Scattering amplitudes in N=6 Chern-Simons-matter theories

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Matías Leoni

UBA - CONICET

Outline

- **AdS/CFT and ABJM**
- **Gluon Scattering at strong coupling**
- **Super Wilson Loops/Scattering in N=4**
- **Results in N=6 Chern-Simons matter theories**
- **Conclusions**

In collaboration with M.Bianchi, A.Mauri, S.Penati, A.Santambrogio **(MI Bicocca- MI Statale)**

AdS/CFT

- IIB Strings on AdS5xS5
- $\sqrt{\lambda} = R^2$
- Perturbation in powers of $\frac{1}{\sqrt{\lambda}}$

- N=4 SYM field Theory
- $\lambda = g_{YM}^2 N$
- Perturbation in powers of λ

- IIA Strings on AdS4xCP3

$\sqrt{\lambda} = R^2$

- N=6 Chern-Simons matter theory

$\lambda = \frac{N}{k}$

ABJM Component Action

$$S_{\text{kin}} = \frac{k}{4\pi}\int \mathrm{d}^3x\, \mathrm{tr}\Big[A_\alpha\, \epsilon^{\alpha\beta\gamma}\partial_\beta A_\gamma - \hat{A}_\alpha\, \epsilon^{\alpha\beta\gamma}\partial_\beta \hat{A}_\gamma + Y^\dagger_A \partial_\mu \partial^\mu Y^A + i\psi^{\dagger B}\partial\!\!\!/\psi_B\Big]$$

$$\begin{aligned}
S_{\text{int}} = \frac{k}{4\pi}\int \mathrm{d}^3x\, \mathrm{tr}\Big[&\frac{2}{3}i\epsilon^{\alpha\beta\gamma}(A_\alpha A_\beta A_\gamma - \hat{A}_\alpha \hat{A}_\beta \hat{A}_\gamma) \\
&- iA_\mu Y^A \overleftrightarrow{\partial^\mu} Y^\dagger_A - i\hat{A}_\mu Y^\dagger_A \overleftrightarrow{\partial^\mu} Y^A + 2Y^\dagger_A A_\mu Y^A \hat{A}^\mu \\
&- \hat{A}_\mu \hat{A}_\mu Y^\dagger_A Y^A - A_\mu A_\mu Y^A Y^\dagger_A \\
&- \psi^{\dagger B} A\!\!\!/ \psi_B + \hat{A}_\mu \psi^{\dagger B}\gamma^\mu \psi_B \\
&+ \frac{1}{12} Y^A Y^\dagger_B Y^C Y^\dagger_D Y^E Y^\dagger_F (\delta^B_A \delta^D_C \delta^F_E + \delta^F_A \delta^B_C \delta^D_E - 6\delta^B_A \delta^F_C \delta^D_E + 4\delta^D_A \delta^F_C \delta^B_E) \\
&- \frac{i}{2}(Y^\dagger_A Y^B \psi^{\dagger C}\psi_D - \psi_D \psi^{\dagger C} Y^B Y^\dagger_A)(\delta^A_B \delta^D_C - 2\delta^A_C \delta^D_B) \\
&+ \frac{i}{2}\epsilon^{ABCD} Y^\dagger_A \psi_B Y^\dagger_C \psi_D - \frac{i}{2}\epsilon_{ABCD} Y^A \psi^{\dagger B} Y^C \psi^{\dagger D}\Big]
\end{aligned}$$

ABJM Component Action

$$S_{\text{kin}} = \frac{k}{4\pi}\int \mathrm{d}^3x\,\mathrm{tr}\Big[A_\alpha\,\epsilon^{\alpha\beta\gamma}\partial_\beta A_\gamma - \hat{A}_\alpha\,\epsilon^{\alpha\beta\gamma}\partial_\beta \hat{A}_\gamma + Y^\dagger_A\partial_\mu\partial^\mu Y^A + i\psi^{\dagger B}\partial\!\!\!/\psi_B\Big]$$

cs

$$S_{\text{int}} = \frac{k}{4\pi}\int \mathrm{d}^3x\,\mathrm{tr}\Big[\frac{2}{3}i\epsilon^{\alpha\beta\gamma}(A_\alpha A_\beta A_\gamma - \hat{A}_\alpha\hat{A}_\beta\hat{A}_\gamma)$$

$$- iA_\mu Y^A \overleftrightarrow{\partial^\mu} Y^\dagger_A - i\hat{A}_\mu Y^\dagger_A \overleftrightarrow{\partial^\mu} Y^A + 2Y^\dagger_A A_\mu Y^A \hat{A}^\mu$$

$$- \hat{A}_\mu\hat{A}_\mu Y^\dagger_A Y^A - A_\mu A_\mu Y^A Y^\dagger_A$$

$$- \psi^{\dagger B} A\!\!\!/ \psi_B + \hat{A}_\mu \psi^{\dagger B}\gamma^\mu\psi_B$$

m.c.

ϕ^6 **-potential**

$$+ \frac{1}{12}Y^A Y^\dagger_B Y^C Y^\dagger_D Y^E Y^\dagger_F(\delta^B_A\delta^D_C\delta^F_E + \delta^F_A\delta^B_C\delta^D_E - 6\delta^B_A\delta^F_C\delta^D_E + 4\delta^D_A\delta^F_C\delta^B_E)$$

$$- \frac{i}{2}(Y^\dagger_A Y^B\psi^{\dagger C}\psi_D - \psi_D\psi^{\dagger C}Y^B Y^\dagger_A)(\delta^A_B\delta^D_C - 2\delta^A_C\delta^D_B)$$

$$+ \frac{i}{2}\epsilon^{ABCD}Y^\dagger_A\psi_B Y^\dagger_C\psi_D - \frac{i}{2}\epsilon_{ABCD}Y^A\psi^{\dagger B}Y^C\psi^{\dagger D}\Big]$$

3d Yukawa

N=2 Action

N=2 Action

$$\mathcal{S}_{CS} = \frac{k}{4\pi}\int d^3x d^4\theta \int_0^1 dt \; \mathrm{tr}\left[V\bar{D}^{\alpha}\left(e^{-tV} D_{\alpha} e^{tV}\right) - V \leftrightarrow \hat{V} \right]$$

$$\mathcal{S}_{mat} = \int d^3x d^4\theta \; \mathrm{tr}\left(\bar{\mathcal{A}}^A e^{\hat{V}} \mathcal{A}_A e^{-V} + \bar{\mathcal{B}}_A e^{V} \mathcal{B}^A e^{-\hat{V}}\right)$$

$$\mathcal{S}_{pot} = \frac{2\pi i}{k}\int d^3x d^2\theta \, \epsilon_{AC}\epsilon^{BD} \mathrm{tr}(\mathcal{B}^A \mathcal{A}_D \mathcal{B}^C \mathcal{A}_B) + h.c.$$

$\mathcal{B}^A \quad \mathcal{A}_D$ $\qquad$ $V \quad \hat{V}$

Chiral Scalars $\qquad$ **Gauge Vectors**

N=2 Action

$$\mathcal{S}_{CS} = \frac{k}{4\pi}\int d^3x d^4\theta \int_0^1 dt \; \mathrm{tr}\left[V \bar{D}^{\alpha}\left(e^{-tV} D_{\alpha} e^{tV}\right) - V \leftrightarrow \hat{V} \right]$$

$$\mathcal{S}_{mat} = \int d^3x d^4\theta \; \mathrm{tr}\left(\bar{\mathcal{A}}^A e^{\hat{V}} \mathcal{A}_A e^{-V} + \bar{\mathcal{B}}_A e^{V} \mathcal{B}^A e^{-\hat{V}}\right)$$

$$\mathcal{S}_{pot} = \frac{2\pi i}{k} \int d^3x d^2\theta \, \epsilon_{AC}\epsilon^{BD} \mathrm{tr}(\mathcal{B}^A \mathcal{A}_D \mathcal{B}^C \mathcal{A}_B) + h.c.$$

$\mathcal{B}^A$ $\mathcal{A}_D$ V $\hat{V}$

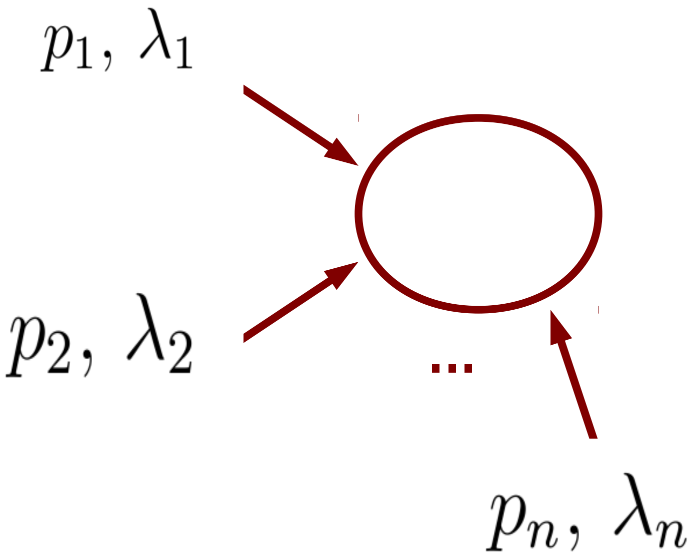
Chiral Scalars **Gauge Vectors**

't hooft coupling: $\hat{\lambda} = \frac{N}{k}$

AdS/CFT Perspective

MHV Gluon-Scattering in N=4

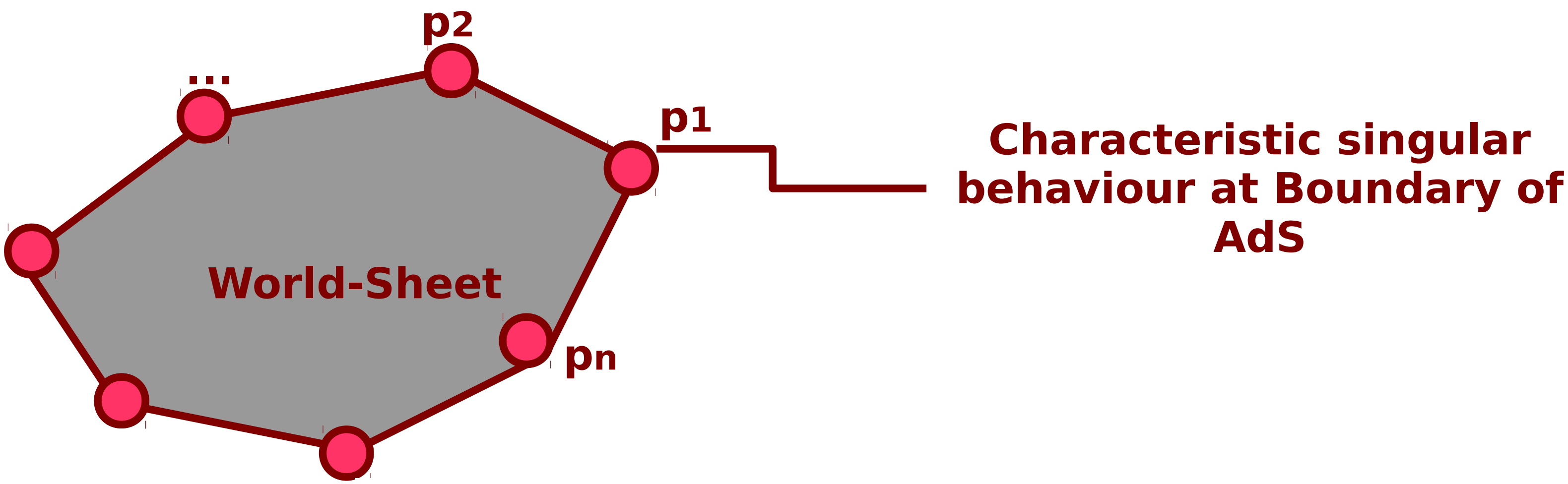
Alday, Maldacena
0705.0303



$$\frac{A}{A^{\text{tree}}} = e^{IR_{\text{div}} + f(\lambda) I(s_i) + Rem(s_i)}$$

$$f(\lambda) = \lambda + \mathcal{O}(\lambda^2)$$

Stringy dual picture



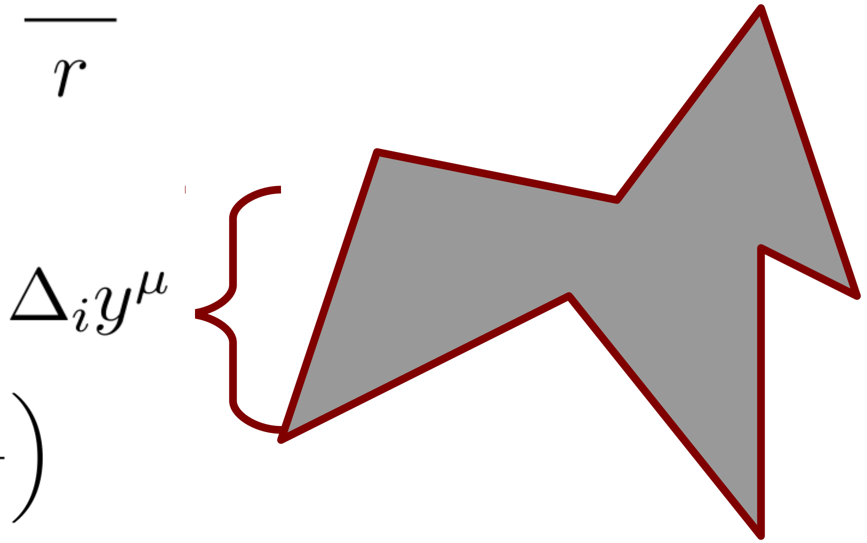
$$S_p = \frac{1}{2\pi}\int d^2\sigma\sqrt{h}h^{ab}g_{\mu\nu}\partial_a\mathcal{X}^\mu\partial_b\mathcal{X}^\nu$$

$$g_{\mu\nu}d\mathcal{X}^\mu d\mathcal{X}^\nu = R^2\left(\frac{dx_\mu dx^\mu + dr^2}{r^2}\right)$$

Stringy dual picture – T duality

$$\partial_a y^\mu = \frac{i}{r^2}\epsilon_{\alpha\beta}\partial_\beta x^\mu \qquad z = \frac{R^2}{r}$$

$$p_i^\mu = \frac{\Delta_i y^\mu}{2\pi}$$



$$g_{\mu\nu} d\mathcal{Y}^\mu d\mathcal{Y}^\nu = R^2 \left(\frac{dy_\mu dy^\mu + dz^2}{z^2}\right)$$

$$\mathcal{A} \sim e^{-\frac{R^2}{2\pi}\mathrm{Area}} = e^{-\frac{\sqrt{\lambda}}{2\pi}\mathrm{Area}}$$

Wilson-Loops/Scattering amplitudes

$$W[c_n] = \frac{1}{N^2}\langle Tr\mathcal{P}\exp\left(ig\int_{C_n} dx_\mu A^\mu(x)\right)\rangle$$

Segments $\Delta_i y^\mu$	⟷	Momenta p_i^μ
Light-like $(\Delta_i y)^2 = 0$	⟷	On-Shell $p_i^2 = 0$
Closed Polygon $\sum_i \Delta_i y^\mu = 0$	⟷	Momentum conservation $\sum_i p_i\mu = 0$
Conformal Symmetry	⟷	Dual Conformal Symmetry

Wilson-Loops

Henn, Plefka, Wiegandt arXiv:1004.0226

$$\langle W_4 \rangle_{\text{ABJM}} = \frac{1}{2N} \left\{ Tr e^{i \int_\gamma A_\mu \, dz^\mu} + Tr e^{i \int_\gamma \hat{A}_\mu \, dz^\mu} \right\}$$

- **1-Loop Vanishing (n=4 analytic, n=6 numerical)**

(analytical) → **Bicocca-Celoria 1103.3675**

- **2-Loop n=4**

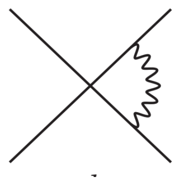
$$\langle W_4 \rangle^{(2)}_{\text{ABJM}} = \left(\frac{N}{k}\right)^2 \left[-\frac{(x_{13}^2 \, \mu'^2)^{2\epsilon}}{(2\epsilon)^2} - \frac{(x_{24}^2 \, \mu'^2)^{2\epsilon}}{(2\epsilon)^2} + \frac{1}{2} \ln^2 \left(\frac{x_{13}^2}{x_{24}^2} \right) + C \right]$$

$$C = \frac{\pi^2}{2} + 2 \ln 2 + 5 \ln^2 2 - \frac{a_6}{4} \qquad \mu'^2 = 8\pi e^{\gamma_E} \mu^2$$

Results: Amplitudes

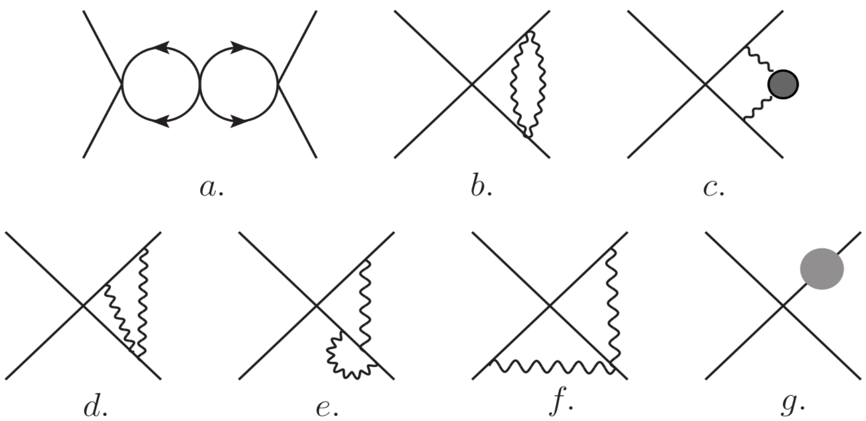
Bicocca-Celoria 1107.3139

- **1-loop n=4 chiral amplitude** $(A^i B_j A^k B_l)$



$\longrightarrow$ **0**

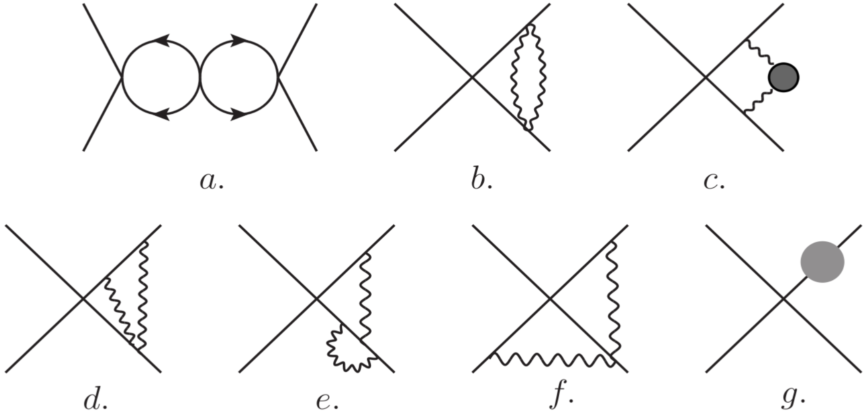
- **2-loop n=4 chiral amplitude**



Results: Amplitudes

Bicocca-Cleoria 1107.3139

2-loop n=4 chiral amplitude $(A^i B_j A^k B_l)$



$$\mathcal{M}^{(2)} \equiv \frac{\mathcal{A}_4^{(2\,loops)}}{\mathcal{A}_4^{tree}} = \lambda^2 \left[-\frac{(s/\mu'^2)^{-2\epsilon}}{(2\,\epsilon)^2} - \frac{(t/\mu'^2)^{-2\epsilon}}{(2\,\epsilon)^2} + \frac{1}{2} \log^2\left(\frac{s}{t}\right) + K_1 + \mathcal{O}(\epsilon) \right]$$

$$K_1 = 4\zeta_2 + 3\log^2 2$$

$$s = -x_{13}^2, t = -x_{24}^2$$

$$1/\mu'^2_{WL} = \mu'^2$$

$\mathcal{M}^{(2)} \equiv \langle W_4 \rangle^{(2)} +$ **Const**

Comments

- **Our Result permits to conjecture a BDS-like ansatz**

$$\frac{\mathcal{A}_4}{\mathcal{A}_4^{tree}} = e^{Div + \frac{f_{CS}(\lambda)}{8}\left(\ln^2\left(\frac{s}{t}\right) + \frac{4\pi^2}{3}\right) + C(\lambda)} \qquad f_{CS}(\lambda) = \frac{1}{2} f_{\mathcal{N}=4}(\lambda)\Big|_{\frac{\sqrt{\lambda}}{4\pi} \to h(\lambda)}$$

- **We derived two-loop WL/Amplitude match without assumptions.**

Assuming dual conformal invariance:

Chen, Huang, arXiv:1107.2710

- **We need to go to higher-loops and more scattered particles to check WL/Correlator duality for N=6**

More particles and superamplitudes

N=4 Superamplitudes

- **Maximally helicity violating gluons**

$$\mathcal{A}(++\ldots++)=0, \quad \mathcal{A}(++\ldots-\ldots++)=0$$

$$\mathcal{A}(++.i^-\ldots j^-..++)=i\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

$$p_{\alpha\dot{\alpha}}=p^\mu(\gamma_\mu)_{\alpha\dot{\alpha}}=\lambda_\alpha\bar{\lambda}_{\dot{\alpha}} \quad \langle 12\rangle=\lambda_1^\alpha\lambda_{2\alpha} \quad [12]=\bar{\lambda}_1^{\dot{\alpha}}\bar{\lambda}_{2\dot{\alpha}}$$

(Parke-Taylor 1986)

N=4 Superamplitudes

- **MHV gluons and superpartners**

$$\Phi(p,\eta) = G^+(p)+\eta^A\Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C\epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D\epsilon_{ABCD}G^-(p)$$

$$\mathcal{A}_n^{\mathrm{MHV};0} = i\frac{\delta^{(4)}(\sum_{i=1}^n \lambda_i^\alpha\tilde{\lambda}_i^{\dot{\alpha}})\ \delta^{(8)}(\sum_{i=1}^n \lambda_i^\alpha\eta_i^A)}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

Drummond et.al. 0807.1095

$$p_i^{\alpha\dot{\alpha}} = \lambda_i^\alpha\tilde{\lambda}_i^{\dot{\alpha}} = x_i^{\alpha\dot{\alpha}} - x_{i+1}^{\alpha\dot{\alpha}}\,, \qquad \lambda_i^\alpha\eta_i^A = \theta_i^{\alpha A} - \theta_{i+1}^{\alpha A}$$

N=4 Superamplitudes

- **Full Superamplitude**

$$\mathcal{A}_n = \mathcal{A}_n^{\rm MHV} + \mathcal{A}_n^{\rm NMHV} + \ldots + \mathcal{A}_n^{\rm N^{n-4}MHV}$$

$$\mathcal{A}_n^{\rm N^kMHV} = i(2\pi)^4 \frac{\delta^{(4)}(p^{\dot\alpha\alpha})\delta^{(8)}(q^A_\alpha)}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}\,\widehat{\mathcal{A}}_{n;k}(\lambda,\tilde\lambda,\eta;a)$$

$$\ln\widehat{\mathcal{A}}_{n;0} = \ln W_n + O(\varepsilon)\,, \qquad W_n = \frac{1}{N_c}\left\langle \mathrm{tr}\, P\exp\left(ig\int_{C_n} dx\cdot A(x)\right)\right\rangle$$

N=4 Superamplitudes

- **Super Wilson-loop**

$$\mathcal{W}_n = \frac{1}{N_c}\left\langle \mathrm{tr}\, P \exp\left(ig \int_{\mathcal{C}_n} dx^\mu \mathcal{A}_\mu(x,\theta) + ig \int_{\mathcal{C}_n} d\theta^{\alpha A} \mathcal{F}_{\alpha A}(x,\theta) \right) \right\rangle$$

$$\mathcal{W}_n = \mathcal{W}_{n;0}(x_i; a) + \mathcal{W}_{n;1}(x_i, \theta_i^A; a) + \ldots + \mathcal{W}_{n;n}(x_i, \theta_i^A; a)$$

$$\mathcal{W}_{n;k}(x,\theta;a) = a^k \widehat{\mathcal{A}}_{n;k}(\lambda, \tilde{\lambda}, \eta; a)$$

Caron-Huot 1010.1167

ABJM Superamplitudes

- **On-Shell Superfields**

$$\Phi(\Lambda) = \phi^4(\lambda) + \eta^A\,\psi_A(\lambda) + \frac{1}{2}\,\epsilon_{ABC}\,\eta^A\,\eta^B\,\phi^C(\lambda) + \frac{1}{3!}\,\eta^A\,\eta^B\,\eta^C\,\epsilon_{CBA}\,\psi_4(\lambda)$$

$$\bar{\Phi}(\Lambda) = \bar{\psi}^4(\lambda) + \eta^A\,\bar{\phi}_A(\lambda) + \frac{1}{2}\,\epsilon_{ABC}\,\eta^A\,\eta^B\,\bar{\psi}^C(\lambda) + \frac{1}{3!}\,\eta^A\,\eta^B\,\eta^C\,\epsilon_{CBA}\,\bar{\phi}_4(\lambda)$$

- **N=6 Superamplitudes**

$$\mathcal{A}_n(\Lambda_1,\cdots,\Lambda_n) = \delta^{(3)}(P)\,\delta^{(6)}(Q)\,\sum_{k=1}^{K} f_{n,k}\,F_{n,k}$$

ABJM Superamplitudes

- **Four-Point superamplitude**

$$\mathcal{A}_{4;l} = \delta^3(P)\delta^6(Q)\, f_l(\lambda)$$

$$\mathcal{A}_{4;0} = \frac{\delta^3(P)\delta^6(Q)}{\langle 21\rangle\langle 14\rangle} \qquad \mathcal{A}_{4;1} = 0$$

$$\mathcal{A}_{4;2} = \mathcal{A}_{4;0}\lambda^2 \left[-\frac{(s/\mu^2)^{-2\epsilon}}{(2\epsilon)^2} - \frac{(t/\mu^2)^{-2\epsilon}}{(2\epsilon)^2} + \frac{1}{2}\log^2(s/t) + C \right]$$

ABJM Superamplitudes

Bargheer et.al.
1003.6120

- **Six Point superamplitude**

$$\mathcal{A}_6^{(l)} = \delta^3(P)\,\delta^6(Q)\ \left[f^{+(l)}(\lambda)\,\delta^3(\alpha) + f^{-(l)}(\lambda)\,\delta^3(\beta)\right]$$

- **Constraints on** $f^{+(0)}(\lambda_i)$ and $f^{-(0)}(\lambda_i)$
- **Tree Level:**

$$\mathcal{A}_6^{(0)} = -\frac{\delta^3(P)\,\delta^6(Q)}{p_{123}^2}\left[\frac{\left(\epsilon_{ijk}\,\langle j\,k\rangle\,\eta_i^I + i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\,\langle \bar{j}\,\bar{k}\rangle\,\eta_{\bar{i}}^I\right)^3}{(\langle 1\,|\,p_{23}\,|\,4\rangle - i\,\langle 2\,3\rangle\,\langle 5\,6\rangle)(\langle 3\,|\,p_{12}\,|\,6\rangle - i\,\langle 1\,2\rangle\,\langle 4\,5\rangle)}\right.$$
$$\left. + \frac{\left(\epsilon_{ijk}\,\langle j\,k\rangle\,\eta_i^I - i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\,\langle \bar{j}\,\bar{k}\rangle\,\eta_{\bar{i}}^I\right)^3}{(\langle 1\,|\,p_{23}\,|\,4\rangle + i\,\langle 2\,3\rangle\,\langle 5\,6\rangle)(\langle 3\,|p_{12}\,|\,6\rangle + i\,\langle 1\,2\rangle\,\langle 4\,5\rangle)}\right]$$

ABJM Superamplitudes

Bicocca- Celoria 1204.4407
Bargheer et.al 1204.4406

- **1-loop**

$$\mathcal{A}_6^{(1)} = i\,\mathcal{C}(P)\,\frac{\delta^3(P)\,\delta^6(Q)}{p_{123}^2}\left[\frac{\left(\epsilon_{ijk}\,\langle j\,k\rangle\,\eta_i^I + i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\,\langle \bar{j}\,\bar{k}\rangle\,\eta_{\bar{i}}^I\right)^3}{\left(\langle 1\,|\,p_{23}\,|\,4\rangle - i\,\langle 2\,3\rangle\,\langle 5\,6\rangle\right)\left(\langle 3\,|p_{12}\,|\,6\rangle - i\,\langle 1\,2\rangle\,\langle 4\,5\rangle\right)} - \frac{\left(\epsilon_{ijk}\,\langle j\,k\rangle\,\eta_i^I - i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\,\langle \bar{j}\,\bar{k}\rangle\,\eta_{\bar{i}}^I\right)^3}{\left(\langle 1\,|\,p_{23}\,|\,4\rangle + i\,\langle 2\,3\rangle\,\langle 5\,6\rangle\right)\left(\langle 3\,|\,p_{12}\,|\,6\rangle + i\,\langle 1\,2\rangle\,\langle 4\,5\rangle\right)}\right]$$

$$\mathcal{C}(P) = \frac{\pi\lambda}{2}\left[\frac{\langle 12\rangle}{\sqrt{\langle 12\rangle^2}}\frac{\langle 34\rangle}{\sqrt{\langle 34\rangle^2}}\frac{\langle 56\rangle}{\sqrt{\langle 56\rangle^2}} + \frac{\langle 23\rangle}{\sqrt{\langle 23\rangle^2}}\frac{\langle 45\rangle}{\sqrt{\langle 45\rangle^2}}\frac{\langle 61\rangle}{\sqrt{\langle 61\rangle^2}}\right]$$

$$f^{+(1)} = -i\,\mathcal{C}(P)\,f^{+(0)} \quad \text{and} \quad f^{-(1)} = i\,\mathcal{C}(P)\,f^{-(0)}$$

ABJM Superamplitudes

- **The f functions at tree level** → **SCI, DSCI**

$$f^{\pm 1} \sim \mathcal{C}(P)\, f^{\pm 0}$$

- **3D** $$\frac{d}{dx} Sg(x) = 2\,\delta(x) \quad \rightarrow \quad \frac{\partial}{\partial \lambda_i^{\alpha}} Sg(\langle ij \rangle) = 2\lambda_{j\alpha}\,\delta(\langle ij \rangle)$$

$$\frac{\partial}{\partial \bar{z}} \frac{1}{z} = \pi\,\delta^2(z) \quad \rightarrow \quad \frac{\partial}{\partial \bar{\lambda}_i^{\dot{\alpha}}} \frac{1}{\langle ij \rangle} = \pi \bar{\lambda}_{j\dot{\alpha}} \delta^2(\langle ij \rangle)$$

- **4D**

Conclusions

- **Four- and Six-points amplitudes in ABJM**
- **Similarities and differences with N=4 SYM**
- **4 points match with bosonic Wilson-Loop**
- **Found 3D-Analogous “holomorphic” anomaly**
- **Super Wilson-Loop is needed**

Thank you!