Magnetized Branes on T^6

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Based on:

- L. De Angelis, R. M., F. Pezzella and R. Troise, '' More About branes on a General Magnetized Torus'', to appear on JHEP (arXiv:1206.3401).

PLAN

1. Introduction.
2. Magnetized D9-branes.
3. Yukawa couplings on T^6.
4. Conclusions.

Introduction

- String Theory is the most promising candidate for a consistent and unified quantum description of all the known interactions.

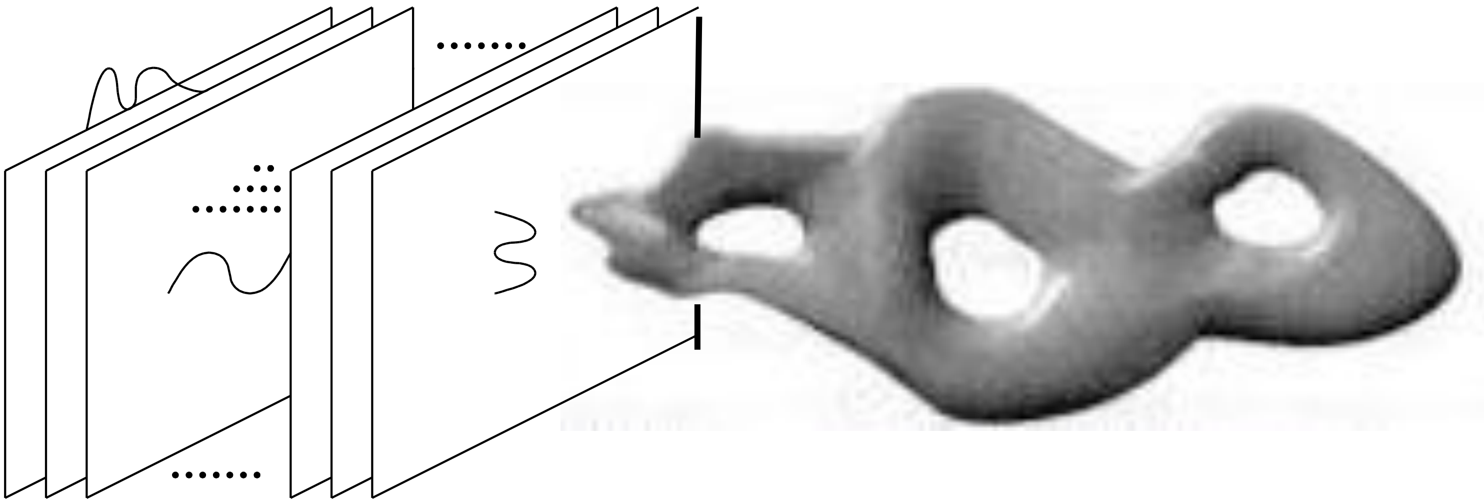
  Gauge interactions and Gravitation.

- How the Standard Model (SM) may be obtained from a low energy limit of string theory.

  SM gauge groups, quark-lepton generations, chirality, Yukawa couplings, Higgs sector etc.

- ✓ String theories require a ten-dimensional space-time. Six of those must be compactified on a manifold having a small volume.
- ✓ When six dimensions are compactified a bunch of scalar fields are generated, corresponding to the components of the metric and other closed string fields in the extra dimensions.
- ✓ Their vacuum expectation values (moduli) , are not fixed at any perturbative order because their potential is flat: Moduli stabilization problem.
  - ✓ In flux compactifications these moduli are frozen to the minimum of the potential, however there are too many consistent compactifications: Landscape

- A corner of the String Landscape is here considered: Orientifold Brane world in Type II toroidal (T^6) compactifications preserving $\mathcal{N}=1$ SUSY.



- A stack of N-branes has a U(N) gauge theory in their word-volume, for example in a stack on N D3-branes lives a $\mathcal{N}=4$ Super Yang-Mills. Non-chiral theory.

⇨ SM is a chiral theory

- Chiral fermions appear in the world-volume of Magnetized Dp-Branes.

Magnetized D-branes are branes having constant magnetic fluxes along the compact directions

$$\int_{\mathcal{A}_2} \mathrm{Tr}_{U(M)}\left[\frac{F}{2\pi}\right] = I \longrightarrow 2\pi\alpha' F = \frac{I}{M}$$

Open string moduli

- The brane is wrapped M-times on the torus and the flux of F is an integer number I (magnetic charge).

The most general motion of an open string in this constant background can be determined and the theory can be explicitly quantized. ⇒ In the fermionic sector, the lowest state is a 4-dimensional massless chiral spinor!!

○ At energies $E \ll M_s$ all the heavy string excitations can be neglected and a low energy four dimensional effective action can be derived

Supegravity (SUGRA) approximation

Local versus Global models

✓ In a local model (Gravity neglected, no RR-tadpole cancellation..) of magnetized D9-branes would be interesting to compute the Yukawa couplings by fully taking in account the effects of the compactification in the case of a manifold non too trivial : the torus T^6 with arbitrary complex structure.

❑ Magnetized D9-branes on a generic torus T^6

- Configuration: M D9-branes in the background $R^{1,3} \times T^6$
- The torus is defined by introducing in $\mathbf{R}^6$ a set of coordinates (x^n, y^n) $n = 1 \ldots 3$ with the identification

$$\Lambda : \quad x^m \equiv x^m + 2\pi R\, m_1^m \quad y^m \equiv y^m + 2\pi\, m_2^m R \qquad \vec{m}_{1,2} \in \mathbf{Z}^3$$

$\mathbf{R}^6/\Lambda$ Real torus

or by dimensionless complex coordinates

$$\Lambda : \quad w^m = \frac{x^m + U^m_{\ n} y^n}{2\pi R} \quad w^m \equiv w^m + m_1^m + U^m_{\ n} m_2^n$$

$\mathbf{C}^3/\Lambda$ Complex torus

Complex Structure

A stack of M D9-branes becomes M D9- magnetized branes when M abelian and constant magnetic fields are presents along the compact directions. ⇒ $U(M) \sim U(1)^M$

The difference between the background fields active on the a, b branes is

$$F^{ab} = \frac{1}{2} F^{(xx)ab}_{mn} dx^m \wedge dx^n + F^{(xy)ab}_{mn} dx^m \wedge dy^n + \frac{1}{2} F^{(yy)ab}_{mn} dy^m \wedge dy^n$$

In the complex system of coordinates, supersymmetry requires F^{ab} to be a (1,1)-form: $F^{ab} = dw^t \wedge F^{(w\bar{w})ab} d\bar{w}$

$$iF^{(w\bar{w})ab} = 2(F^{(xx)t}U + F^{(xy)})\mathrm{Im}U^{-1}$$

⇒ Hermitian matrix, diagonalized by a Unitary matrix

$$iF^{(w\bar{w})ab} \cdot \bar{C}^{-1}_{ab} = \frac{\mathcal{I}^{ab}_{\lambda}}{(2\pi R)} \cdot \bar{C}^{-1}_{ab} \quad ; \quad \mathcal{I}^{ab}_{\lambda} \equiv \mathrm{diag}(\lambda^{ab}_1, \ldots \lambda^{ab}_3)$$

The diagonalization naturally introduces new coordinates

$$w^m = (C^{-1})^m_{\ r}\, z^r$$

with:

$$ds^2 = (2\pi R)^2 \delta_{rs} dz^s_{ab} d\bar{z}^r_{ab}$$

From the unitary condition of the C^{-1}

In this frame both the metric and the background magnetic field are diagonal matrices in their non-vanishing blocks.

$$\mathbf{C}^3/\Lambda \quad \Rightarrow \quad z \equiv z + C \cdot \vec{m}_1 + C \cdot U\, \vec{m}_2$$

Effective action

Starting Point: Low-energy limit of the DBI-action of M D9-branes ⇒ $\mathcal{N}=1$ super Yang-Mills in ten dimensions

$$S = \frac{1}{g^2}\int d^{10}X\,\mathrm{Tr}_{U(M)}\left(-\frac{1}{4}\mathcal{F}^2 + \frac{i}{2}\bar{\lambda}\Gamma^M D_M\lambda\right)$$

Field strength of the gauge field A

Gaugino

The gauge breaking $U(M)\approx U(1)^M$ due to the magnetic field is realized by first separating the generators of the Cartan subalgebra U_a from the other ones e_{ab} and later giving a vev to the abelian fields

$$A_N = B_N^a U_a + \Phi_M^{ab} e_{ab} = (\langle B_N^a\rangle + \delta B_N^a)U_a + \Phi_N^{ab} e_{ab}$$

Chiral scalars

Adjoint scalars $N = 6\ldots 9$

KK-reduction - The four dimensional theory is obtained by expanding the bosonic and fermionic fields in a base of eigenvectors of the internal respectively Laplace-Beltrami and Dirac equation

$$-\tilde{D}_N \mathcal{D}^N \Phi^{ab}_{\mathcal{M}}(X) = m^2_{\mathcal{M}} \Phi^{ab}(X) \quad ; \quad \Phi^{ab}_{M}(X) = \sum_{\mathcal{M}} \varphi^{ab}_{\mathcal{M},N} \otimes \phi^{ab}_{\mathcal{M}}(X)$$

$$\tilde{D}_N \phi^{ab}_{\mathcal{M}} = \partial_N \phi^{ab}_{\mathcal{M}} - i(\langle B^a_N \rangle - \langle B^b_N \rangle)\phi^{ab}_N$$

$$i\gamma^M_{(6)} \tilde{D}_M \eta^{ab}_n = m_n \eta^{ab}_n \quad ; \quad \Psi^{ab} = \sum_n \psi^{ab}_n \otimes \eta^{ab}_n$$

Boundary conditions: identification up to gauge ($\chi_{1,2}$) transformations of the fields under the torus translations.

$$(\phi_{\mathcal{M}}\,;\eta_n)\,(z + C\,\vec{m}_1\,, \bar{z} + C\,\vec{m}_2) = e^{i\chi_1}\,(\phi_{\mathcal{M}}\,;\eta_n)\,(z,\ \bar{z})$$

$$(\phi_{\mathcal{M}}\,;\eta_n)\,(z + C\cdot U\,\vec{m}_2\,, \bar{z} + C\cdot U\,\vec{m}_2) = e^{i\chi_2}\,(\phi_{\mathcal{M}}\,;\eta_n)\,(z,\ \bar{z})$$

- The eigenvalues of the mass operators turn out to be:

KK-spectrum of the scalars: $$M_s^2 = \sum_{r=1}^{3} \frac{|\lambda_r|}{(2\pi R)^2}(2N_r + 1) \pm 2\frac{\lambda_s}{(2\pi R)^2}$$

- When the $N = 1$ SUSY - condition $|\lambda_r| + |\lambda_s| = |\lambda_t|$ is imposed, the lightest state ($N_r = 0$) becomes massless and the spectrum is contained in the expression

$$M_k^2 = \frac{2}{(2\pi R)^2}\sum_{r=1}^{3} |\lambda_r|(N_r + k)$$

$k = 0,1$

$N^f = 0,1$

KK-spectrum of the fermions: $$M_k^2 = \frac{2}{(2\pi R)^2}\sum_{r=1}^{3} |\lambda_r|(N_r + N^f)$$

Bose-Fermi degeneracy peculiar of a SUSY theory

The solution for the lightest bosonic and fermionic fields valid when $F^{(xx)ab} = F^{(yy)ab} = 0$ is:

Bosons

$$\phi_{\vec{j}} = C e^{i\vec{y}^t \frac{I_{ab}}{(2\pi R)^2}\vec{x} + i\vec{y}^t \Omega_{ab} \frac{I^t_{ab}}{(2\pi R)^2}\vec{y}} \Theta \begin{bmatrix} \vec{j} \\ 0 \end{bmatrix} \left(I_{ab} \left(\frac{\vec{x} + \Omega_{ab}\vec{y}}{2\pi R} \right) \middle| I_{ab}\,\Omega_{ab} \right)$$

Constant spinor

$$\eta_0 = \prod_{r=1}^{3} \gamma_{(6)}^{\left(\frac{1+\text{sign}(\lambda_r)}{2}\right) z^r + \left(\frac{1-\text{sign}(\lambda_r)}{2}\right) \bar{z}^r} \chi_0 \otimes \phi_{\vec{j}}$$

→ Fermions

$\vec{j}$ is an integer vector which label the degeneracy of the ground state which turns out to be $\det[I_{ab}]$.

$$\Omega_{ab} = (C^{(\lambda)})^{-1}\tilde{C}^{(\lambda)}$$

"Generalized complex structure" coincides with U or $\bar{U}$ when all the λs have the same signs.

$$C^{(\lambda)}{}^{r}{}_{m} = \left(\frac{1+\mathrm{sign}(\lambda)_r}{2}\right) C^{r}{}_{m} + \left(\frac{1-\mathrm{sign}(\lambda_r)}{2}\right) \bar{C}^{r}{}_{m}$$

$$\tilde{C}^{(\lambda)}{}^{r}{}_{m} = \left(\frac{1+\mathrm{sign}(\lambda)_r}{2}\right) C^{r}{}_{n} U^{n}{}_{m} + \left(\frac{1-\mathrm{sign}(\lambda_r)}{2}\right) \bar{C}^{r}{}_{n} \bar{U}^{n}{}_{m}$$

The Yukawa couplings are obtained by reducing the ten dimensional super Yang-Mills action to four dimensions and considering the terms trilinear in the fields

Lightest boson

$$S_3^{\Phi} = \int d^4x \sqrt{G_4}\, \bar{\psi}_0^{ca} \gamma_{(4)}^5 \phi_{Z^1,0}^{ab} \psi_0^{bc}\, Y^s$$

Massless fermion fields

Being the Yukawa couplings $Y^s = \frac{1}{2g^2}[(u_0^{ac})^\dagger \gamma_{(6)}^{Z_{ab}^1} u_0^{bc}]\, \mathcal{Y}^s$

The spinor product determines which fermions have non vanishing couplings.

$$\mathcal{Y}^s = \int_{T^6} d^3x d^3y \sqrt{G_6} (\phi_0^{ac})^\dagger \phi_0^{ab} \phi_0^{bc}$$

When all the first Chern classes are independents the Yukawa couplings turn out to be:

$$\mathcal{Y}^s = \mathcal{N}\sqrt{G_6}\,[\det(-i(I_{ac}\Omega_{ac}+I_{ab}\Omega_{ab}+I_{bc}\Omega_{bc})]^{-1/2}$$

$$\sum_{\substack{\vec{p}\in\mathbf{Z}^3_{\det[I_{bc}]I_{bc}^{-1}} \\ \vec{\tilde{p}}\in\mathbf{Z}^3_{\det[I_{ab}]I_{ab}^{-1}}}} \Theta\begin{bmatrix} \frac{I_{ab}^t}{\det[I_{ab}I_{bc}]}(\vec{j}_3-\vec{j}_2)+\frac{I_{bc}^t}{\det[I_{bc}]}\vec{p}+\frac{I_{ab}^t}{\det[I_{bc}]}\vec{\tilde{p}} \\ 0 \end{bmatrix}(0|(\det[I_{ab}I_{bc}])^2\Pi)$$

$$\Pi=(\Omega_{ab}I_{ab}^{-t}+\Omega_{bc}I_{bc}^{-t})-(\Omega_{ab}-\Omega_{bc})(I_{ca}\Omega_{ca}+I_{ab}\Omega_{ab}+I_{bc}\Omega_{bc})^{-1}(\Omega_{ab}-\Omega_{bc})^t$$

Valid when $F^{(xx)ab}=F^{(yy)ab}=0$

This expression simplifies when all the magnetic fields living on the three stacks of magnetized branes commute and when evaluated on the factorized torus $T^2\otimes T^2\otimes T^2$ coincides with the results given in literature.

Conclusions

- The field theory approach is a very efficient tool in the determination of the low-energy effective action of branes with oblique magnetizations.
    - Extension of these results to higher couplings.

      (Quadrilinear couplings)

    - Extension of the results in the case of magnetizations along the non Cartan generators.

      (F-theory connection)

    - Generalization to the case of a constant and completely arbitrary magnetic field, both in field and in string theory.