Impact of QCD and SUSY-QCD Corrections on the Neutralino Dark Matter Relic Density.

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TR33 - Summer Institute: Particles and the Universe, Corfu

Overview

1. Interplay of particle and astro particle physics
2. Calculating the dark matter relic density
3. DM@NLO - an extension for numerical relic density calculators
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Constraining the SUSY parameter space

Neutralino - LSP and cold dark matter candidate

Particle physics bounds

- Collider searches
e.g.
\[ m_{\tilde{\chi}_1^0} > 46 \text{ GeV}, \quad m_{\tilde{t}} > 95.7 \text{ GeV} \]
PDG (2012)

- Precision measurements
e.g.
\[ \text{Br}(b \rightarrow s\gamma) = (3.55 \pm 0.26) \times 10^{-4} \]
HFAG collaboration (2010)

Cosmology bounds

- 7 year data of WMAP
\[ \Omega h^2 = 0.1123 \pm 0.035 \]
WMAP collaboration (2011)

Interplay between LHC and PLANCK
data will be even more interesting
Overview

1. Interplay of particle and astro particle physics

2. Calculating the dark matter relic density

3. DM@NLO - an extension for numerical relic density calculators
Calculating the relic density

Today’s number density can be calculated via Boltzmann equation.

\[
\dot{n} + 3Hn = -\langle \sigma v \rangle \left( n^2 - n_{eq}^2 \right)
\]

\( \sigma \) cross section of annihilation and coannihilation

\[
\langle \sigma v \rangle = \sum_{ij} \frac{2}{g_j} \left\langle \sigma_{ij} v_{ij} \frac{n_{eq}^i}{n_{eq}^j} \right\rangle \quad \text{with} \quad \frac{n_{eq}^i}{n_{eq}^j} \propto \exp \left[ -\frac{(m_i - m_\chi)}{T} \right]
\]

Coannihilation...
gets important, when masses of LSP and NLSP almost degenerate

Calculating the relic density

On the basis of the number density one can calculate the relic density.

\[ \Omega_{CDM} \ h^2 = \frac{m_\chi n_0}{\rho_C} \propto \frac{1}{\langle \sigma v \rangle} \]

Public computational programs (e.g.):

DarkSUSY
Gondolo, Edsjö, Ullio, Bergström, Bringmann et al. [astro-ph/0406204]

MicrOMEGAs
Bélanger, Boudjema, Pukhov et al. [hep-ph/1004.1092]

SuperIso Relic
Arbey, Mahmoudi [hep-ph/0906.0369]
Theoretical uncertainties in the relic density prediction

In cosmology
- Choice of cosmological model
- Variation in Hubble expansion rate

In particle physics
- Precision of masses
- Uncertainties of spectrum calculators
  Bélanger, Kraml, Pukhov (2005) [hep-ph/0502079]
- Precision in the calculation of (co)annihilation cross section

Current status in calculating the relic density:
- Calculation in public programs only on effective tree level
- Current theoretical uncertainties bigger than future precision of PLANCK
- Significant impact of NLO-corrections on the relic density
Current packages at next-to-leading order

SLOOPS
- electroweak corrections
  - Huge number of processes and NLO-diagrams
  - Automatized calculation
  - Interface to micrOMEGAs

http://code.sloops.free.fr

DM@NLO
- strong corrections
  - SUSY-QCD corrections to (co)annihilation processes
  - Efficient infrared treatment (dipole subtraction method)
  - Interface to micrOMEGAs (and DarkSUSY in the future)

http://dmnlo.hepforge.org
Overview

1. Interplay of particle and astro particle physics
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3. DM@NLO - an extension for numerical relic density calculators
**DM@NLO - relic density including QCD corrections**

- **SPheno**
  - Calculation of sparticle masses, mixings & precision observables
  - W. Porod.

- **MicrOMEGAs**
  - Integration of Boltzmann equation, etc.
  - G. Bélanger, F. Boudjema, A. Pukhov, A. Semenov.

- **CalcHEP**
  - Calculation of (co)annihilation processes at tree level
  - A. Pukhov, A. Belyaev, N. Christensen.

- **DM@NLO**
  - Calculation of cross sections at NLO level for relevant processes
  - J. Harz, B. Herrmann, M. Klasen, K. Kovarik, Q. Le Boulc’h.

**Input**
- Setting parameters for cMSSM, pMSSM, ...

**Output**
- Neutralino relic density

http://projects.hepforge.org/dmnlo
Impact of SUSY-QCD-correctors to annihilation

Example: Dominant Z-exchange

- Enhancement of annihilation cross section into quarks by 50% through QCD-corrections
- Reduction of the predicted relic density
- Significant shift of the WMAP favoured region

\[
\tan \beta = 10, \ A_0 = 0, \ m_0 = 1500, \ M_2 = 600, \ \mu > 0
\]


⇒ Effect of corrections to the relic density larger than current experimental uncertainties!

B. Herrmann, M. Klasen, K. Kovarik
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Impact of SUSY-QCD-corrections to annihilation

Extension: annihilation of other gauginos

- Especially relevant for wino- or higgsino-like neutralinos
- Realisation e.g. in pMSSM
- Generalisation of neutralino annihilation code
- Addition of few new contributions concerning charginos


B. Herrmann, M. Klasen, K. Kovarik, M. Meinecke (in progress).
Coannihilation with Squarks

- Contribution if squarks and neutralino almost degenerate
- Dominant in different regions of parameter space
- Extendable to coannihilation of gauginos with sleptons in general
Coannihilation with Squarks

Interesting scenario: pMSSM8

- 8 free parameters:
  - $\mu$, $\tan\beta$, $A_t$, $M_2$, $m_A$, $M_{\tilde{q}_{1,2}}$, $M_{\tilde{q}_3}$, $M_l$
  - $A_b = A_{\tau} = 0$
  - $M_2 = 2M_1 = M_3/3$

![Graph showing relative contributions of different particles to the dark matter relic density.](image)
Coannihilation with Squarks

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- 8 free parameters:
  - $\mu$, $\tan\beta$, $A_t$, $M_2$, $m_A$, $M_{\tilde{q}_{1,2}}$, $M_{\tilde{q}_3}$, $M_1$
  - $A_b = A_\tau = 0$
  - $M_2 = 2M_1 = M_3/3$
Coannihilation with Squarks

One example point in pMSSM8

- $\mu = 421$, $\tan\beta = 14$, $T_t = 1017$
- $m_A = 928$, $M_{\tilde{q}_{1,2}} = 434$, $M_{\tilde{q}_3} = 388$
- $M_{\tilde{t}} = 627$
- $A_b = A_\tau = 0$
- $M_2 = 2M_1 = M_3/3 = 325$

Fully dominated by coannihilation processes:

- $\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow bW^+ : 29\%$
- $\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t\tau : 26\%$
- $\tilde{t}_1 \tilde{t}_1 \rightarrow gg : 15\%$
- $\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow tZ : 8\%$
- $\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow th : 6\%$

In agreement with experimental bounds:

- $m_h = 125.4\, \text{GeV}$
- $\Omega h^2 = 0.1159$

$\Rightarrow$ Precise relic density prediction needs next-to-leading order corrections for coannihilation
Full NLO calculation for coannihilation with Squarks

J.H., B. Herrmann, M. Klasen, K. Kovarik, Q. Le Bouc'h (in progress)
Some technicalities

Handling of IR-divergencies - I

- Phase-space-slicing

\[ \sigma^{NLO} = \int_{2\to3} d\sigma^R + \int_{2\to2} d\sigma^V \]

\[ \sigma^{\text{brems}}_{2\to3}(\lambda) = \sigma^{\text{soft}}(\Delta E, \lambda) + \sigma^{\text{hard}}(\Delta E) \]

\[ \mathcal{M} = \frac{i(2p - k)^\mu \varepsilon_\mu(k)}{(p - k)^2 - m^2} (-ig_s T^a) A_0(p - k) \]

\[ \mathcal{M} = - (g_s T^a) \frac{\varepsilon.p}{p.k} \mathcal{M}_0 \quad \text{with} \quad k^\mu \ll p^\mu \]

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{soft}} = - \left( \frac{d\sigma}{d\Omega} \right)_0 \times \frac{g_s^2 C_F}{(2\pi)^3} \int_{|\vec{k}| \leq \Delta E} \frac{d^3 k}{2\omega} \frac{-2k_1.p_2}{(p_2.k)(k_1.k)} \]
Some technicalities

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\]

\[\tilde{\chi}_1^0 \to t + h\]

\[\tilde{\chi}_2^0 \to t + h\]

\[\tilde{t}_1 \to g + t\]

Julia Harz

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Some technicalities

Handling of IR-divergencies - I

- Phase-space-slicing

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\sigma^{NLO} = \int_{2\rightarrow3} d\sigma^R + \int_{2\rightarrow2} d\sigma^V
\]

\[
\sigma_2^{\text{Brems}}(\lambda) = \sigma^{\text{soft}}(\Delta E, \lambda) + \sigma^{\text{hard}}(\Delta E)
\]

\[
\mathcal{M} = \frac{i(2p - k)^\mu \varepsilon_\mu(k)}{(p - k)^2 - m^2} (-ig_s T^a) A_0(p - k)
\]

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\mathcal{M} = -\left(g_s T^a\right) \frac{\varepsilon.p}{p.k} \mathcal{M}_0 \quad \text{with} \quad k^\mu \ll p^\mu
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\]
Some technicalities

Handling of IR-divergencies - I

- Phase-space-slicing

![Graph showing the impact of SUSY-QCD Corrections on the Dark Matter Relic Density](image-url)
Some technicalities

Handling of IR-divergencies - I

- Phase-space-slicing

![Graph](image-url)
Some technicalities

Handling of IR-divergencies - II

- Dipole-Subtraction-formalism

\[
\sigma^{NLO} = \int_{2 \to 3} \left[ d\sigma^R - d\sigma^A \right] |_{\epsilon=0} + \int_{2 \to 2} \left[ d\sigma^V + \int_1 d\sigma^A \right] |_{\epsilon=0}
\]

Handling of UV-divergencies

- input parameters:
  \[m_{b}^{\overline{MS}}, m_{t}^{OS}, A_b^{\overline{DR}}, A_t^{\overline{DR}}, m_{b_1}^{OS}, m_{b_2}^{OS}, m_{t_1}^{OS}\]

\[
\begin{pmatrix}
M_Q^2 + m_q^2 + M_Z^2 c_{2\beta} (T_q^3 - Q_q s_w^2) & m_q (A_q - \mu^* \kappa) \\
q (A_q - \mu \kappa) & M_Q^2 + m_q^2 + M_Z^2 c_{2\beta} Q_q s_w^2
\end{pmatrix}
= \mathcal{U}_q \begin{pmatrix}
m_{q_1}^2 & 0 \\
0 & m_{q_2}^2
\end{pmatrix} \mathcal{U}_q^\dagger
\]

- consistent renormalization scheme for all (co-)annihilation processes
Impact of SUSY-QCD-corrections to coannihilation

Example scenario: cMSSM

\[ \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t h \quad 34.3 \% \]
\[ \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t g \quad 16.1 \% \]
\[ \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow b W^+ \quad 3.4 \% \]
\[ \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t Z \quad 1.4 \% \]
\[ \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t \gamma \quad 0.4 \% \]

For \( \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t h @ p_{cm} = 50 \text{ GeV} \): 
\( \rightarrow \) relative correction: -14.8 \%
Impact of SUSY-QCD-corrections to coannihilation

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For \( \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow th \) @ \( p_{cm} = 50 \text{ GeV} \):
\( \rightarrow \) relative correction: -14.8 %

For \( \tilde{\chi}_1^0 \tilde{t}_1 \rightarrow tH \) @ \( p_{cm} = 1000 \text{ GeV} \):
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Impact of SUSY-QCD-corrections to coannihilation

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\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow bW^+ \quad 3.4 \%
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\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow tZ \quad 1.4 \%
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\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t\gamma \quad 0.4 \%
\]

For \(\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t h \) @ \(p_{cm} = 50 \text{ GeV}\):
→ relative correction: -14.8 %

For \(\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t H \) @ \(p_{cm} = 1000 \text{ GeV}\):
→ relative correction: -32.0 %

For \(\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t Z \) @ \(p_{cm} = 50 \text{ GeV}\):
→ relative correction: -2.2 %

\(\Rightarrow\) NLO-corrections to coannihilation have significant impact on cross section
Consequences on the relic density

- huge impact on relic density through
  \[ \Omega_{CDM} \, h^2 = \frac{m_\chi \, n_0}{\rho_C} \propto \frac{1}{\langle \sigma v \rangle} \]

- vector boson and gluon final states will be soon finished

- similar effects up to 50% through corrections expected

⇒ SUSY-QCD-corrections to coannihilation will have sizeable impact on relic density prediction!

Conclusion and Outlook

- Public codes do not take into account full NLO corrections
- **DM@NLO** will contain SUSY-QCD corrections to Gaugino annihilation and Neutralino-Squark coannihilation
- corrections up to 50% expected and thus a huge impact on the relic density prediction
- Package **DM@NLO** allows to link SUSY-QCD corrections to the public codes

→ Code will be public available!

http://dmnlo.hepforge.org