Non-perturbative aspects of gauge/gravity duality

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Corfu, September 24th, 2012
XVIII European Workshop on String Theory

Talk based on:
Billò, Frau, Fucito, L.G., Lerda, Morales, Ricci Pacifici,
arXiv 1206.3914
Billò, Frau, L.G., Lerda,
arXiv 1105.1869, 1201.4231

Introduction

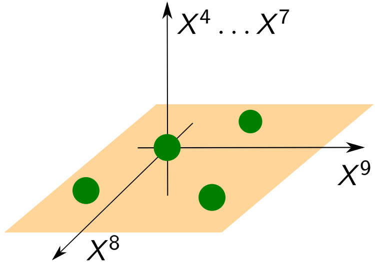
- Gauge/gravity correspondence analyzed through dual open/closed string description of D-branes
- Long ago, gravitational solutions dual to $\mathcal{N} = 2$ gauge theories realized through fractional branes on orbifolds were found at the perturbative level [Bertolini *et al.* '01, Polchinski '01, ...]
- Non-perturbative (D-instanton) effects are needed to remove singularities
- Recently the exact axio-dilaton profile in an orientifold model has been microscopically derived [Billò *et al.* '11, '12]
- Here the same program is pursued for the twisted scalar emitted by fractional branes in an orbifold background

Outline

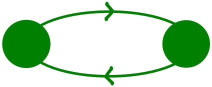
Brane Setup

Type IIB string theory on $\mathbb{Z}_2$ orbifold background

$$\mathbb{R}^4 \times \mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{C}$$



- Consider (N_0, N_1) fractional D3 branes sitting at the orbifold fixed point
- Their 4d worldvolume supports $\mathcal{N} = 2$ $SU(N_0) \times SU(N_1)$ quiver theory



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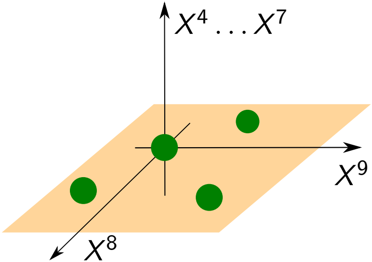

- Adjoint vector multiplet

$$\Phi = \phi + \theta\lambda + \frac{1}{2}\,\theta\gamma^{\mu\nu}\theta\, F_{\mu\nu} + \cdots\,, \quad \Phi = \begin{pmatrix} \Phi_0 & 0 \\ 0 & \Phi_1 \end{pmatrix}$$

- Two bifundamental hypers
- $b_0 = -2(N_0 - N_1), \quad b_1 = -2(N_1 - N_0)$

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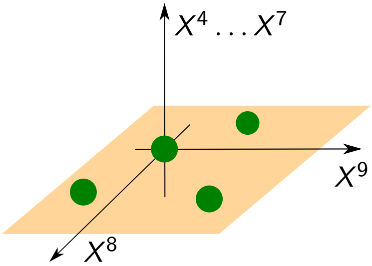
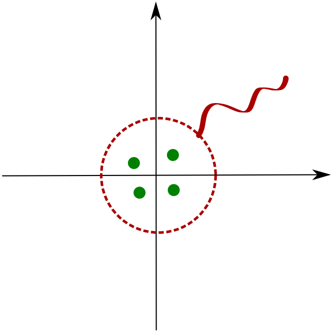


- Microscopic analysis can be done in the full quiver theory
- To simplify the following discussion, discard dynamics on $D3_1$ branes and set $N_0 = N_1 = N$
  $\Rightarrow$ Conformal $\mathcal{N} = 2$ $SU(N)$ gauge theory with $2N$ flavours

The twisted scalar

Branes are sources for closed string fields ⇒ classical solutions for sugra



Untwisted sector

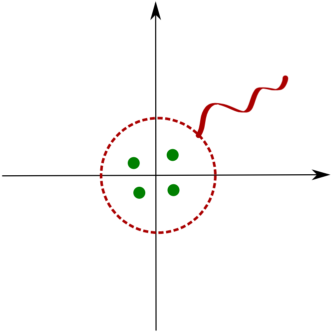
- Non-trivial metric and F_5
- Constant axio-dilaton $C_0 + \mathrm{i}e^{-\varphi} \to \frac{\mathrm{i}}{g_s}$

The twisted scalar

Branes are sources for closed string fields ⇒ classical solutions for sugra

- In the twisted sector, two scalars from NS and R sector b, c complexified into a holomorphic field

$$t = c + \frac{\mathrm{i}}{g_s} b$$



- Lowest component of chiral superfield

$$T = t + \ldots + \theta^4 \frac{\partial^2}{\partial z^2} \bar{t} + \ldots$$

The classical t profile

Consider the worldvolume action of fractional D3's

$$S_{\mathrm{D3}} \propto (N_0 - N_1) \int d^6x \, \bar{t} \, \delta^2(z) + \ldots + \int d^4x \, t \, \mathrm{tr}\, F^2 + \ldots$$

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- The twisted scalar is classically the gauge coupling on fractional D3 branes

$$t_{\mathrm{sugra}} = \tau_{\mathrm{gauge}} = \frac{\theta_{YM}}{\pi} + \mathrm{i}\frac{8\pi}{g_{YM}^2}$$

- S_{D3} generates source terms in the e.o.m. for t

$$\Box t + \frac{\delta}{\delta \bar{t}} S_{\mathrm{D3}} = 0$$

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$$\Box t + \frac{\delta}{\delta \bar{t}} S_{\mathrm{D3}} = 0$$

- For $N_0 = N_1$ and all the branes sitting at the origin, t is constant

$$\mathrm{i}\pi t(z) = \mathrm{i}\pi t_0$$

The classical t profile

- When branes are away from the origin, scalars get non-vanishing vevs

$$\langle\phi\rangle = \mathrm{diag}\,(a_1, \cdots, a_N), \quad \langle m\rangle = \mathrm{diag}\,(m_1, \cdots m_N)$$

- Conformal symmetry broken $\Rightarrow$ non-trivial profile for t

$$\Rightarrow \mathrm{i}\pi t(z) = \mathrm{i}\pi t_0 - 2\,\mathrm{tr}\log\frac{z-\langle\phi\rangle}{\mu} + 2\,\mathrm{tr}\log\frac{z-\langle m\rangle}{\mu}$$

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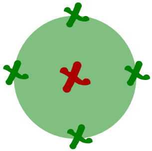
- Non-trivial source in e.o.m. for t

$$\Box t = J_{\mathrm{cl}} \delta^2(z)$$

- J_{cl} is encoded in effective action for massless open string fields

The classical source and prepotential

- Compute disk diagrams involving l adjoint scalars on the boundary and a twisted scalar, e.g. b, in the bulk



$$\sum_{l=0}^{\infty} \frac{1}{l!} \langle \underbrace{V_\phi \dots V_\phi}_{l} V_b \rangle_{\mathrm{D3}_0} = \frac{\pi}{g_s} \sum_{l=0}^{\infty} \frac{1}{l!} \mathrm{tr}\langle \phi \rangle^l (\mathrm{i}\bar{p})^l b$$

  - Taking into account c and the susy completion yields the linear part of the classical prepotential

$$F_{\mathrm{cl}} = \mathrm{i}\pi \sum_{l=1}^{\infty} \frac{(\mathrm{i}\bar{p})^l}{l!} \left(\mathrm{tr}\langle \Phi \rangle^l - \mathrm{tr}\langle M \rangle^l \right) \frac{T}{\bar{p}^2} + \cdots$$

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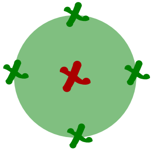

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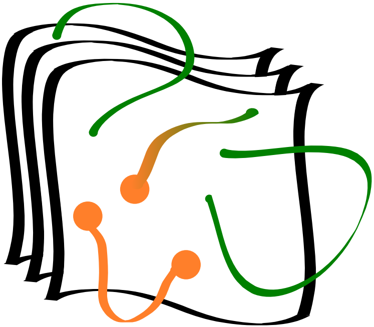
  - Integration over fermionic superspace variables yields the classical source through

$$J_{\mathrm{cl}}=\frac{\bar{p}^2}{\pi}\frac{\delta F_{\mathrm{cl}}}{\delta T}\bigg|_0=\sum_{l=1}^{\infty}\frac{\mathrm{i}}{l!}\Big(\mathrm{tr}\langle\phi\rangle^l-\mathrm{tr}\langle m\rangle^l\Big)$$

D(−1) branes as instantons

- The exact t profile gets contributions from non-perturbative effects on the source D3 branes
- Instantonic configurations of the gauge theory are realized by adding k D(−1) branes on top of the D3's [Douglas '95, Witten '95, Green and Gutperle '00, Billò *et al.* '02, . . .]



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- Physical excitations of $-1/-1$ and $-1/3$ strings correspond to instanton moduli

(ϕ, ψ)	$U(k) \times SU(N_0)_g \times SU(N_1)_f$
$(a^\mu, M^\mu = M^{\alpha\dot{a}})$	(adj, 1, 1)
$(\bar{\chi}, \eta = \epsilon_{\dot{\alpha}\dot{a}}\lambda^{\dot{\alpha}\dot{a}})$	(adj, 1, 1)
$(\eta^c = (\tau^c)_{\dot{\alpha}\dot{a}}\lambda^{\dot{\alpha}\dot{a}}, D^c)$	(adj, 1, 1)
$(w_{\dot{\alpha}}, \mu_{\dot{a}})$	(k, $\bar{N}$, 1)+h.c.
(μ'_a, h_a)	(k, 1, $\bar{N}$)+h.c.

Non-perturbative corrections

- The non-perturbative source can be extracted from the non-perturbative prepotential

$$F_{\text{n.p.}} = \sum_k \int d\widehat{\mathcal{M}}_k \mathrm{e}^{-S_{\text{inst}}(\mathcal{M}_k, \Phi, T)}$$

- $S_{\text{inst}}(\mathcal{M}_k, \Phi, T)$ computed by D(-1) diagrams with insertions of T and moduli

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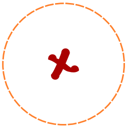
- Simplest diagrams are k D(-1) disks with only t inserted, corresponding to classical instanton action

$$S_{\text{cl}} = -\mathrm{i}\pi k t$$



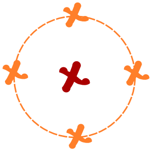
- S_{cl} weighs k-instanton contribution to $F_{\text{n.p.}}$

$$\mathrm{e}^{-S_{\text{cl}}} = q^k, \quad q = \mathrm{e}^{\mathrm{i}\pi t}$$

Moduli interactions

- Relevant contributions to S_{inst} come from diagrams with the insertion of e.g. b and l χ moduli

$$\sum_{l=0}^{\infty} \frac{1}{l!} \langle \underbrace{V_\chi \ldots V_\chi}_{l} V_b \rangle_{\mathrm{D}(-1)_0}$$
$$= -\frac{\pi}{g_s} \sum_{l=0}^{\infty} \frac{1}{l!} \mathrm{tr}_k\, \chi^l (\mathrm{i}\bar{p})^l b$$



- This gives a linear non-perturbative prepotential

$$F_{\text{n.p.}} = \mathrm{i}\pi T \sum_k q^k \int d\widehat{\mathcal{M}}_k \mathrm{e}^{-S'_{\text{inst}}(\mathcal{M}_k,\Phi)} \sum_{l=1}^{\infty} \frac{1}{l!} \mathrm{tr}_k\, \chi^l (\mathrm{i}\bar{p})^l + \ldots$$

The exact t profile

- The integrals over moduli space appearing in $F_{\text{n.p.}}$ compute the k-th instanton contribution to the $(l+2)$-th element $\langle \operatorname{tr} \phi^{l+2} \rangle$ of the chiral ring of the gauge theory:

$$F_{\text{n.p.}} = \mathrm{i}\pi \sum_{l=1}^{\infty} \frac{(\mathrm{i}\bar{p})^l}{l!} \langle \operatorname{tr} \Phi^l \rangle_{\text{n.p.}} \frac{T}{\bar{p}^2} + \dots$$

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- The inclusion of instantonic corrections thus amounts to promote classical vevs to full quantum vevs in the source

$$\operatorname{tr}\langle\phi^n\rangle \Rightarrow \langle \operatorname{tr}\phi^n\rangle$$

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- The inclusion of instantonic corrections thus amounts to promote classical vevs to full quantum vevs in the source

$$\Rightarrow J = \frac{\bar{p}^2}{\pi}\frac{\delta F}{\delta T}\bigg|_0 = \mathrm{i}\sum_{l=1}^{\infty}\frac{(\mathrm{i}\pi)^l}{l!}\Big(\langle \operatorname{tr}\phi^l\rangle - \operatorname{tr}\langle m\rangle^l\Big)$$

yielding the exact t profile

$$\mathrm{i}\pi t(z) = \mathrm{i}\pi t_0 - 2\Big\langle \operatorname{tr}\log\frac{z-\phi}{\mu}\Big\rangle + 2\operatorname{tr}\log\frac{z-\langle m\rangle}{\mu}$$

The exact t profile

- Exact description from the SW curve for the D3 gauge theory [Fucito *et al.* '11]

$$y^2 = P(z)^2 - g^2 Q(z),$$

$$P = \prod_{i=1}^{N}(z - e_i),\, Q = \prod_{k=1}^{N}(z - m_k)^2,\, g^2 = \frac{4q}{(1+q)^2}$$

- The correlator appearing in the t profile is computable from the SW curve

$$\left\langle \operatorname{tr}\log\frac{z-\phi}{\mu}\right\rangle = \log\frac{P(z)+\sqrt{P(z)^2-g^2Q(z)}}{\mu^N(1+\sqrt{1-g^2})}$$

- Exact t profile emitted from brane system (agrees with [Witten '97, Cremonesi '09])

$$\pi \mathrm{i} t(z) = \log\frac{P(z)-\sqrt{P^2(z)-g^2Q(z)}}{P(z)+\sqrt{P^2(z)-g^2Q(z)}}$$

t and gauge couplings

- Focus on special vacuum of gauge theory having only one scale left $\mathbf{v} = \langle \mathrm{tr}\, \phi^N \rangle$ [Argyres and Pelland '00]
- Corresponds to symmetric arrangement of D3's

$$a_i = a\omega^{i-1}, \quad m_i = m\omega^{i-1}, \quad \omega = \mathrm{e}^{2\mathrm{i}\pi/N}$$

- On the gravity side, evaluating t with all invariants set to zero leaves only one scale, z
- How is the twisted scalar evaluated at z related to the gauge coupling on this slice of moduli space?

t and gauge couplings

- In the massless case t is constant

$$\mathrm{i}\pi t(z)\big|_{\mathbf{v}=0} = \log q = \mathrm{i}\pi t_0$$

- But the effective gauge coupling gets corrections even in conformal case $\Rightarrow$ t can't simply be τ
- Classically, the matrix of couplings takes the simple form

$$2\pi\mathrm{i}\,\tau^{ij}_{\mathrm{tree}} = \begin{pmatrix} 2 & 1 & 1 & \dots \\ 1 & 2 & 1 & \dots \\ 1 & 1 & 2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \log q_0$$

- At the quantum level, additional matrix structures emerge for $N > 3$

t and τ for $SU(2)$

- From explicit instanton computations, UV t_0 and IR τ couplings for $m = 0$ are related by

$$\mathrm{i}\pi\tau = \log q_0 + \mathrm{i}\pi - \log 16 + \frac{1}{2} q_0 + \frac{13}{64} q_0^2 + \dots$$

- Inverting it,

$$q_0 = \mathrm{e}^{\mathrm{i}\pi t_0} = -16\left(\mathrm{e}^{\mathrm{i}\pi\tau} + 8\mathrm{e}^{2\mathrm{i}\pi\tau} + \dots\right) = -16\frac{\eta^8(4\tau)}{\eta^8(\tau)}$$

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- When $m \neq 0$, substituting t_0 with the non-constant $t(z)$, the equation still holds for the corresponding $\tau(\mathbf{v})$ after identyfing $z^2 \leftrightarrow \mathbf{v}$
- Still true for non-conformal cases reached by decoupling some flavours

t and τ for $SU(3)$

- For $SU(3)$, the matrix of couplings retains its form at the exact level

$$2\pi\mathrm{i}\,\tau^{ij}_{\mathrm{SU}(3)} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \pi\mathrm{i}\,\tau \ .$$

- After identifying $z^3 \leftrightarrow \mathbf{v}$, t and τ related by

$$\mathrm{e}^{\mathrm{i}\pi t} = -27\left(\mathrm{e}^{\mathrm{i}\pi\tau} + 12\mathrm{e}^{2\mathrm{i}\pi\tau} + \dots\right) = -27\frac{\eta^{12}(3\tau)}{\eta^{12}(\tau)}$$

t and τ for $SU(4)$

- For $SU(4)$ 1-loop and instantonic corrections spoil classical matrix structure

$$2\pi\mathrm{i}\,\tau^{ij}_{\mathrm{SU}(4)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \pi\mathrm{i}\frac{\tau_+ + \tau_-}{2} + \begin{pmatrix} 0 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & 0 \end{pmatrix} \pi\mathrm{i}\frac{\tau_+ - \tau_-}{2}$$

- Again t and τ_+, τ_- related through modular functions after identifying $z^4 \leftrightarrow \mathbf{v}$

$$\mathrm{e}^{\mathrm{i}\pi t} = -16\left(\mathrm{e}^{\mathrm{i}\pi\tau_+} + 8\,\mathrm{e}^{2\mathrm{i}\pi\tau_+} + \ldots\right) = -16\,\frac{\eta^8(4\tau_+)}{\eta^8(\tau_+)}\,,$$

$$\mathrm{e}^{\mathrm{i}\pi t} = -64\left(\mathrm{e}^{\mathrm{i}\pi\tau_-} + 24\,\mathrm{e}^{2\mathrm{i}\pi\tau_-} + \ldots\right) = -64\,\frac{\eta^{24}(2\tau_-)}{\eta^{24}(\tau_-)}\,.$$

t and τ from the SW curves

- The SW curve for the massless conformal theory is

$$Y^2 = (X^N + 1)^2 - g^2 X^{2N}, \; g^2 = \frac{4q_0}{(1+q_0)^2}$$

- Making the substitution $q_0 \to q(\frac{m^N}{\mathbf{v}}, q_0)$ and changing variables one recovers the SW curve for the massive theory with UV coupling q_0

$$y^2 = (x^N + u^N)^2 - g^2 (x^N - m^N)^2$$

- The massive couplings can thus be related to massless ones by

$$\tau^{ij}\left(\frac{m^N}{\mathbf{v}}, q_0\right) = \tau^{ij}\left(0, q\left(\frac{m^N}{\mathbf{v}}, q_0\right)\right)$$

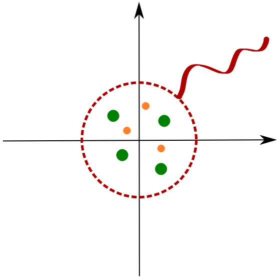
Conclusions

- We gave an explicit microscopic derivation of the exact twisted scalar profile through emission from D(−1)/D3 disks: the supergravity solution is expressed in terms of quantum correlators of the gauge (quiver) theory

$$\mathrm{tr}\langle \phi^n \rangle \Rightarrow \langle \mathrm{tr}\, \phi^n \rangle$$

- We found non-trivial relationships between the twisted scalar profile and the effective gauge couplings of the gauge theory, involving modular functions and showing

$$t_{\mathrm{sugra}} \neq \tau_{\mathrm{gauge}}$$



Thanks for your attention!