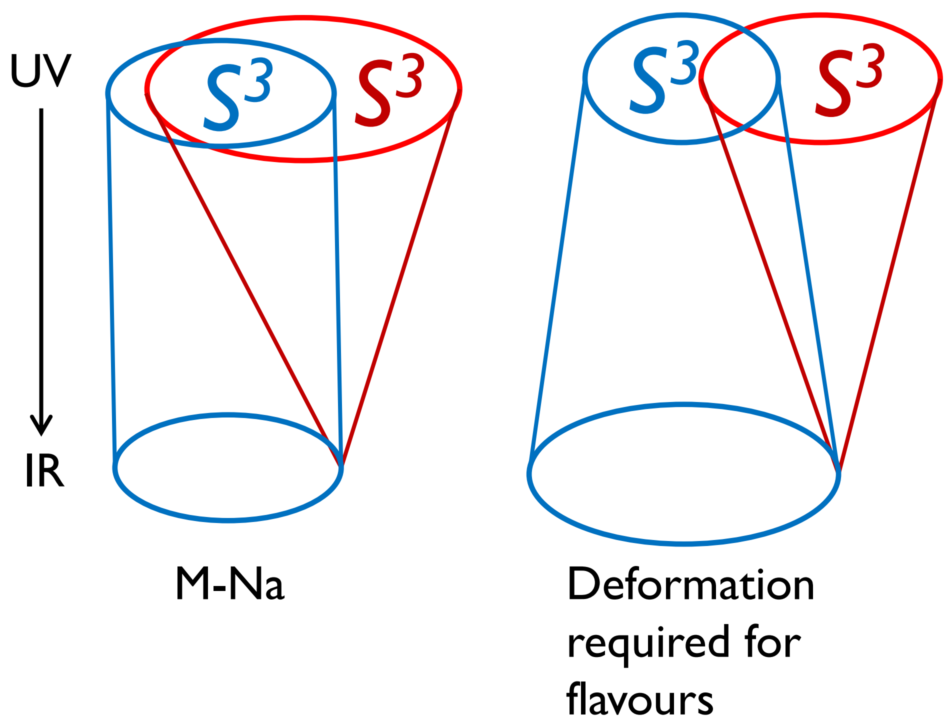
UV		IR
5+1d	⟶	2+1 d

Dual field theory

$$\mathcal{S} = \mathcal{S}_{\rm SYM} + \underbrace{\int d^3x L_{KK}}_{\text{KK Tower}}$$

The Maldacena-Nastase Solution





D5 branes wrap the intersection of the S^3's

The Maldacena-Nastase Solution

D5 branes wrapping a 3-cycle in a G_2 Manifold.

$R^{1,2} \times R \times S^3 \times S^3$ - fibrated:

$$ds^2 = e^{\phi/2} N_c \left(\frac{dx_{1,2}^2}{N_c} + dr^2 + \frac{e^{2h}}{4}(\sigma^i)^2 + \frac{1}{4}(\omega^i - A^i)^2 \right)$$

SU(2) left invariant 1-forms: $d\sigma^i = -\frac{1}{2}\epsilon_{ijk}\sigma^j \wedge \sigma^k; \; d\omega^i = -\frac{1}{2}\epsilon_{ijk}\omega^j \wedge \omega^k$

SU(2) Gauge field: $A^i = \frac{1+w(r)}{2}\sigma^i$

RR 3-form: $F_3 = N_c \left(-\frac{1}{4} \bigwedge_{i=1}^{3} (\omega^i - A^i) + \frac{1}{4} F^i \wedge (\omega^i - A^i) + H \right); \quad dF_{(3)} = 0$

Let: $\rho = e^{2h}$ Then: $\underset{\rho \to 0}{w_{\mathrm{IR}} = 1; \; \phi_{IR} = \phi_0}$ $\underset{\rho \to \infty}{w_{\mathrm{UV}} \sim \frac{1}{\rho}; \; \phi_{IR} \sim \rho}$

The Maldacena-Nastase Solution

D5 branes wrap: $\Sigma = \{\sigma^i | \omega^i = \sigma^i\}$ Canoura, Merlatti, Ramallo

Σ vanishes in the IR: $F_{(3)}\Big|_{\Sigma} = 0 \implies$ Non singular

S3's parameterised by σ^i and ω^i are non vanishing:

Flux quantisation: $-\frac{1}{2\kappa_{10}^2 T_5}\int_{\omega^i} F_3 = N_c$

Probe D5: $\Xi = (x, y, t, \sigma^i)$ Maldacena, Nastase

$$-\frac{1}{16\pi^3}\int_{\Xi} F_3 \wedge tr[\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}] = -\frac{\tilde{k}}{4\pi}\int_{\mathbb{R}^{1,2}} tr[\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}]$$

$$\frac{1}{2\kappa_{10}^2 T_5}\int_{\sigma^i} F_3 = N_c \implies \tilde{k} = N_c$$

Need to integrate out KK states in IR: $\implies$ $k = \tilde{k} - \frac{N_c}{2} = \frac{N_c}{2}$

IR Gauge theory $SU(N_c)_{\frac{N_c}{2}}$ in 2+1 d

Adding Unquenched Massive flavours

Veneziano limit: $N_c \to \infty; \;\; N_f \to \infty; \;\; \frac{N_c}{N_f} \sim 1$

Add smeared flavour D5 branes

$\Longrightarrow$ Back react on geometry; $S = S_{\rm IIB} + S_{\rm branes}$; $dF_{(3)} = \Xi_{(4)}$

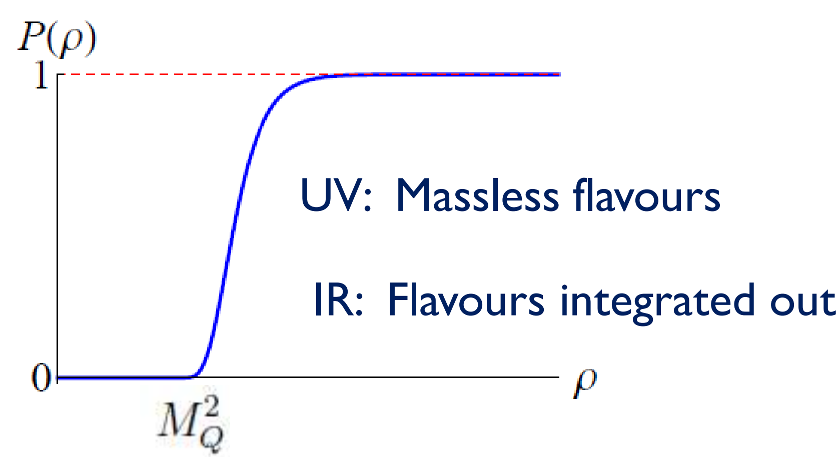
G_2 structure $\Longrightarrow$ Associative 3-form: $\Phi_{(3)}$

Calibration condition $\Longrightarrow$ SUSY cycle: $X^*\Phi_{(3)} = \sqrt{-\hat{g}}d\xi^3$

DBI+WZ : $S_{\rm branes} = -N_f \int e^{\phi/2}\left[e^{3/4\phi} Vol_3 \wedge \Phi_{(3)} \; - \; C_{(6)}\right] \wedge \Xi_{(4)}$

Massive flavours, branes that don't reach the IR:

$N_f \to P(\rho) N_f$



Adding Unquenched Massive Flavours

System gets modified: NTM

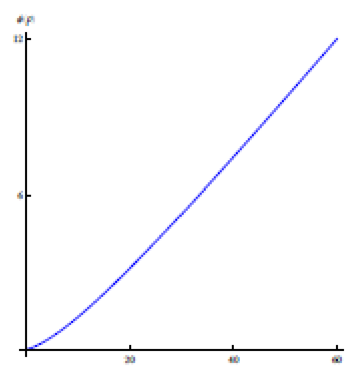
$$ds^2 = e^{\phi/2} N_c \left(\frac{dx_{1,2}^2}{N_c} + dr^2 + \frac{e^{2h}}{4}(\sigma^i)^2 + \frac{e^{2g}}{4}(\omega^i - A^i)^2 \right)$$

$$F_3 = N_c \left(-\frac{1}{4} \bigwedge_{i=1}^{3} (\omega^i - B^i) + \frac{1}{4}(F^i + F_f^i(P, P')) \wedge (\omega^i - B^i) + H + H_f(P) \right)$$

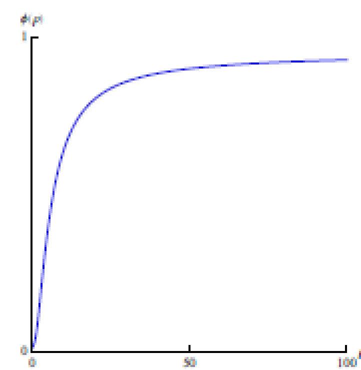
$$A^i = \frac{1}{2}(1+w)\sigma^i \,;\; B^i = \frac{1}{2}(1+\gamma)\sigma^i$$

2 types of UV consistent with IR:
-Linear Dilaton
-Constant Dilaton

Canoura, Merlatti, Ramallo



Fundamental Flavours



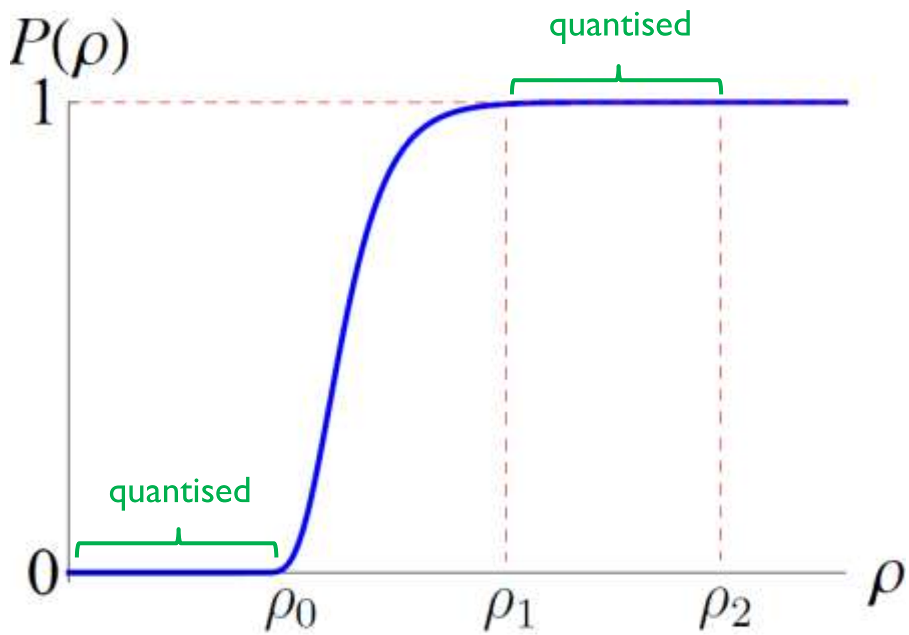
Gauged Flavours

New 6d Chern-Simons tem: $\tilde{k} = N_c + \frac{3N_f}{2}(C-1)P.$

What about quantisation???

The Chern-Simons level

Linear Dilaton case:



$$\rho_0 \sim M_Q^2$$
$$\rho_2 \sim M_{KK}^2$$
$$P(\rho_1) \sim 1$$

$\rho \gg \rho_2$	5+1d field theory:	$k = \tilde{k} = N_c + \frac{3N_f}{2}(C-1)$
$\rho_1 < \rho < \rho_2$	Integrate out KK modes, 2+1d field theory:	$k = \frac{N_c}{2} + \frac{3N_f}{4}(C-1)$
$\rho_0 < \rho < \rho_1$	Interpolation region:	$k = k(\rho)$
$\rho < \rho_0$	Flavours integrated out induces shift in k	$k = \frac{N_c}{2}$

NTM

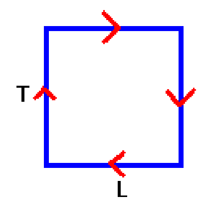
On the field theory (linear Dilaton):

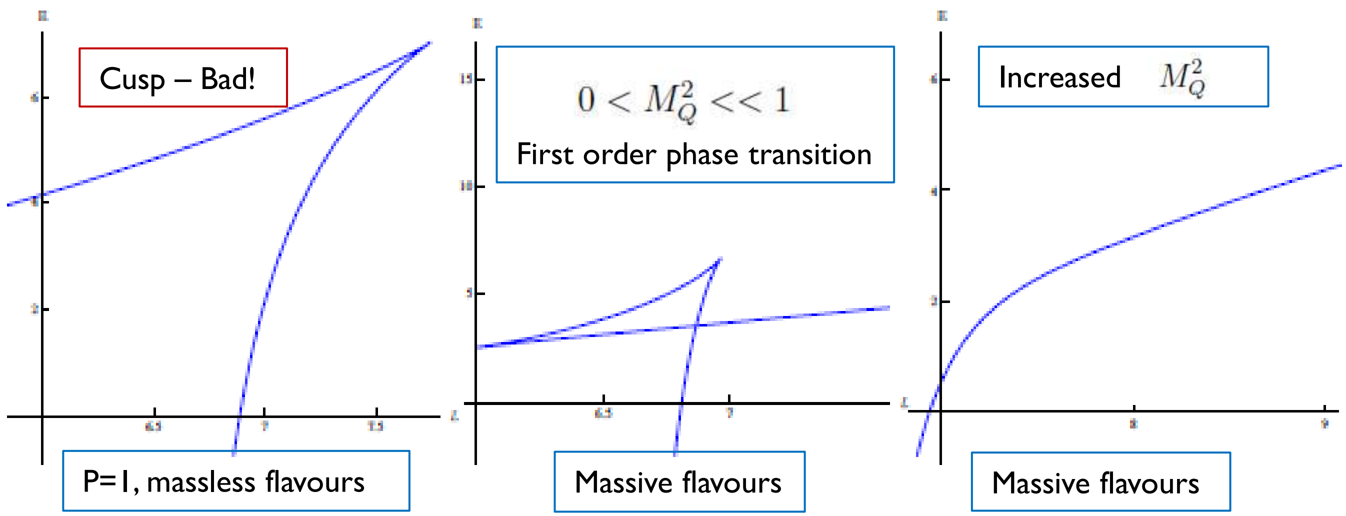
- $N_c < 2N_f$ Theory develops a Landau pole in the UV

  $N_c \geq 2N_f$ Theory is asymptotically free. cf: 3+1d SQCD

- Wilson Loops: Probe string suspended from UV

  Rectangular Loop – quark, anti-quark potential:





Large L $\Longrightarrow$ IR:

$$E = e^{\phi_0} L + O\left(e^{-\sqrt{\frac{7}{24F_0^3}}N_c L}\right) \Rightarrow \; < W_{\mathcal{C}} > \propto e^{-e^{\phi_0}\mathrm{Area}(\mathcal{C})} \Longrightarrow$$ Confinement.

NTM

Rotated Type IIA Solution:

G$_2$ structure, Asymptotically constant Dilaton solution → Solution Generating Technique -Rotation → Type-IIA with NS5, D2, D4 branes, modified Dilaton and interpolating G$_2$ structure.

Gaillard, Martelli

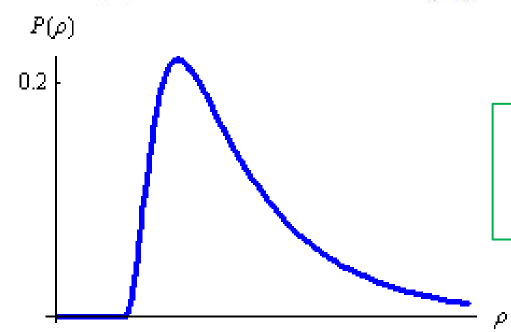
Result after further rescaling of coords:

$$ds^2_{\rm str} = N_c \left(H^{-1/2} s^{-1} dx^2_{1,2} + H^{1/2}(dr^2 + \frac{e^{2h}}{4}(\sigma^i)^2 + \frac{e^{2g}}{4}(\omega^i - A^i)^2 \right); \quad s = e^{2g}\Big|_{\rm IR}$$

$H = 1 - \tanh \beta e^{2(\phi - \phi_\infty)}$, New Dilaton couples to D2 branes: $e^{2\hat{\phi}} = \frac{sH^{1/2}}{\tanh \beta} e^{2(\phi_\infty - \phi)}$

Source for NS5 branes: $dH_{(3)} = \Xi_{(4)}$; $dF_{(4)} = 0$

Well behaved UV requires modified profile:



G$_2$ Analogue of BB of KS with addition scale

NTM

Rotated Type IIA Solution:

Rescale: $\rho \to \rho s$

Define: $U = \sqrt{\rho} = e^h$ $\Rightarrow$ $$ds^2_{\rm str} = U^2 dx^2_{1,2} + \frac{dU^2}{U} + \frac{(\sigma^i)^2}{12} + \frac{1}{9}(w^i - A^i)^2 + O(\frac{1}{sU^2})$$

Limit: $\beta \to \infty$

AdS₄ ✗ Tip of G₂ cone

Additional limit: $s \to \infty \Rightarrow$ G₂ analogue of KS with additional scale

Compare with interpolating SU(3) structure Conifold backgrounds

Maldacena, Martelli, Nunez, Piai, Ramallo …

$$C = -\frac{1}{4\pi^2}\int_\Sigma C_{(3)} \sim N_c U$$

cf: Running $B_{(2)}$ in KS

Suggests dual is a cascading, higgsing quiver

$$SU(\tilde{N}_c) \times SU(N_c + \tilde{N}_c + \frac{N_f}{2})$$ for $\tilde{N}_c \sim c N_c$

↓ Cascade

$$SU(N_c)$$

NTM

Summary

- Dual of 2+1 d SCQD-CS with Massive flavours

  - Non singular IR

  - $N_c \geq 2N_f$: Confinement and asymptotic freedom

  - $N_c < 2N_f$: Landau pole in the UV

  - More can be learnt about field theory

- Generated new type IIA solutions

  - Speculate that its dual to a cascading higgsing quiver

  - More work on this is underway

JHEP06(2012)136 [hep-th/1204.4222]

Rotation Details

Solution generating technique: Gaillard, Martelli

Type IIA: $H_{(3)}$

$$ds^2_{\text{str}} = dx^2_{1,2} + ds^2_7$$

$\Rightarrow$

Type IIA: $F_{(4)}$; $H_{(3)}$

$$ds^2_{str} = e^{2\Delta+2\phi/3}\left(dx^2_{1,2} + ds^2_7\right)$$

SUSY Conditions of complicated system:

$$\Phi \wedge d\Phi = 0$$
$$d(e^{6\Delta} *_7 \Phi) = 0$$
$$d(e^{2\Delta+2\phi/3}\cos\zeta) = 0$$
$$2d\zeta - e^{-3\Delta}\cos\zeta d(e^{3\Delta}\sin\zeta) = 0$$
$$\frac{1}{\cos^2\zeta}e^{-4\Delta+2\phi/3} *_7 d(e^{6\Delta}\cos\zeta\Phi) = H_{(3)}$$
$$Vol_3 \wedge d(e^{3\Delta}\sin\zeta) - \frac{\sin\zeta}{\cos^2\zeta}e^{-3\Delta}d(e^{6\Delta}\cos\zeta\Phi) = F_{(4)}$$

$\zeta = 0.$ →

$$2\Delta = -2\phi/3$$
$$\Phi \wedge d\Phi = 0$$
$$d(e^{-2\phi} *_7 \Phi) = 0$$
$$e^{2\phi} *_7 d(e^{-2\phi}\Phi) = H_{(3)}$$
$$F_{(4)} = 0$$

S-duality;

SUSY conditions of IIB Solution with D5 branes in string frame

$$\hat{\Phi} = \left(\frac{\cos\zeta}{c_1}\right)^3 \Phi^{(0)},\quad e^{3\hat{\Delta}} = \left(\frac{c_1}{\cos\zeta}\right)^2 e^{-\phi^{(0)}};$$
$$e^{2\hat{\phi}} = \frac{\cos\zeta}{c_1}e^{2\phi^{(0)}};\quad d\hat{s}^2_7 = \frac{\cos^2\zeta}{c_1^2}ds_7^{(0)2};$$
$$\sin\zeta = c_2 e^{-\phi^{(0)}};$$