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EXTRA DIMENSIONS

I and II

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Outline

- Generalities

- extra dimensions and Kaluza-Klein unification

- chirality, orbifolds

- I TeV size extra dimensions

- experimental signatures

- universal extra dimensions

- non-universal extra dimensions

- II mm dimensions and the ADD scenario

- D-branes (Kiritsis lectures ?) and Brane Universes

- ADD scenario and TeV scale strings
- Experimental signatures: graviton emission, virtual graviton exchange
- Black holes production at LHC ?
- Neutrino masses

- III Randall-Sundrum models, warped compactifications
  - Holography and warped compactifications (Santamaria)
  - Bulk fields, composite Higgs (G. Ross lectures ?)
  - Experimental signatures

- Conclusions, Prospects

GENERALITIES

- Extra dimensions and Kaluza-Klein unification

The electromagnetic interaction and gravity can be described in a unified manner starting from Einstein gravity in $4 + 1$ dimensions (Nordstrom,1914;Kaluza,1921)

five dimensions	four dimensions		
g_{MN}	$g_{\mu\nu}$,	$A_\mu = g_{\mu 5}$,	$g_{55} = \Phi^{2/3}$
(graviton)	(graviton)	photon	scalar

More precisely,

$$ds^2 = \Phi^{-1/3}(g_{\mu\nu} - \Phi A_\mu A_\nu) dx^\mu dx^\nu - 2\Phi^{2/3} A_\mu dx^\mu dy - \Phi^{2/3} dy^2$$

For y-independent fields, we find the 4d action

$$-\frac{M_5^3}{2}\int d^5x\sqrt{G}\mathcal{R}_5 = -\frac{M_4^2}{2}\int d^4x\sqrt{g}\left[\mathcal{R}_4 + \frac{1}{4}\Phi F_{\mu\nu}F^{\mu\nu} - \frac{\partial_\mu\Phi\partial^\mu\Phi}{6\Phi^2}\right]$$

There are two problems:

• if the new space dimension is infinite, the gravity attractive force is

$F \sim \frac{m_1 m_2}{r^3}$ instead of $F \sim \frac{m_1 m_2}{r^2} \Rightarrow$ needs to compactify the extra dimension

• There is a massless dilaton-like fields Φ; needs to give it a mass $\Rightarrow$ moduli stabilization.

$\longrightarrow$ the new dimension y has to be compact (Klein,1926) (ex. circle of radius R) and small

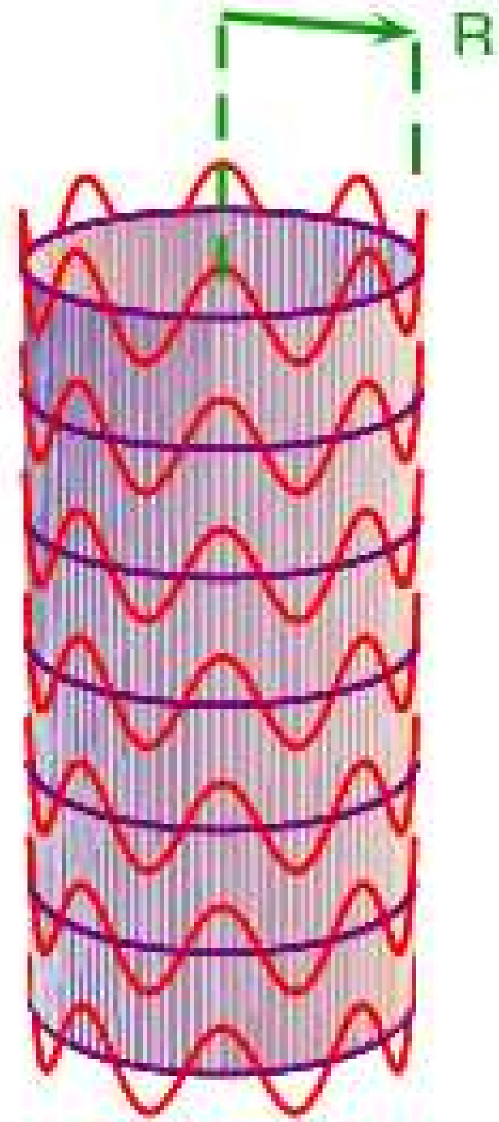
The observed particles are vibrational modes $\phi^{(n)}$

$$\phi(x^\mu, y) \;=\; \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{\frac{iny}{R}} \, \phi^{(n)}(x^\mu)$$

Same quantum numbers (charge,spin, etc) as ordinary particles $\phi^{(0)}$. Mass

$$M_n^2 = m_0^2 + \frac{n^2}{R^2}$$

We did not see KK photons or KK electrons $\rightarrow$ typically only dimensions $R < 10^{-17} cm$ $(M_1 \geq TeV)$ are allowed.


R

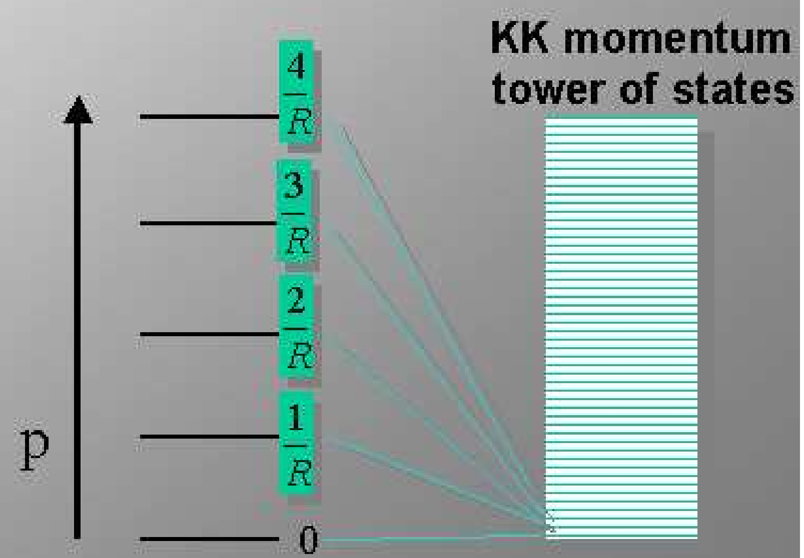
Kaluza-Klein modes

if spatial dimension is compact
then momentum in that
dimension is quantized:

$$p = \frac{n}{R}$$

from our point of view we see new massive particles!

$$m^2 = m_0^2 + \frac{n^2}{R^2}$$



Are fundamental scale and compactification radius experimentally accessible ?

Start with a 5d theory containing graviton + 5d Yang-Mills

$$\int d^4x \; dy \left(M_5^3 \mathcal{R} - \frac{1}{g_5^2} F_{MN}^2 \right) \; ,$$

where M_5 is the 5d Planck scale and g_5 is the 5d (dimensionfull) gauge coupling. The 4d parameters are

$$M_P^2 \; = \; R \; M_5^3 \quad , \quad \frac{1}{g_4^2} \; = \; \frac{R}{g_5^2} \sim R M_5$$

where we used the natural value of 5d coupling

$$g_5^2 \sim M_5^{-1}$$

We find therefore

$$M_P^2 \; = \; \frac{1}{g_4^2} M_5^2 \quad , \quad R^{-1} \sim g_4^3 M_P$$

which imply $M_5 \lesssim M_P$. We cannot get $M_5 \sim$ TeV in this standard KK setup.

• The natural compactification scale $R^{-1} \sim M_P^{-1} \sim 10^{-33}$ cm ! No hope to discover KK states at colliders.

The way out is to have gravity propagating in higher dimensions than the gauge fields

$\Rightarrow$ brane worlds (2nd lecture).

The KK field theory is valid for

$$M_c \equiv R^{-1} \ll M_5 \quad \text{and for} \quad E \ll M_5$$

- Chirality, Orbifolds

• Toroidal compactifications of KK theories to 4d give rise to non-chiral fermions.

Ex : there is no chirality in 5d. "Minimal" fermion is

$$\Psi(y, \mathbf{x}) = \frac{1}{\sqrt{2\pi R}} \begin{pmatrix} \sum_{k=-\infty}^{\infty} e^{\frac{iky}{R}} \psi_1^{(k)}(\mathbf{x}) \\ \sum_{k=-\infty}^{\infty} e^{\frac{iky}{R}} \bar{\psi}_2^{(k)}(\mathbf{x}) \end{pmatrix} ,$$

in a Weyl basis. From a 4d perspective, Ψ is a collection of Dirac fermions

$$\Psi^{(k)} = \begin{pmatrix} \psi_1^{(k)} \\ \bar{\psi}_2^{(k)} \end{pmatrix} , \text{ of mass } \frac{k}{R} .$$

In particular the massless mode $\Psi^{(0)}$ is non-chiral.

Orbifolds = A simple way of producing fermion chirality (Dixon,Harvey,Vafa,Witten, 86)

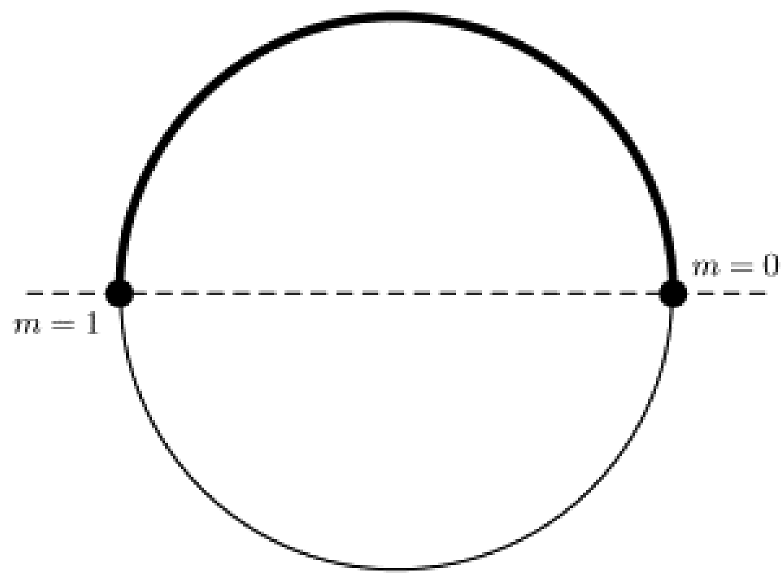
• A d-dimensional orbifold O^d is a d-dimensional torus T^d with identified points as

$$O^d \ = T^d/P \quad ,$$

where P is a discrete Z_N identification.

Popular field theory example : S^1/Z_2 orbifold $\Leftrightarrow$ an interval. This is the circle $y = y + 2\pi R$ with the identification $y = -y$.

There are two fixed points $y = 0$ and $y = \pi R$.



Physical states are even or odd :

$$\phi_e(-y,\mathbf{x}) = \phi_e(y,\mathbf{x}) \;\Rightarrow\; \phi_e(y,\mathbf{x}) = \frac{1}{\sqrt{2^{\delta_{k0}}\pi R}}\sum_{k=0}^{\infty}\cos(\frac{ky}{R})\phi_e^{(k)}(\mathbf{x}),$$

$$\phi_o(-y,\mathbf{x}) = -\phi_o(y,\mathbf{x}) \;\Rightarrow\; \phi_o(y,\mathbf{x}) = \frac{1}{\sqrt{\pi R}}\sum_{k=1}^{\infty}\sin(\frac{ky}{R})\phi_o^{(k)}(\mathbf{x})$$

Orbifold reduction of a fermion

$$\Psi(-y,\mathbf{x}) \;=\; \gamma_5\;\Psi(y,\mathbf{x})\;,$$

implies a **chiral** reduction

$$\Psi(y,\mathbf{x}) \;=\; \begin{pmatrix} \frac{1}{\sqrt{2^{\delta_{k0}}\pi R}}\sum_{k=0}^{\infty}\cos(\frac{ky}{R})\psi_1^{(k)}(\mathbf{x}) \\ \frac{1}{\sqrt{\pi R}}\sum_{k=1}^{\infty}\sin(\frac{ky}{R})\bar{\psi}_2^{(k)}(\mathbf{x}) \end{pmatrix}\;.$$

Massless fermion $\psi_1^{(0)}$ chiral. Massive modes non-chiral.

There are two ways of looking at S^1/Z_2:

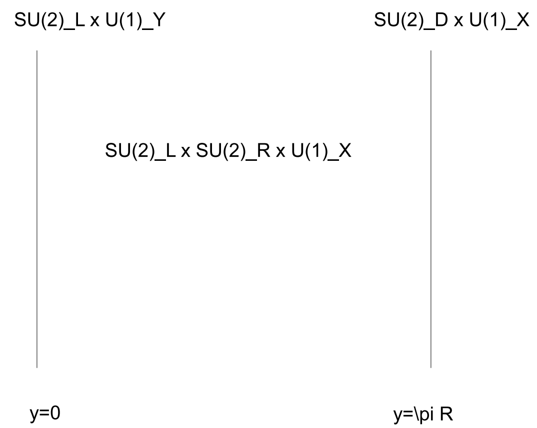
- circle with identification: fields are even or odd
- interval + boundary conditions (BC).

Ex (for gauge fields):

- even field $\Leftrightarrow$ Neumann BC (zero mode: 4d gauge symmetry) $\quad \partial_y A_\mu(y=0) = \partial_y A_\mu(y=\pi R) = 0$
- odd field $\Leftrightarrow$ Dirichlet BC (only massive KK modes) $A_\mu(y=0) = A_\mu(y=\pi R) = 0$

There are however more general BC (Homework: work out wavefunctions and KK masses; lightest mode ?) : $(\partial_y A_\mu + m_1^2 A_\mu)_{y=0} = 0$, $(\partial_y A_\mu + m_2^2 A_\mu)_{y=\pi R} = 0$.

They were widely used for symmetry breaking by BC.



Example of symmetry breaking by BC (*Homework: write BC*):

- bulk left-right gauge symmetry, broken to SM on $y = 0$ boundary

- broken to custodial $SU(2)_D \times U(1)_X$ on $y = \pi R$ boundary.

- Hypercharge is $Y = T_3^R + X$. Only electric charge $U(1)_Q$ is 4d gauge symmetry.

Momentum conservation in the fifth dimensions $\Leftrightarrow$ KK momentum conservation $\sum_n p_n = 0$.

Orbifolds have

- untwisted (UT) fields, propagating in the whole space
- twisted (T) fields, living in the fixed points of the orbifold.

In an orbifold there are two types of interactions :

1) Bulk interactions for UT fields : KK momentum is conserved

Ex : gauge interactions

$$g_{mnp}\bar{\Psi}^{(m)}\Gamma^{M}\Psi^{(n)}A_{M}^{(p)}$$

where $g_{m,n,p} \sim g$ if

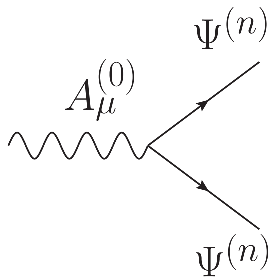
$$m \pm n \pm p = 0$$

and zero otherwise.

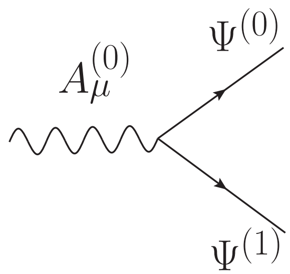
2) Localized interactions (at the fixed points), typically of the form UUU , TTU and TTT. They violate KK momentum conservation.

But they can preserve the KK-parity : $R_{\mathsf{KK}} = (-1)^n$.

Vertices allowed/forbidden by KK momentum conservation and KK parity

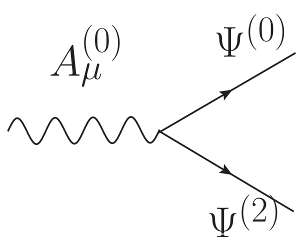


allowed by KK momentum conservation



forbidden by KK momentum conservation

forbidden by KK parity



forbidden by KK momentum conservation

allowed by KK parity

I : TeV size extra dimensions

Universal extra dimensions

(Appelquist,Cheng,Dobrescu, 01)

UED = All particles propagate in a TeV extra dimension.

Bulk interactions : preserve KK number conservation

"Brane" interactions : break KK conservation.

There is a KK parity left unbroken in UED ,

$$P_{KK} \;=\; (-1)^n$$

for the n^{th} KK mode, if exchange symmetry

$y = 0 \leftrightarrow y = \pi R$.

• Due to quantum corrections, KK masses of various SM particles are different.

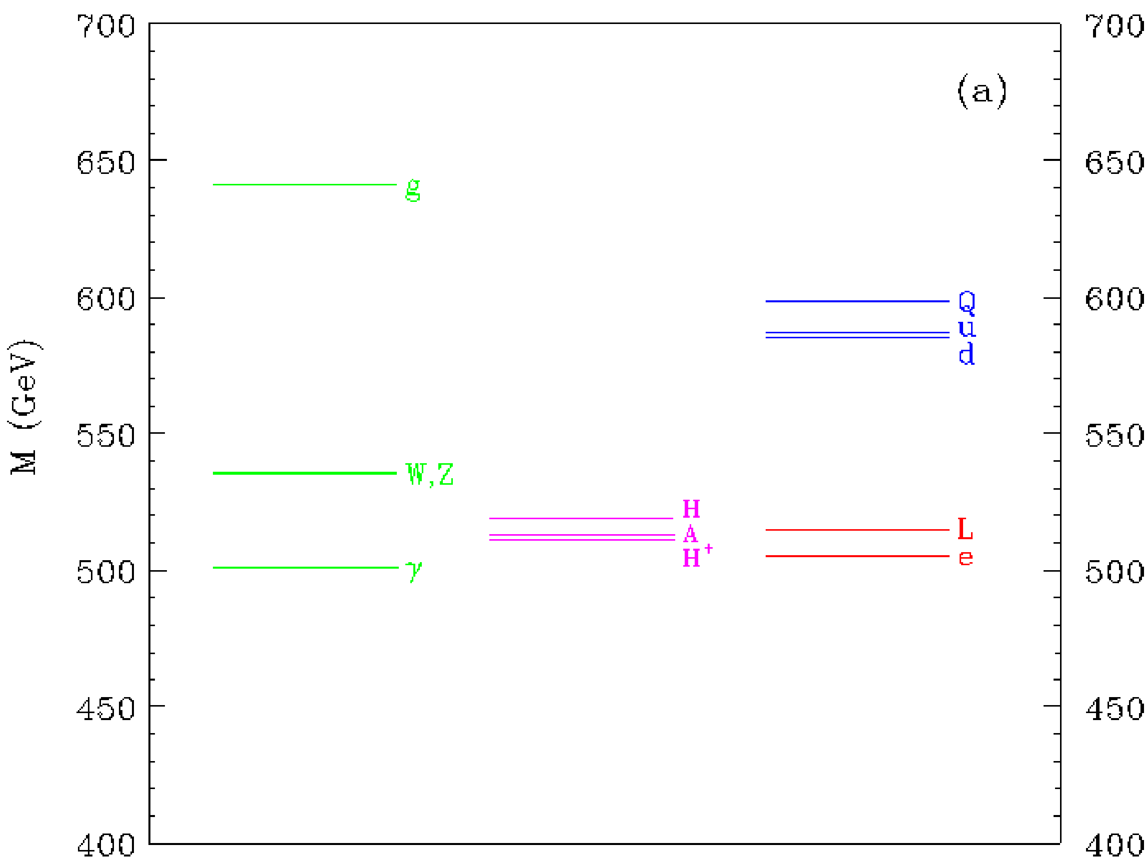
For example,

$$\delta M_W^2(n) = \frac{g^2}{16\pi^2 R^2}\left[-\frac{5\zeta(3)}{2\pi^2} + 15n^2 \ln(\Lambda R)\right]$$

where Λ is the UV cutoff of the theory.

• 1st KK photon becomes the lightest KK particle and the 1st gluon KK the heaviest.

First level (n=1) masses in UED with $R^{-1} = 500\ GeV$ look like



(a)
M (GeV)
700
650
600
550
500
450
400
g
W,Z
γ
H
A
H+
Q
u
d
L
e

Consequences of UED with KK parity :

- the lightest KK state (first KK "photon" state $B^{(1)}$) is stable. It can play the role of the dark matter of the universe. Annihilation cross section

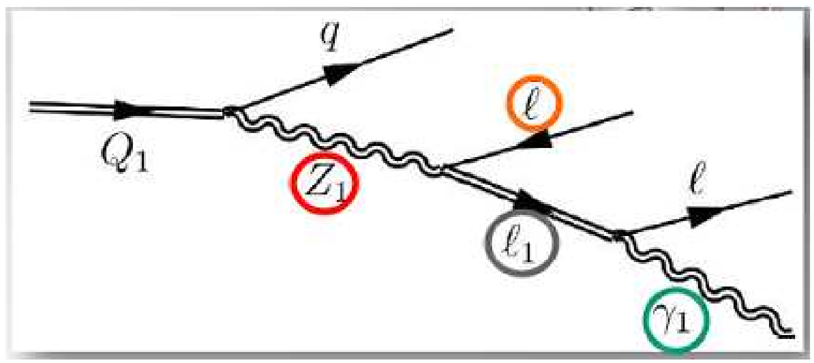
$$\langle \sigma v \rangle \sim \frac{g_Y^4}{m_{B^{(1)}}^2}$$

gives relic density $\Omega h^2 \sim 0.11$ for 600 $GeV < m_{B^{(1)}} <$ 2 TeV.

- odd-level KK modes can only be produced in pairs, then cascade decay to the LKP.

- lightest KK states cannot be single produced $\rightarrow$ experimental limits on the UED are weak $R^{-1} \geq 700$ GeV.

- certain number of jets + leptons and photons + missing $\not{E}_T$.



Non-UED TeV extra dimensions

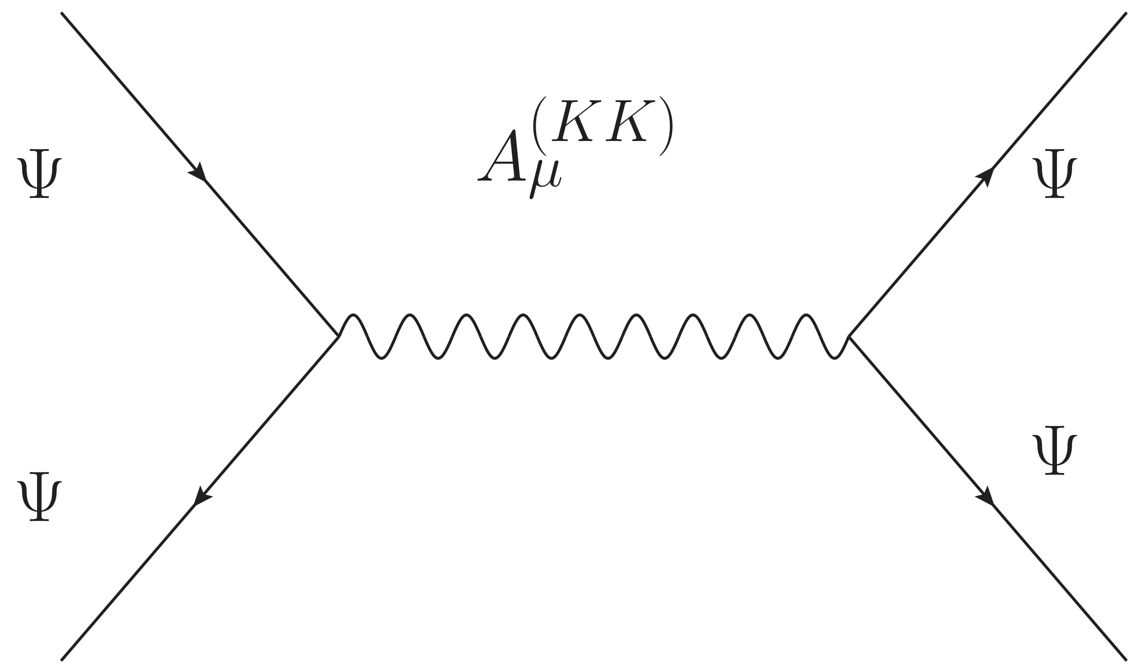
- Direct production of single KK states, if $E > R^{-1}$

- Virtual exchange of KK states leading to effective operators of the type

$$\frac{\lambda}{M_c^2} \, (\bar{\Psi}\gamma_\mu\Psi)^2$$

where $M_c = R^{-1}$. Present exp. limits are $R^{-1} \sim 3 - 5$ TeV.

Effective operators generated by exchange of KK states



Accelerated Unification

(K.Dienes,E.D.,T.Gherghetta, 1998)

• Gauge coupling unification seems to predict a very high unification scale M_s, unaccessible to colliders.

Is there's a way to get unification at low energies ?

- YES The elementary particles: electron, quarks, etc propagate in the extra dimensions . Their Kaluza-Klein states produce an accelerated evolution of the couplings.

$\longrightarrow$ accelerated unification .

Start with MSSM in 4d and extend it to $R^4 \times S^1$, a circle of radius $R_{||}$. In this case the RG running of gauge couplings is

$$\frac{1}{\alpha_a(\mu)} = \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi} \ln \frac{\mu}{M_Z} - \frac{\tilde{b}_a}{2\pi} \int_{1/\mu^2}^{1/M_Z^2} \frac{dt}{t} \theta_3^\delta \left(\frac{it}{\pi R_{||}^2} \right)$$
$$\simeq \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi} \ln \frac{\mu}{M_Z} + \frac{\tilde{b}_a}{2\pi} \ln(\mu R_{||})$$
$$- \frac{\tilde{b}_a}{2\pi} [(\mu R_{||})^\delta - 1] \ .$$

Obs : The coefficients $\tilde{b}_a$ are one-loop beta-function coefficients of the massive KK modes.

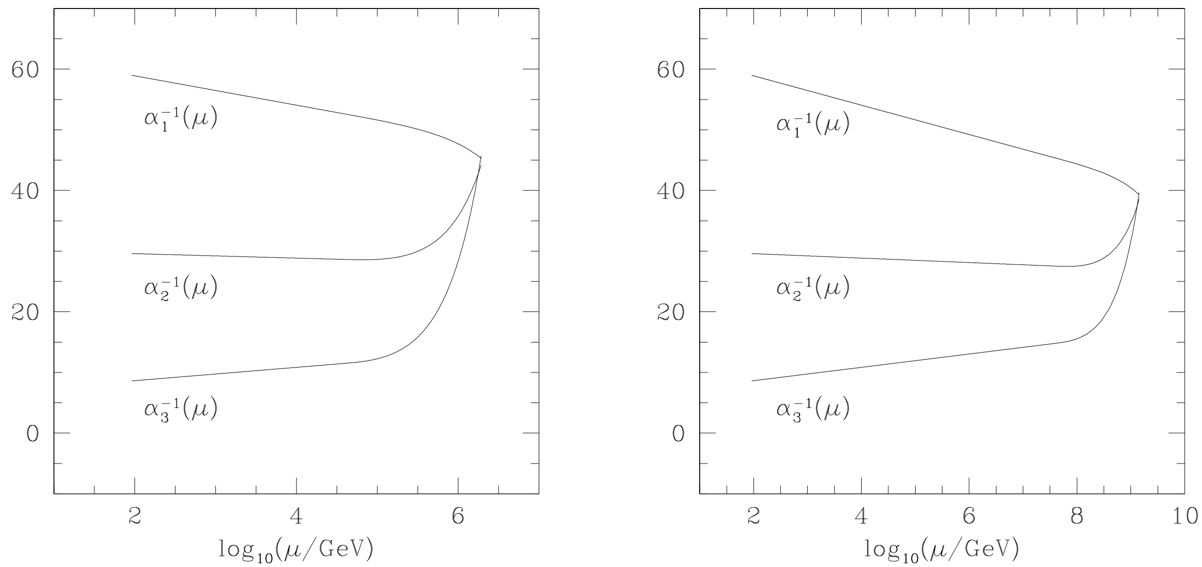
Consider the simplest embedding of the MSSM in 5d.

The matter fermions of MSSM can either contain only zero modes or, alternatively, can have associated mirror fermions and KK excitations for $\eta = 0, 1, 2, 3$ families. The unification pattern does not depend on η.

$\longrightarrow$ the couplings unify for any compact radius 10^3 GeV $\leq R_{||}^{-1} \leq 10^{15}$ GeV, at a energy scale roughly a factor of 20 above the compactification scale $R_{||}^{-1}$.

• The results are UV sensitive. Nice field-theory UV completion by Hebecker-Westphal.

Others: deconstruction, string theory (Kiritsis lecture).



Unification of gauge couplings in the presence of extra spacetime dimensions. We consider two representative cases: $R^{-1} = 10^5$ GeV (left), $R^{-1} = 10^8$ GeV (right). In both cases we have taken $\delta = 1$ and $\eta = 0$.

Summary TeV / UED extra dimensions

- KK modes of quarks and leptons are vectorlike fermions.

-Orbifolds can generate chirality

- Ex: q_L, u_R, d_R , $(q^{(k)}, \tilde{q}^{(k)})$, $(u^{(k)}, \tilde{u}^{(k)})$, $(d^{(k)}, \tilde{d}^{(k)})$

- Fast variation of couplings due to KK states

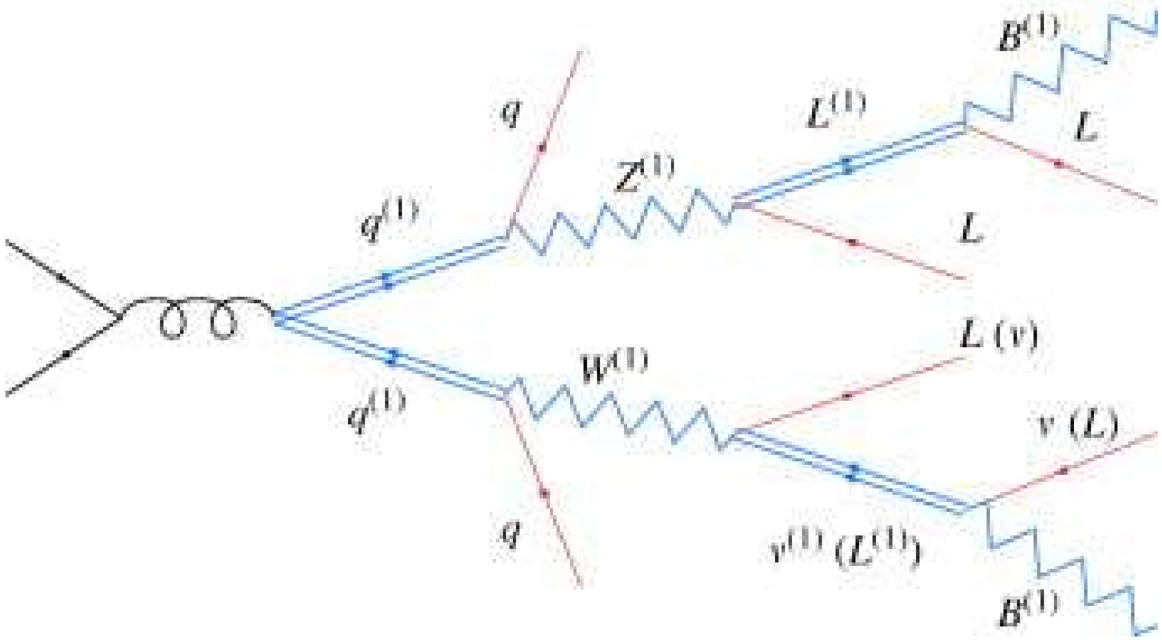
- Search for direct production and virtual effects of individual KK resonances

- UED : KK parity $\rightarrow$ lowest KK stable

($B^{(1)}$ dark matter candidate)

- lowest KK produced in pairs

Ex : $q\bar{q} \rightarrow g \rightarrow q^{(1)}\bar{q}^{(1)}$: jets,leptons + missing E_T



$B^{(1)}$
q
$L^{(1)}$
L
$Z^{(1)}$
$q^{(1)}$
L
$L\ (\nu)$
$W^{(1)}$
$q^{(1)}$
$\nu\ (L)$
q
$\nu^{(1)}\ (L^{(1)})$
$B^{(1)}$

loop corrections to electroweak observables $\longrightarrow$

$R^{-1} > 500 - 700$ GeV

• Nontrivial to distinguish experimentally between UED and SUSY (need spin).

• Non-UED case: direct production and virtual effects of single KK states

Ex : $q\bar{q} \rightarrow \gamma^{(k)} \rightarrow f\bar{f}$, $R^{-1} > 3 - 5$ TeV

II mm dimensions: the ADD scenario

• **Strings and D-branes**

- Consistency conditions superstrings: $\longrightarrow$ six Xtra dims

- Superstrings are characterized by

l_s $(= M_s^{-1})$ = length (mass) of the string

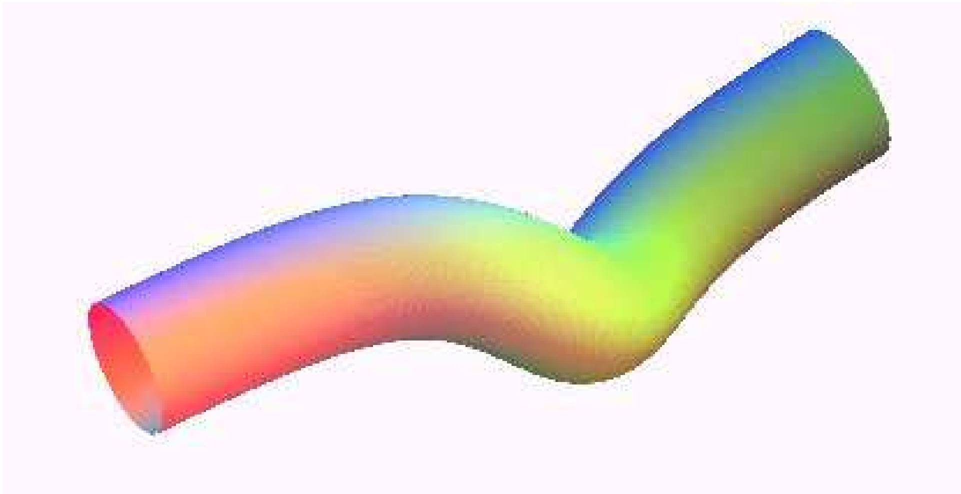
$g_s = e^{\langle \phi \rangle}$ string coupling, ϕ = dilaton

M_s is a free parameter, whereas g_s is dynamically determined.

After compactification, there are moduli fields : dilaton, volume moduli, shape moduli. Most of them are massless $\rightarrow$ need to stabilize moduli them.

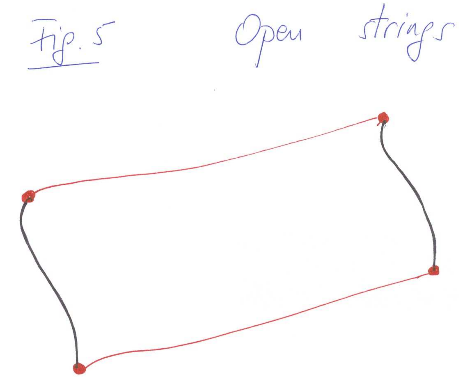
There are two types of strings (see E.Kiritsis lectures):

closed strings : excitations : gravitons , etc



They propagate everywhere (in the "bulk").

open strings excitations : electrons, etc



Their end points are :

- free to move : $p+1$ Neumann BC

- fixed : $9-p$ Dirichlet BC $\Rightarrow$ Dp brane.

Strings have surfaces of p space-dims. : D-branes (Polchinski, 95), which contain gauge fields (coupling g) and matter fields.

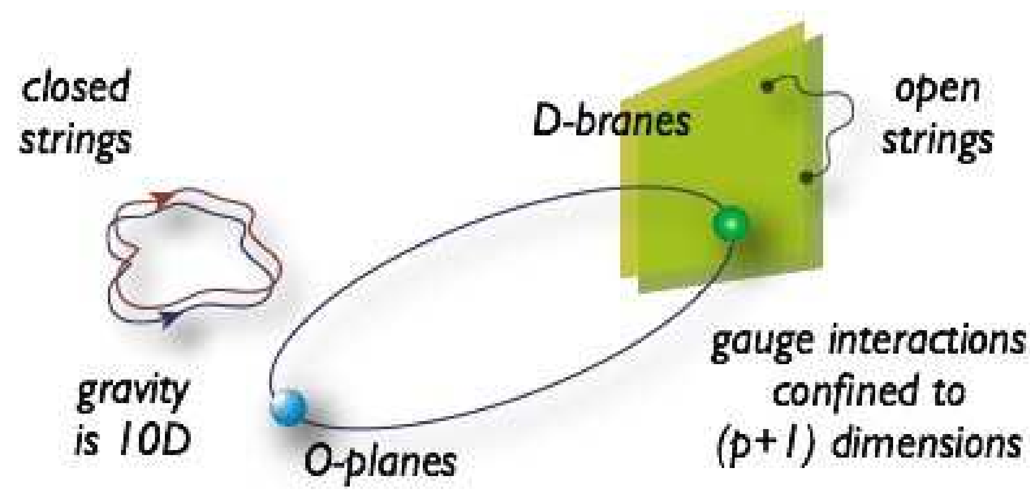
- D-branes carry mass and charges. They source gravitational fields and curve the internal space.
- Bulk (gravity) fields interact with the brane fields.

$$T^{\mu\nu}_{\text{brane}}\ g_{\mu\nu}(\mathbf{y}=0,\mathbf{x}) \ = \ \frac{1}{\sqrt{V}} T^{\mu\nu}_{\text{brane}}\ \sum_k g^{(k)}_{\mu\nu}(\mathbf{x})$$

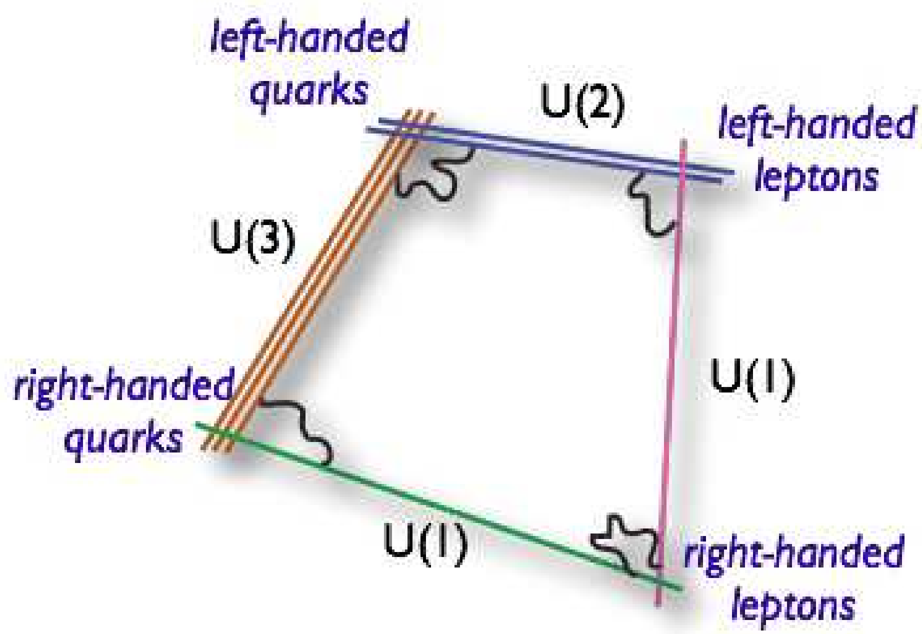
- Branes interact via the exchange of bulk fields.

Gauge groups for N coincident Dp-branes: $U(N)$ (type II strings) or $SO(N)$ (or $USP(N)$) for orientifolds.

Brane world model in string theory. We live on the green D-branes



Standard Model realization on D-branes



Strings have no point-like interactions$\rightarrow$ no UV divergences ! Quantum corrections to the Higgs mass should be UV finite

$$\delta m_h^2 \sim M_s^2$$

Solution to hierarchy problem (G. Ross lectures) if $M_s \sim$ TeV.

5. Brane universes and TeV scale strings : ADD scenario

(Arkani-Hamed,Dimopoulos,Dvali + Antoniadis, 98)

• If SM lives on D-branes (open strings), constraints on unification and fundamental scale weaken considerably.

ADD Brane Universe :

- the three SM gauge interactions and matter (open strings) are localized on a Dp (ex. D3) brane
- The gravitation (closed strings) lives everywhere in (“in the bulk”).

The relevant effective action is

$$\int d^4x \, \left(\int d^6y \frac{M_s^8}{g_s^2} \mathcal{R} - \int d^\delta y \frac{M_s^\delta}{g_s} F_{MN}^2 \right) \ , \text{ where}$$

δ = parallel dimensions = $p - 3$ for a Dp brane, $V_{||}$.

$n = 6 - \delta = 9 - p$ = perpendicular dimensions, $V_\perp$.

We get the relations

$$M_P^2 \ = \ \frac{1}{g_s} V_\perp \ M_s^{2+n} = V_\perp M_*^{2+n}$$
$$g_4^2 \ = \ g_s V_{||} \ M_s^{6-n}$$

M_* is the higher-dim. Planck scale.

If $M_s \sim TeV$, the hierarchy problem is solved. Need a huge volume $V_\perp \sim 10^{30} \Rightarrow$ reformulation of the hierarchy problem.

The n perpendicular extra dimensions can be of macroscopic size

$$R_\perp \ \leq \ 10^{-1} \ mm \ ,$$

constraint coming from eventual deviations from Newton's law .

- If SUSY breaking on the branes, scale $M_{SUSY} \sim M_s$

$$m_{\text{bulk-moduli}} \sim \frac{M_{SUSY}^2}{M_P} \sim 10^{-3} eV$$

$\rightarrow$ also possible modifications of Newton law.

Newtonian potential between two bodies of masses m_1, m_2 is

$$V(r) = -\frac{1}{M_P^2}\frac{m_1 m_2}{r} \quad , \quad \text{for } r > R \ ,$$

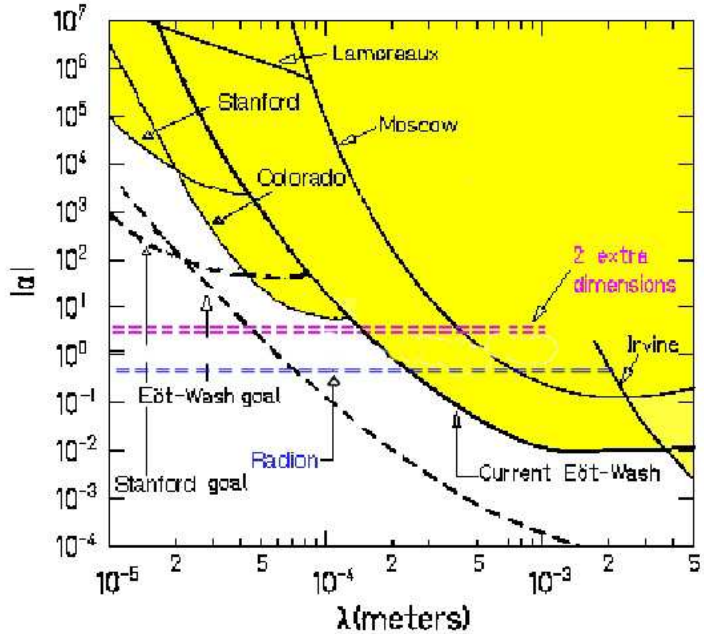
$$V(r) = -\frac{1}{M_*^{2+n}}\frac{m_1 m_2}{r^{1+n}} \quad , \quad \text{for } r < R \ .$$

Two dims. of extreme size $R_\perp \sim 10^{-1}$ mm give a fundamental string mass scale

$$M_s \sim 3 - 10 \quad TeV$$

$\longrightarrow$ strings could be accessible at LHC !

Hierarchy problem translated into the problem of finding a very large transverse volume.



Searches for deviations from Newton law, with

$$V(r) = G_N \frac{m_1 m_2}{r} (1 + \alpha \, e^{-r/\lambda})$$

C. D. Hoyle et al, [hep-ph/0405262]

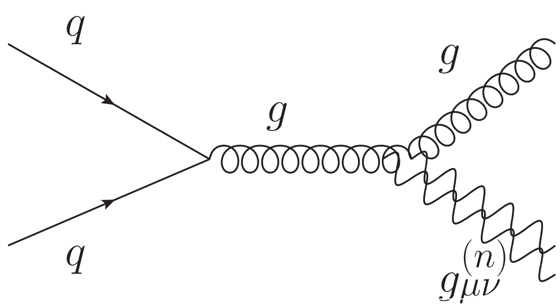
Gravity becomes strong at energies

$$M_* \;=\; (\frac{1}{g_s})^{1/(2+n)} M_s \;>\; M_s$$

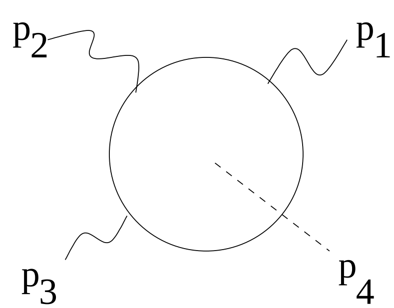
$\longrightarrow$ string effects are observable at LHC, if TeV strong gravity.

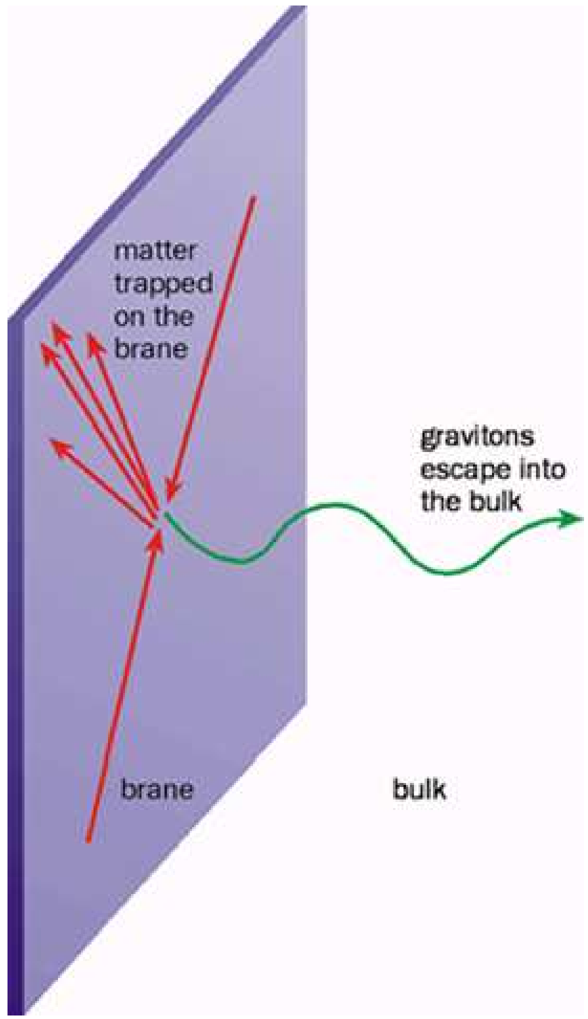
- **Experimental signatures**

Graviton emission in the bulk (missing E_T)



String viewpoint: three open + one closed string particles :




matter trapped on the brane
gravitons escape into the bulk
brane
bulk

The inclusive cross-section (sum over huge number of KK gravitons compensates $1/M_P^2$ suppression)

$$\sigma_{FT} \sim \frac{1}{M_P^2} \sum_{m_i=0}^{RE} \sim \frac{E^n}{M_*^{2+n}}$$

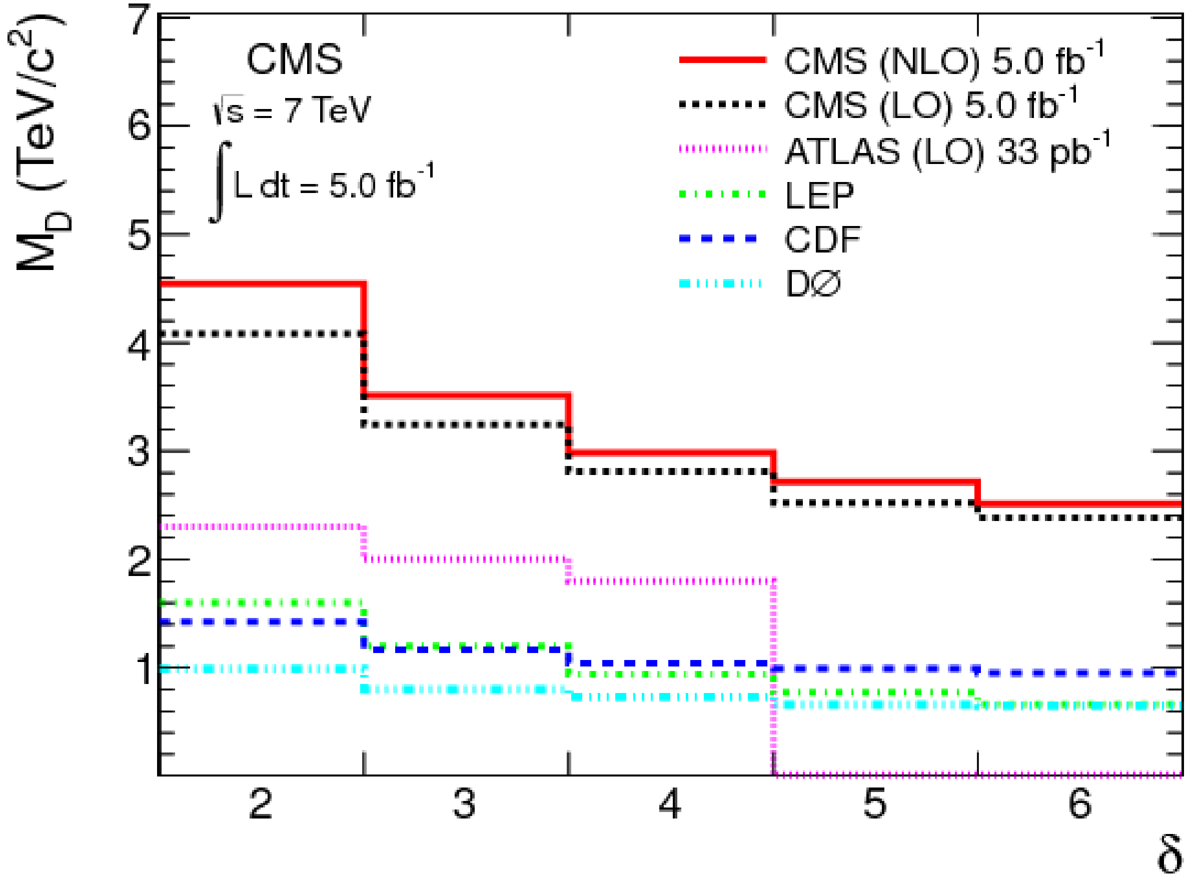
is reliable at the field-theory level.

However, string effects appear at $M_s < M_*$. Explicit computation (m =graviton mass) :

$$A_4 = \frac{2^{-\frac{m^2}{M_s^2}}}{\sqrt{\pi}} \frac{\Gamma(-\frac{m^2}{2M_s^2}+\frac{1}{2})\Gamma(1-\frac{s}{2M_s^2})\Gamma(1-\frac{t}{2M_s^2})\Gamma(1-\frac{u}{2M_s^2})}{\Gamma(1+\frac{s-m^2}{2M_s^2})\Gamma(1+\frac{t-m^2}{2M_s^2})\Gamma(1+\frac{u-m^2}{2M_s^2})} A_4^{FT}$$

which at low energy can be expanded

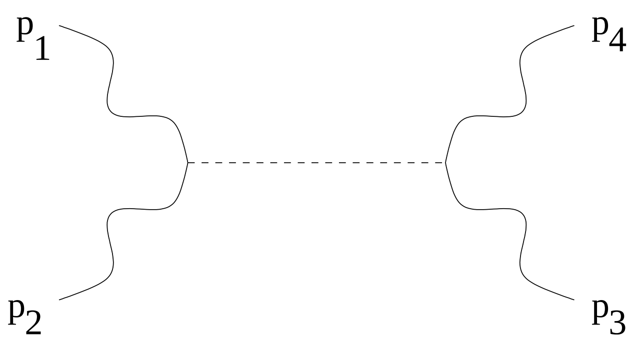
$$\frac{\sigma - \sigma_{FT}}{\sigma_{FT}} \sim \frac{E^4}{M_s^4}$$



Recent CMS constraints on M_* from pp collisions (monojet events) at LHC at 7 TeV

Virtual graviton exchange

Another important process for the large Xtra dim. scenario: virtual graviton exchange



This leads to $\mathcal{L}_{\text{int}} = \frac{4}{M_T^4}(T_{\mu\nu}T^{\mu\nu} - \frac{1}{n-2}T^{\mu}_{\mu}T^{\nu}_{\nu})$

For $n \geq 2$, summation is UV divergent

$$\frac{4}{M_T^4} \sim \frac{1}{M_P^2} \sum \frac{1}{s - (m_1^2 + \cdots m_n^2)/R_\perp^2} \sim \frac{1}{M_P^2} R_\perp^2 (R_\perp \Lambda)^{n-2}$$

where the summation was cut at KK masses lighter than Λ. The result can be written

$$A \sim \frac{\Lambda^{n-2}}{M_*^{2+n}} \sim \frac{c^{n-2}}{M_*^4}$$

It is believed that string theory regulates the divergence, so $\Lambda = cM_*$. Slight subtlety: this is a one-loop diagram in string theory.

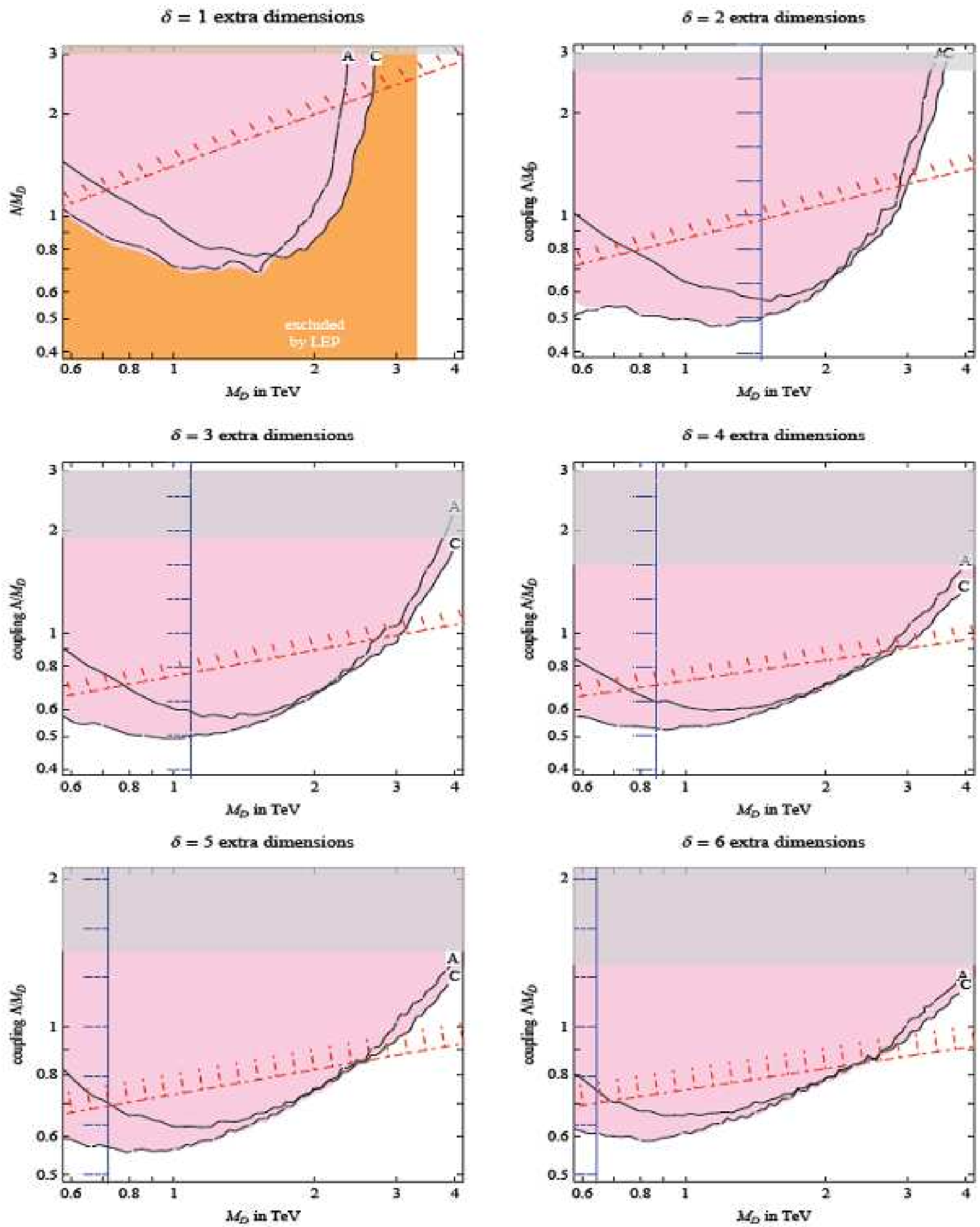


Figure 6: *The shaded area is the bound from virtual graviton exchange at CMS (continuous line denoted as 'C', data after 36/pb), ATLAS (long-dashed line denoted as 'A', data after 36/pb). Vertical blue line: bound from graviton emission (as summarized in table 1 of [5]). Red line: Naive Dimensional Analysis estimate of LEP bound from loop graviton exchange.*

(from R. Franceschini et al, [arXiv:1101.4919 [hep-ph]])

String theory: main corrections not graviton exchange,

but tree-level string oscillators exchange:

$$A(1,2,3,4) \;\sim\; g^2 \,\frac{\Gamma(1-\frac{s}{M_s^2})\Gamma(1-\frac{t}{M_s^2})}{\Gamma(1-\frac{s+t}{M_s^2})} K(1,2,3,4)$$

Field-theory amplitude $\nwarrow$

Black hole production at LHC ?

(Dimopoulos,Landsberg;Giddings,Thomas,2001)

• Interesting feature in the transplackian regime $E >> M_*$: possible production of black holes (BH).

• In a collision of energy $\sqrt{s}$ a BH of mass $M_{BH} \sim \sqrt{s}$ can form, with Schwarzschild radius

$$R_S = \frac{1}{\sqrt{\pi} M_*} \left[\frac{M_{BH}}{M_*} \left(\frac{8\Gamma(\frac{n+3}{2})}{n+2} \right) \right]^{\frac{1}{n+1}}$$

If the impact parameter is less than R_S, a BH of mass M_{BH} is expected to form.

Total cross section can be estimated from geometrical arguments

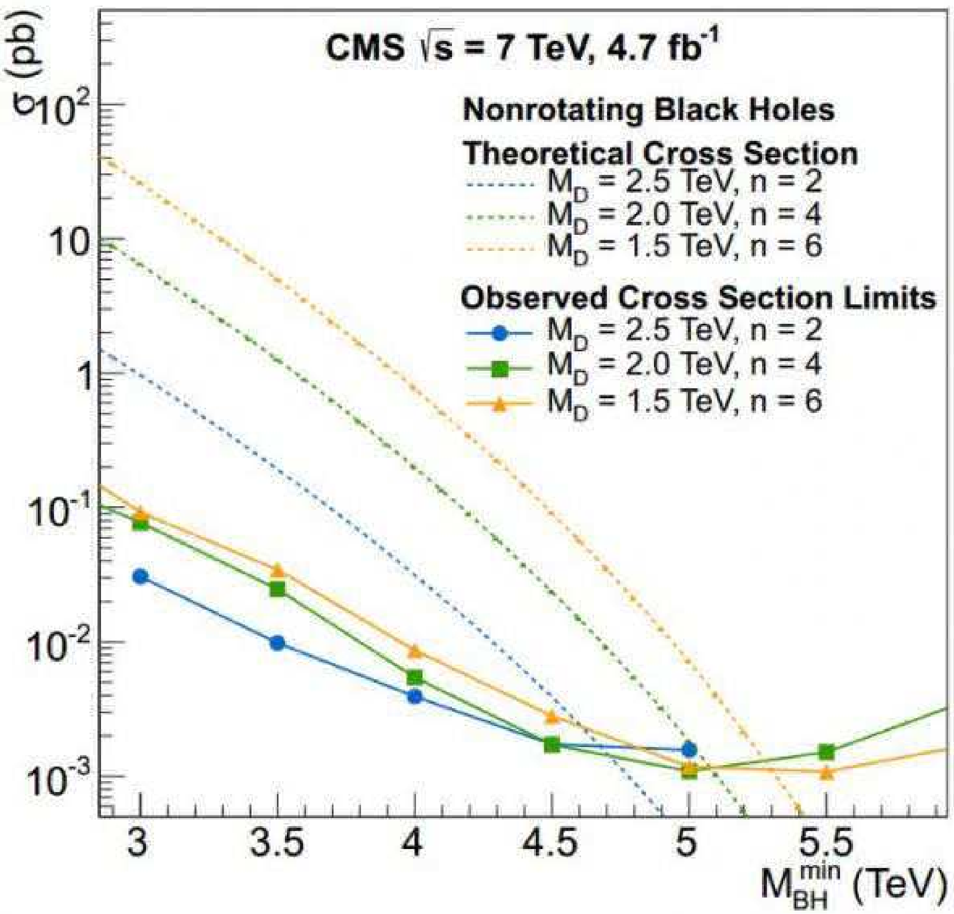
$$\sigma(M_{BH}) \sim \pi R_S^2$$

For $M_* \sim$ TeV, this gives a production of 10^7 BH at $\sqrt{s} = 14$ TeV LHC with an integrated luminosity of 30 fb^{-1}.

Experimental signatures rely on two properties :

- the absence of small couplings
- A BH is expected to thermally radiate with a Hawking temperature $T_H = (n+1)/(4\pi R_S)$ and would therefore evaporate "democratically" (flavor independently).

Search for microscopic black holes, CMS [arXiv:1202.6396 [hep-ex]] $\Rightarrow M_{BH} \geq 5$ TeV



- **Neutrino masses in the ADD scenario**

(Dienes,E.D.,Gherghetta; Arkani-Hamed,Dimopoulos,Dvali, March-Russell, 98)

Neutrino masses and mixings $\longrightarrow$ new physics at a high mass scale M, seesaw mechanism ?

$$m_\nu \sim \frac{v^2}{M} \quad \text{Santamaria lectures}$$

There is no large M in ADD, but singlet fields can also propagate in perpendicular dimensions $\rightarrow$ bulk sterile neutrinos = infinite tower of KK sterile neutrinos

Simple model : SM fields (Higgs H,active neutrino ν_L) = brane fields singlet neutrino = 5d bulk field $\Psi = (\Psi_1, \bar{\Psi}_2)^T$, where Ψ_1 (Ψ_2) is Z_2 even (odd). Localized Dirac mass

$$(\nu_L \Psi_1 H)(y=0) \to \frac{v}{\sqrt{2\pi R}}\, \nu_L \left[\Psi_1^{(0)} + \sqrt{2} \sum_m \Psi_1^{(m)}\right]$$

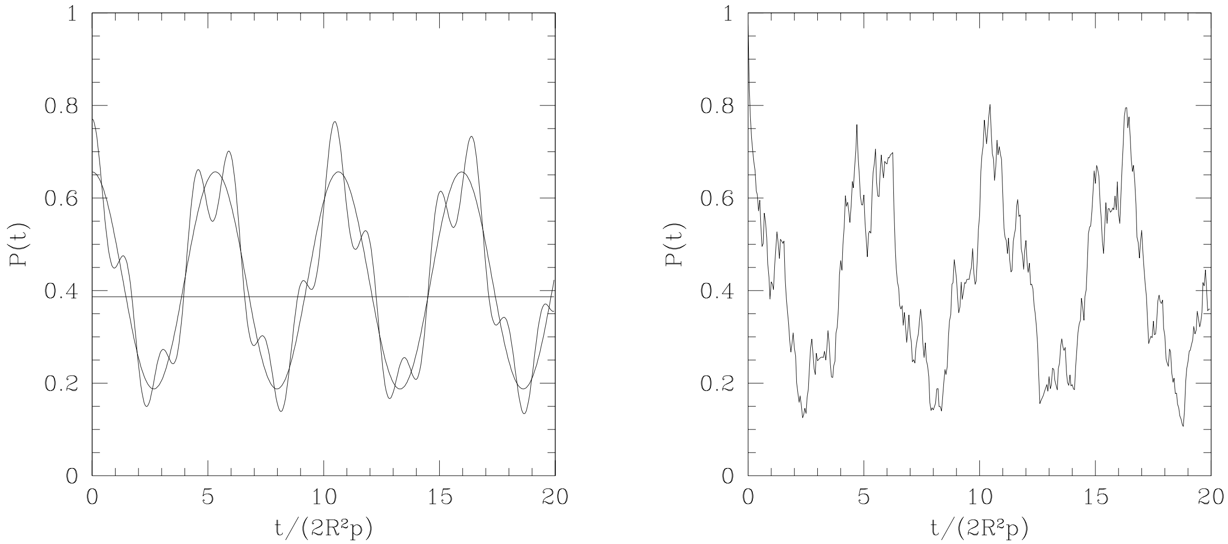
• The right-handed neutrino has a dense tower of KK states. If $R_\perp^{-1} \sim 10^{-2}$ eV, the active neutrino oscillates into the infinite tower of KK sterile states !

• Original motivation : explaining solar oscillations $\Delta m^2 \sim 8 \times 10^{-5} eV^2$ by oscillations in the bulk $\nu_e \to \Psi^{(m)}$.

The naive (Dirac) mass of the zero mode

$$m_\nu \sim \frac{v}{(RM_*)^{n/2}} \equiv m$$

is volume suppressed; this generates sub-eV masses.



Higher-dim. neutrino oscillations (a) The evolution of the survival probability as more and more KK states are included in the sum. (b) The final result, when all KK states are included. The resulting neutrino deficits and regenerations are never total.

Summary ADD scenario :

Experimental constraints:

• perpendicular dimensions : $R_\perp \leq 37\ \mu m$ for $n = 2$.

• Search for inclusive effects of gravitons, not individual resonances. Main collider processes :

- $q\bar{q} \rightarrow g\ G$, $q\bar{q} \rightarrow \gamma\ G$ (missing energy +jet, photon)

- contact interactions $q\ \bar{q} \rightarrow q\ \bar{q}$

• Possible black hole production (controversial)

• There are astrophysics/cosmology constraints :

- supernovae cooling constraints : $M_* > 50\ TeV$ for $n = 2$

III Warped compactifications

(Randall,Sundrum, 99)

D-branes curve internal space $\rightarrow$ warped extra dimensions. 5d metric is

$$ds^2 \ = \ a^2(y) \ \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 \ ,$$

where $a(y)$ is the "warp" factor. Physics depends drastically on warp factor and localisation of the fields in the extra dimensions. Mass scales are red-shifted

$$M \ \rightarrow M(y) \ = \ a(y) \ M$$

The RS1 model

5d model on $R^4 \times S^1/Z_2$, with two branes :

- Planck brane, at $y = 0$.

- TeV brane, at $y = \pi R$. SM lives on the TeV brane.

The action is

$$S \;=\; S_B \;+\; S_0 \;+\; S_{\pi R} \;,$$

where

$$\begin{aligned} S_B \;&=\; \int d^4x \; dy \; \sqrt{g} \; (M_*^3 R \;-\; \Lambda) \;, \\ S_0 \;&=\; \int_{y=0} d^4x \; \sqrt{g} \; V_0 \;, \\ S_{\pi R} \;&=\; \int_{y=\pi R} d^4x \; \sqrt{g} \; (V_{\pi R} \;+ \mathcal{L}_{\mathrm{SM}}) \;, \end{aligned}$$

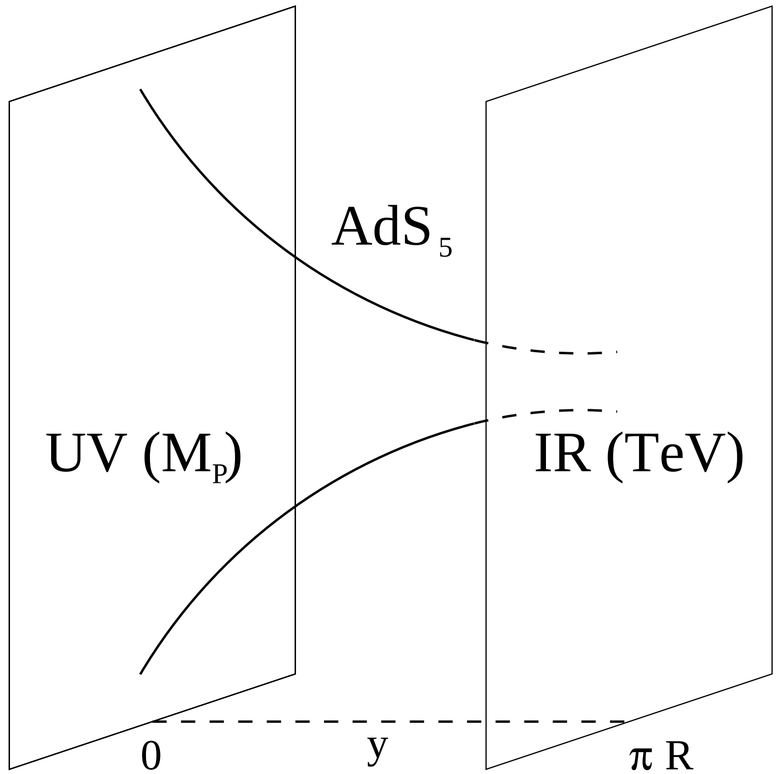
Tensions and 5d cosmological constant are

$$V_0 = -V_{\pi R} \ = \ 12kM_*^3 \quad , \quad \Lambda \ = \ -kV_0 \ .$$

and warp factor is

$$ds^2 \ = \ e^{-2k|y|} \ \eta_{\mu\nu}dx^\mu dx^\nu + R^2 dy^2 \ .$$

- The TeV brane has negative tension.
- k=anti de-Sitter radius $k \le M_* \sim M_P$. Small radius, but fundamental scale is red-shifted along the extra dim. $M(y) \sim e^{-k|y|}M_*$


AdS 5
UV (MP)
IR (TeV)
0
y
π R

Examine closer the SM Higgs action

$$S_{SM} \;=\; \int_{y=\pi R} d^4x\ \sqrt{g}\ [g^{\mu\nu} D_\mu \bar{H} D_\nu H - \lambda(|H|^2 - v^2)^2]$$

Due to warping, the physical Higgs vev is

$$\tilde{v} \;=\; e^{-k\pi R}\, v \sim TeV\ .$$

whereas the 4d Planck scale is

$$M_P^2 \;=\; \frac{M_*^3}{k}\,(1 - e^{-2k\pi R})$$

RS solution of the hierarchy problem is

$$M_* \sim k \sim M_P \quad , \quad e^{-k\pi R} k \sim TeV$$

if all of the SM fields live on the TeV brane.

The masses of KK graviton modes are **not equally spaced**

$$m_n \; = \; x_n \; k \; e^{-k\pi R} \quad , \quad \text{where } J_1(x_n) \; = \; 0 \; .$$

Wave functions of the gravitons are localized :

- zero mode on the Planck brane : $\Psi_0 \sim e^{-3/4k|y|}$

- KK modes on the TeV brane $\longrightarrow$ they have strong (TeV) couplings to SM fields.

RS model does not explain flavor; proton decay on TeV brane $\Rightarrow$ realistic models use geometric localization.

- bulk scalar, mass $m_\phi^2 = ak^2$. Zero mode:

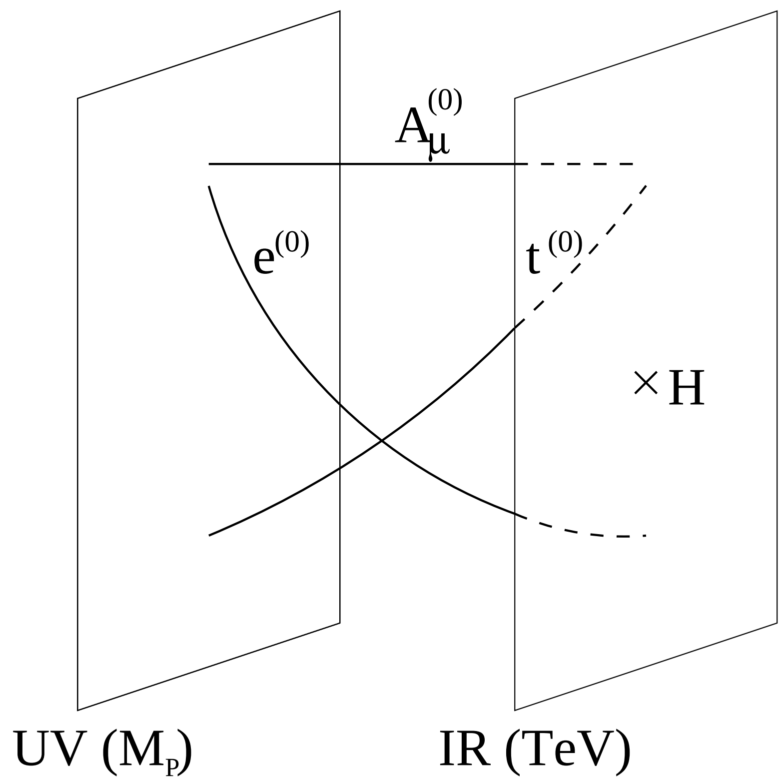
$$\phi^{(0)}(y) \sim e^{(b-1)ky} \;=\; e^{(1\pm\sqrt{4+a})ky}$$

$b < 1$ ($b > 1$) $\rightarrow$ zero mode localized towards the UV (IR) brane.

- bulk fermion, bulk mass $m_\psi = ck\epsilon(y)$. Zero mode:

$$\Psi^{(0)}(y) \sim e^{(1/2-c)ky}$$

$c > 1/2$ ($< 1/2$) $\rightarrow$ localization on the UV (IR) brane.

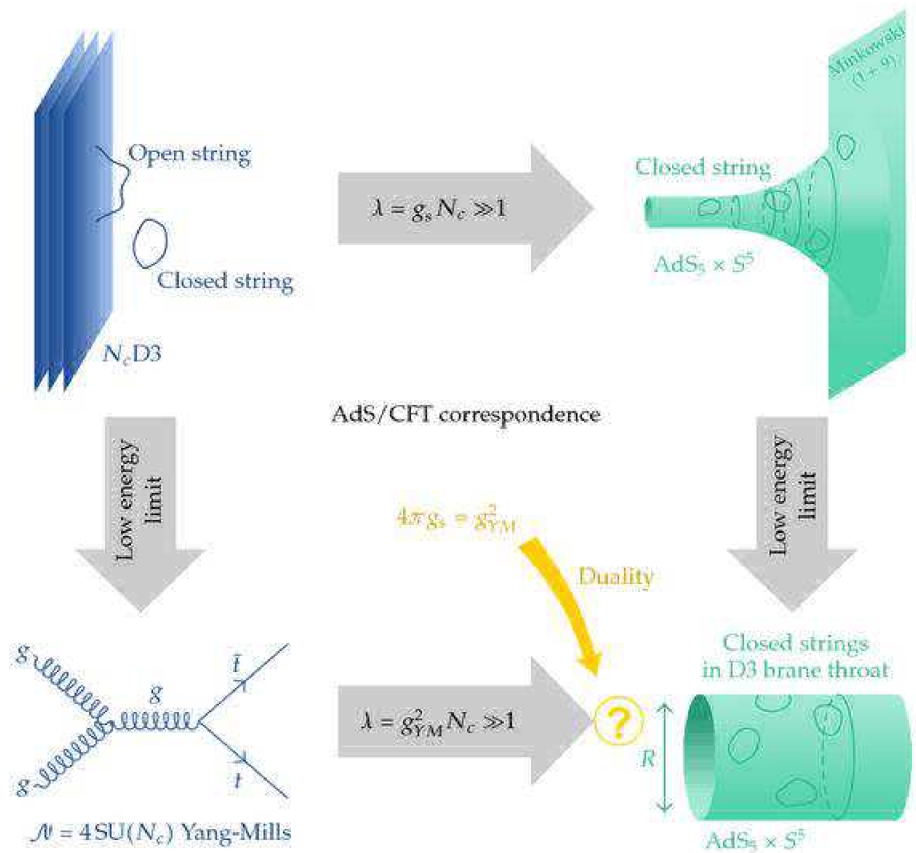


A_μ^(0)
e^(0)
t^(0)
× H
UV (M_P)
IR (TeV)

. Holography and warped compactifications

(Maldacena, 97; Papadodimas lectures)

field theory		string theory
in 4d	$\Leftrightarrow$	in 10d, comp.
(strong coupling)		on $AdS_5 \times X_5$
SUSY, conformal		(weak coupling)



Nonperturbative methods in field theory. The 4d CFT theory has no gravity.

There is a holographic interpretation of RS models:

• Due to the Planck brane, CFT couples to 4d gravity. Planck brane breaks conformal symmetry by higher-dim. (irrelevant) operators.

• The TeV brane generates a spontaneous breaking of CFT. States localized towards:

- Planck brane : are elementary states.
- TeV brane : are composite (bound) states.

• Global symmetries (ex: custodial) are gauge symmetries in the bulk.

5d models (argued to be) dual to 4d strongly coupled ones.

Ex : Composite Higgs models (Agashe et al, 2005):

- Bulk gauge group $G = SU(3)_c \times SO(5) \times U(1)_X$.

Gauge symmetry broken by boundary conditions to:

- G_{SM} on the Planck brane.

- $SU(3)_c \times SO(4) \times U(1)_X$ on the TeV brane.

• Higgs is the 5th component of a gauge field, pseudo-goldstone boson of $SO(5)/SO(4)$.

• It behaves as a composite state, since localized on IR brane. Lightest KK states in the model are color fermions with electric charges $-1/3, 2/3$ and $5/3$.

Experimental signatures of warped models

i) For original RS1 setup, specific LHC processes :

$$q\ \bar{q}\ ,\ \ gg \rightarrow G^{(1)} \rightarrow l^{+}l^{-} \quad (leptonic\ events)$$

$$q\ \bar{q}\ ,\ \ gg \rightarrow G^{(1)} \rightarrow q\ \bar{q}\ ,\ \ gg \quad (jet\ pairs)$$

$\gamma\gamma, ee, \mu\mu, WW, ZZ$ final states put limits $m^{(1)} \geq 2$ TeV.

ii) If SM also propagate into the bulk, check first KK masses of various fields

$$m_n \ \simeq \ (n + \frac{\alpha}{2} - \frac{1}{4})\pi k \exp^{-\pi k R}$$

where $\alpha = \{|c_f - 1/2|, 0, 1\}$ for fermions, KK gauge bosons and KK gravitons, respectively.

• First KK gauge bosons (gravitons) are expected to be the lightest (heaviest).

• Experimental searches for KK gauge bosons and fermions are more appropriate discovery channels. KK gauge bosons mainly decay into top quarks, longitudinal W/Z and Higgs boson.

• Couplings to $ee, \mu\mu, \gamma\gamma$ are suppressed and the bounds of RS1 do not apply. Couplings to light SM fermions are suppressed by a factor $g_4/\sqrt{g_5^2 k} \sim 0.2$.

• Recent LHC limits $m^{(1)}_{\text{(gluons)}} \geq 1.5$ TeV.

- In warped models of electroweak symmetry breaking, there are colored states with charges $-1/3, 2/3$ and $5/3$ and masses between 0.5 and 1.5 TeV.
- The $q = 5/3$ state decays mainly into $W^+ t \to W^+ W^+ b$, giving a pair of same-sign leptons in the final state.
- Masses below 500 GeV are excluded from recent LHC searches.

Conclusions, Prospects

• Xtra dims. models attracted considerable attention as experimentally testable alternatives to SUSY to solve/reformulate the hierarchy problem.

• They are testable in colliders:

- missing (but different) E_T signatures in UED and ADD
- spin-two resonances in RS
- microscopic black holes (ADD and RS)

• All needed ingredients (branes, extra dims.,orbifolds, dualities) exist in string theories, that are often needed to UV complete xtra dims. models (UV sensitivity, nonrenormalisability).

• Xtra dims. opened new perspectives on 4d pheno model building: little and composite Higgs, sequestering, deconstruction, strong-coupling dynamics.

• 5d holographic models became a useful tool to study electroweak symmetry breaking (holographic technicolor), AdS/QCD, superconductors, superfluids...

• LHC will tell us if nature was kind to us to have chosen low-scale values for the string and compactification scales !