Boundary State from Open String Field Theory Invariants

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Based on arXiv:1207.4785
with M. Kudrna and M. Schnabl

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Motivation

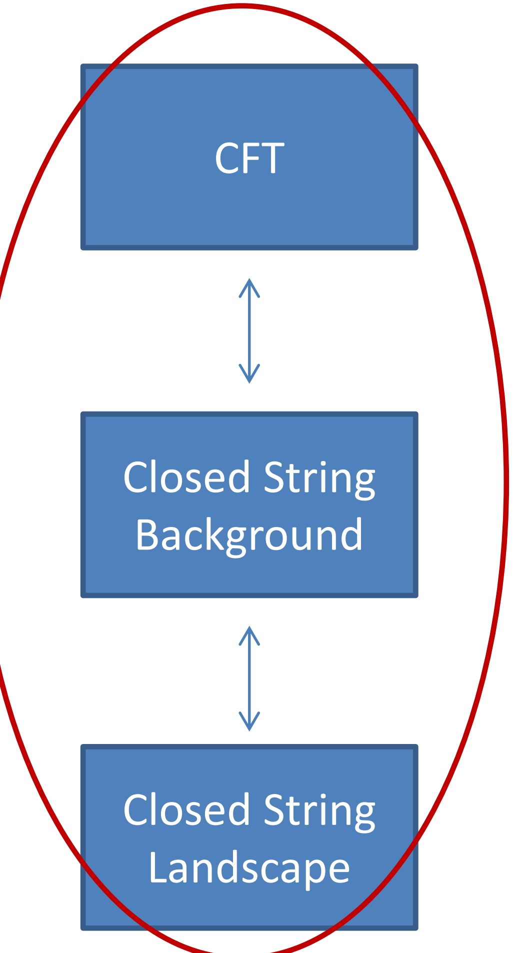
String Field Theory is an in principle complete non-perturbative approach to String Theory

- Closed String Field Theory is complicated and non polynomial

- Open String Field Theory is ‘simple’ and under control (bosonic).
- Closed string physics is typically encoded into the gauge invariant content of open string dynamics (e.g. AdS/CFT)

The Landscape: CFT's and BCFT's

CLOSED STRING FIELD THEORY



$\longrightarrow$

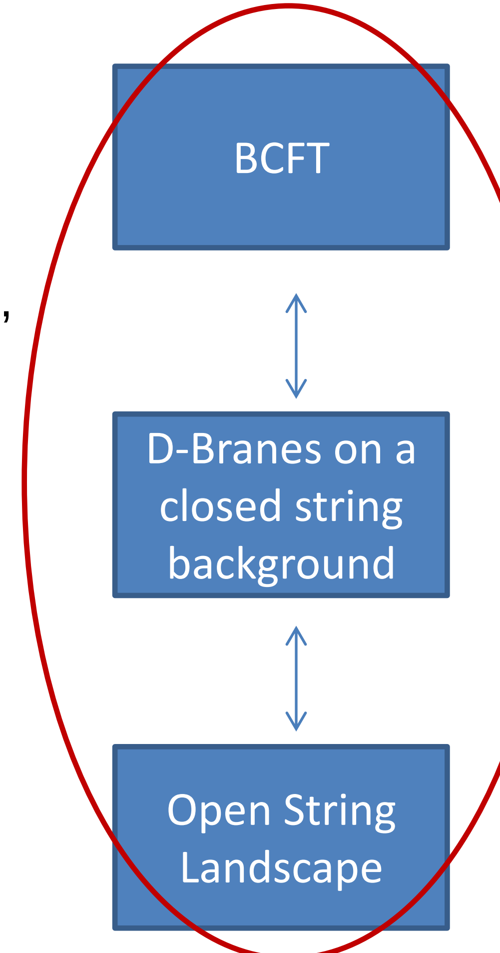
Conformal Boundary conditions
(isomorphic left/right moving sectors)

For a given CFT, a choice of BCFT is encoded in a peculiar closed string state, called the BOUNDARY STATE

$$|B_*\rangle \in \mathcal{H}_{\rm CFT}$$

It is defined as a path integral on the disk with given boundary conditions

OPEN STRING FIELD THEORY



$$\langle B|\phi\rangle \equiv \langle\phi(0)\rangle^{\rm BCFT_*}_{\rm disk}, \qquad \forall|\phi\rangle = \phi(0)|0\rangle_{SL(2,C)} \in \mathcal{H}_{\rm CFT}$$

The boundary state is the stringy generalization of the energy momentum tensor, source for closed strings (gravity)

- The worldsheet boundary conditions are conformal (holomorphic and antiholomorphic components of the stress tensor are identified)

$$(L_n - \bar{L}_{-n})|B_*\rangle = 0$$

- Take the set of spinless (non-singular) primaries

$$|V_\alpha\rangle = V_\alpha(0)|0\rangle_{SL(2,C)} \qquad \begin{aligned} L_0|V_\alpha\rangle &= \bar{L}_0|V_\alpha\rangle = h_\alpha|V_\alpha\rangle \end{aligned}$$

$$\langle V^\beta|\, V_\alpha\rangle = \delta^\alpha_\beta \qquad \begin{aligned} L_n|V_\alpha\rangle &= \bar{L}_n|V_\alpha\rangle = 0 \end{aligned}$$

- For every primary there is an Ishibashi state in its Verma module, which is uniquely defined by

$$(L_n - \bar{L}_{-n})|V_\alpha\rangle\rangle = 0$$

- Every boundary state can thus be written as

$$|B_*\rangle = \sum_\alpha n_*^\alpha\, |V_\alpha\rangle\rangle$$

$$n_*^\alpha = \langle V^\alpha|B_*\rangle = \langle V^\alpha(0)\rangle_{\rm disk}^{\rm BCFT_*}$$

I will show how to get these one-point functions from

Open String Field Theory solutions

bypass of Cardy and Factorization constraints!

OPEN STRING FIELD THEORY

- Fix a bulk CFT (closed string backgorund, for example flat D=26 space-time)
- Fix a reference $BCFT_0$ (open string background, for example Neumann everywhere, D25-brane)
- The string field is a state in $BCFT_0$

$$|\psi\rangle = \psi(0)|0\rangle_{SL(2,R)}$$

- There is a non-degenerate inner product (bpz)

$$\langle\psi, \chi\rangle = \langle\psi(-1)\,\phi(1)\rangle_{disk}$$

- The bpz-inner product allows to write a space-time action

$$S[\psi] = \frac{1}{2}\langle\psi, Q\psi\rangle + \frac{1}{3}\langle\psi, \psi * \psi\rangle$$

- Witten product: peculiar way of gluing surfaces through the midpoint in order to have associativity

(Selected) Facts in OSFT

- 1985: **Witten** writes down the action: star product, associative non-commutative algebra
- 1990: **Zwiebach** proves that the Feynmann rules gives a complete covering of the moduli space of Riemann surfaces with boundary (loop-amplitudes are guaranteed to be reproduced), on shell closed strings are automatically accounted for (**Shapiro-Thorn)** (more on this later…)
- 1999: **Sen** uses OSFT to formulate his conjectures on the tachyon vacuum (empty closed string background, no D-branes)
- 2005: **Schnabl** solves the equation of motion and finds the first analytic solution describing the tachyon vacuum
- 2005-Today: **Analytic revolution**, new analytic solutions are found (general marginal deformations) **Schnabl, Kiermaier, Okawa, Rastelli, Zwiebach, Fuchs, Kroyter, Potting, Noumi, Erler**. Progresses for other backgrounds (relevant deformations, **Bonora, CM, Tolla, Erler**, multiple branes, **Murata, Schnabl, Kojita, Hata** etc...)
- Non trivial gauge structure **(Okawa, Ellwood)**, solutions are organized in a Category, **Erler, CM**
- Numerically a quite large landscape of solutions is seen to emerge, (multiple lower dimensional branes, intersecting branes,…) **Kudrna, Schnabl**

→ **Need for gauge invariant observables to identify the physical properties of the solutions**

- Equation of motion

$$Q\psi_o + \psi_o * \psi_o = 0$$

- Gauge invariance

$$\psi = U(Q+\psi)U^{-1}$$

- BASIC CONJECTURE: *Any non-trivial solution defines (up to gauge transformations) a new BCFT: a different D-brane system from the one desrcibed by* $BCFT_0$ *.*
- Ideally we expect to have

- 2008, **Kiermaier, Okawa and Zwiebach:** very elegant (but involved) construction of the boundary state up to BRST exact terms.

Gauge invariants of OSFT

- Action: For static solutions it evaluates the energy of the solution (D-brane tension, Sen's conjectures)
- More interesting: Closed String Overlap **(Shapiro-Thorn, Gaiotto-Rastelli-Sen-Zwiebach, Itzhaki-Hashimoto, Ellwood)**

$$\langle I|V(i,-i)|\Psi\rangle \equiv W_V(\Psi), \quad (Q+\bar{Q})V(z,\bar{z})=0$$

- This is a gauge invariant operator that can be consistently added to the action. All amplitudes between **off-shell open strings** and **on-shell closed strings** can be computed at all genera (the moduli space is correctly covered).
- They are also important non-perturbatively. **Ellwood**: for all known analytic solutions one has

$$W_V(\Psi) = \frac{1}{2\pi i}\left(\langle V(0)c(1)\rangle_{\mathrm{disk}}^{\mathrm{BCFT}_0} - \langle V(0)c(1)\rangle_{\mathrm{disk}}^{\mathrm{BCFT}_\Psi}\right)$$

- The invariant computes the shift in the **(on shell)** closed string tadpole between the reference BCFT and the new background described by the solution.
- Since the tachyon vacuum is a solution which `cancels' the reference BCFT, this important relation can also be written as

$$-2\pi i W_V(\Psi-\Psi_{TV}) = \langle V(0)c(1)\rangle_{\mathrm{disk}}^{\mathrm{BCFT}_\Psi} \equiv \frac{1}{2}\langle V|c_0^-|B_\Psi\rangle$$

ELLWOOD CONJECTURE (a rigorous proof is lacking but no counterexamples are known)

- Instead of trying to prove it, we will extract all the consequences from this conjecture, to get the complete boundary state.
- Main limitation: the closed string must be on-shell, a lot of information on the boundary state is missing if we can only contract with on-shell closed strings.
- We need a modification of the invariant which computes the tadpoles of generic closed string primaries, not just on shell primaries.
- This can be done by tensoring the original BCFT with an auxiliary BCFT of **vanishing central charge** and lifting the solutions of the original theory to solutions in the tensor theory.

$$\Psi = \sum_{i;\,M,N} a^i_{N,M}\, L^{\mathrm{matter}}_{-N} L^{\mathrm{ghost}}_{-N}\, c\phi^i(0)|0\rangle_{SL(2,R)} \qquad L_{-N} \equiv L_{-n_N} \ldots L_{-n_1}$$

$$L^{\mathrm{matter}} \to \tilde{L}^{\mathrm{matter}} \equiv L^{\mathrm{matter}} + L^{\mathrm{aux}}$$

$$\Psi \to \tilde{\Psi} \equiv \Psi\Big|_{L^{\mathrm{matter}} \to L^{\mathrm{matter}} + L^{\mathrm{aux}}},$$

$$Q \to \tilde{Q} \equiv \sum_n :c_{-n}\left(L^{\mathrm{matter}}_n + L^{\mathrm{aux}}_n + \frac{1}{2} L^{\mathrm{ghost}}_n\right):$$

$$Q\Psi + \Psi * \Psi = 0 \quad \Longrightarrow \quad \tilde{Q}\tilde{\Psi} + \tilde{\Psi} * \tilde{\Psi} = 0$$

- The boundary state for the lifted solution will be

$$|B_{\tilde{\Psi}}\rangle = |B_\Psi\rangle \otimes |B_{\mathrm{aux}}\rangle$$

- In the auxiliary theory I need spinless bulk primaries with unit one point function on the disk

$$\langle w^\alpha(0)\rangle_{\mathrm{disk}}^{\mathrm{BCFT_{aux}}} = 1,\ \forall\alpha \qquad w^\alpha = e^{2i\sqrt{1-h_\alpha}\,Y}\, e^{\frac{2i}{\sqrt{3}}\varphi}$$

- For any bulk primary in the original BCFT consider the total weight zero combination in the tensor theory

$$\tilde{\mathcal{V}}^\alpha \equiv c\bar{c}\,V^\alpha \otimes w^\alpha \longrightarrow \tilde{Q}\tilde{\mathcal{V}}^\alpha = 0$$

- Then use the Ellwood conjecture

$$\begin{aligned}
&-4\pi i\langle\, I|\tilde{\mathcal{V}}^\alpha(i)|\tilde{\Psi}\,\rangle \\
&= \langle\tilde{\mathcal{V}}^\alpha|\, c_0^-\,|\tilde{B}_{\tilde{\Psi}}\rangle - \langle\tilde{\mathcal{V}}^\alpha|\, c_0^-\,|\tilde{B}_0\rangle \\
&= \Big(\langle c\bar{c}V^\alpha|\otimes\langle w^\alpha|\Big)\, c_0^-\, \Big(|B_\Psi\rangle\otimes|B_{\mathrm{aux}}\rangle\Big) - \Big(\langle c\bar{c}V^\alpha|\otimes\langle w^\alpha|\Big)\, c_0^-\, \Big(|B_0\rangle\otimes|B_{\mathrm{aux}}\rangle\Big) \\
&= \Big(\langle\, c\bar{c}V^\alpha|\, c_0^-\,|B_\Psi\,\rangle - \langle\, c\bar{c}V^\alpha|\, c_0^-\,|B_0\,\rangle\Big)\ \langle\, w^\alpha|B_{\mathrm{aux}}\,\rangle \\
&= \Big(\langle\, c\bar{c}V^\alpha|\, c_0^-\,|B_\Psi\,\rangle - \langle\, c\bar{c}V^\alpha|\, c_0^-\,|B_0\,\rangle\Big)\ \langle\, w^\alpha(0)\,\rangle_{\mathrm{disk}}^{\mathrm{BCFT^{aux}}} \\
&= \langle\, c\bar{c}V^\alpha|\, c_0^-\,|B_\Psi\,\rangle - \langle\, c\bar{c}V^\alpha|\, c_0^-\,|B_0\,\rangle\,.
\end{aligned}$$

- So we find

$$\langle\, c\bar{c}V^\alpha|\, c_0^-\,|B_\Psi\,\rangle = -4\pi i\langle\, I|\tilde{\mathcal{V}}^\alpha(i)|\tilde{\Psi}-\tilde{\Psi}_{TV}\,\rangle \equiv -4\pi i\, W_{\tilde{\mathcal{V}}^\alpha}(\tilde{\Psi}-\tilde{\Psi}_{TV}).$$

$\Longrightarrow$ **GENERALIZED ELLWOOD CONJECTURE**

- The string theory boundary states are factorized in matter and ghost (the Polyakov path integral doesn't mix them)

$$|B_\Psi\rangle = |B_\Psi\rangle^{\rm matter} \otimes |B_{bc}\rangle = \sum_\alpha n_\Psi^\alpha \, |V_\alpha\rangle\rangle \otimes |B_{bc}\rangle$$

- It is easy to compute (consistent with modular invariance of bc-system)

$$\langle c\bar{c}|c_0^-|B_{bc}\rangle = -2$$

- To finally get

$$n_\Psi^\alpha \equiv \langle V^\alpha|B_\Psi\rangle \equiv \langle V^\alpha(0)\rangle_{\rm disk}^{\rm BCFT_\Psi} = 2\pi i\, W_{\tilde{\mathcal{V}}^\alpha}(\tilde\Psi - \tilde\Psi_{TV})$$

The main advantage is that these invariants are very easily computable!

EXAMPLE: LUMPS IN SIEGEL GAUGE

- Inhomogeneous tachyon condensation, RG flow from Neumann to Dirichlet boundary conditions
- Solution known only numerically **Moeller, Sen, Zwiebach** (although remarkable analytic progresses, **Bonora, CM, Tolla; Erler, CM**)
- The known solution is schematically (level truncation)

$$\Psi = \sum_{n;\,M,N} a^n_{N,M}\, L^{\rm matter}_{-N} L^{\rm ghost}_{-N} \cos\left(\frac{nX}{R}\right)(0) c_1|0\rangle + \sum_{j;\,M,N} b^j_{N,M}\, L^{\rm matter}_{-N} L^{\rm ghost}_{-N}\, P_j(0) c_1|0\rangle_{SL(2,R)}$$

- Energy density profile (off-shell part of the boundary state)

$$E(x) \equiv T^{00}(x) = \frac{1}{\pi R}\left(\frac{1}{2}E_0 + \sum_{n=1}^{\infty} E_n \cos\frac{nx}{R}\right) \quad \boxed{= \sum_{i=1}^{N} \delta(x - x_i)}$$

- The coefficients of the Ishibabshi states are given by

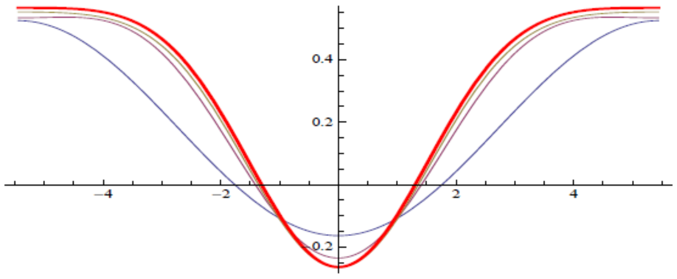
$$E_n \equiv -4\pi i \left\langle I \middle| c\bar{c}\partial X^0 \bar{\partial} X^0\, e^{i\frac{nX}{R}+\frac{nY}{R}}(i) \middle| \Psi - \Psi_{TV} \right\rangle$$

They are easily computable in terms of the coefficients of the solution

For exact lump solutions we expect exact delta function profiles (generation of Dirichlet boundary conditions)

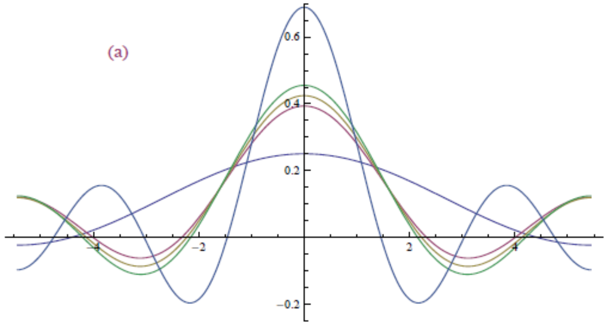
SINGLE LUMP

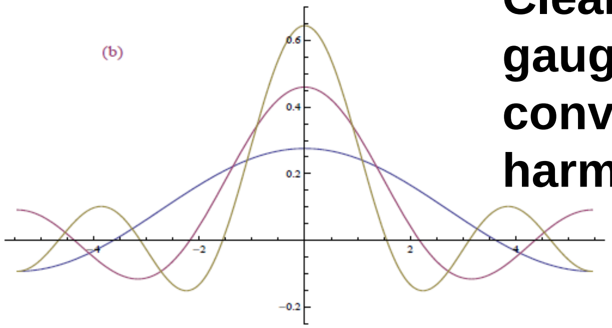
- The level-truncated solution at (for example) compactification radius R=sqrt3 (MSZ) gives the tachyon profile (which is not a gauge invariant quantity). The profile is not localized (physical puzzle....)



- Computing the (gauge invariant) Ellwood invariants

L $(R=\sqrt{3})$	Action	E_0	E_1	E_2	E_3	D	P
1/3	1.32002	1.23951	0.743681	–	–	1.23951	−1.23951
4/3	1.25373	1.14776	0.741903	0.825738	–	1.14776	−1.14776
2	1.11278	1.10298	0.830459	0.927894	–	−0.574734	−0.264122
7/3	1.07358	1.07489	0.899585	1.0405	–	−0.992768	−0.0410632
3	1.06421	1.0645	0.89973	1.07981	1.23776	−1.08289	0.00919196
Expected	1	1	1	1	1	−1	0

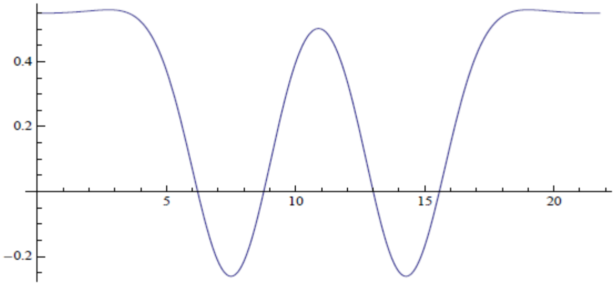




Clear indication that the gauge invariant profile converges to a delta function harmonic by harmonic

DOUBLE LUMP

- The level-truncated solution at compactification radius R=2sqrt3 gives the tachyon profile (which is not a gauge invariant quantity). The profile clearly shows two lumps (two D-branes)... At which distance?

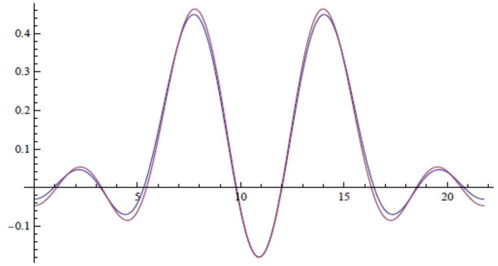


- Computing the (gauge invariant) Ellwood invariants

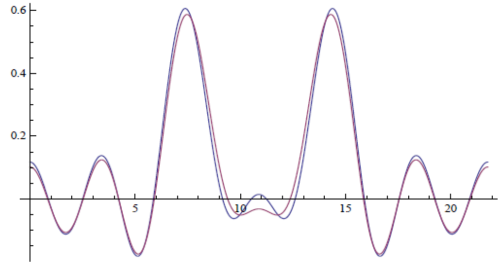
L	Action	D	E_0	E_1	E_2	E_3	E_4	E_5
1	2.57014	2.4209	2.4209	− 0.816955	− 0.54184	1.3133	−	−
2*	2.21165	−1.69337	2.1897	− 0.848747	− 0.60583	1.89707	− 1.62092	−
3	2.19355	−2.50001	2.11767	− 0.908501	− 0.838798	1.84278	− 1.24372	− 0.987367
4*	2.06874	−1.39183	2.08709	− 0.919667	− 0.850043	1.88425	− 1.0523	− 1.02488
5	2.05531	−1.37542	2.07382	− 0.983959	− 0.812633	1.91245	− 1.15202	− 0.57724
6	2.03894	−2.09185	2.05368	− 1.00138	− 0.788653	1.92175	− 1.30591	− 0.518028
7	2.03494	−2.1419	2.04912	− 1.03283	− 0.765547	1.90846	− 1.35827	− 0.488344
8	2.0269	−1.71527	2.04119	− 1.04599	− 0.743696	1.90879	− 1.35485	− 0.42022
9	2.02525	−1.70495	2.03899	− 1.06273	− 0.734362	1.91644	− 1.37781	− 0.37505
10	2.02052	−2.07063	2.03154	− 1.07229	− 0.717661	1.91526	− 1.44161	− 0.329759
11	2.01969	−2.08504	2.03029	− 1.08369	− 0.709787	1.90937	− 1.45664	− 0.295048
12	2.01658	− 1.81655	2.02687	− 1.09091	− 0.696749	1.90744	− 1.45907	− 0.256288

We use the first invariant to compute the distance

$$a_1 = \frac{1}{\pi} \arccos\left(-\frac{E_1}{2}\right)$$



up to 4-th harmoinc



up to 6-th harmoinc

The energy profile is nicely reproducing a double delta
(need to extrapolate to infinite level to get better precision)

Discussion

- The boundary state is a useful tool to identify different BCFT’s
- We found a simple way to get the boundary state from the gauge invariant content of OSFT solutions
- Our construction is infact a limit of the more involved construction of Kiermaier, Okawa and Zwiebach, but we provided a way to get rid of the BRST exact components of their construction (useful for generic solutions!)
- How Cardy and sewing conditions emerge from the OSFT equation of motion? (KOZ construction seems to suggest how this happens)
- Coefficients of the solutions are linearly mapped to the coefficients of the Ishibashi states: useful information to find new solutions!
- Are all boundary conditions reachable? Can we increase the boundary entropy??? Old question: IS THE STRING FIELD BIG ENOUGH?

We have now a simple workable framework to address these things!

Thank you