A New Road to Massive Gravity?

Eric Bergshoeff

Groningen University

TR33 – Summer Institute: Particles and the Universe

Corfu, September 20 2012

rijksuniversiteit groningen

Outline

Introduction

Outline

Outline

Outline

Outline

Outline

Massive Spin-2 by Higher Derivatives

Einstein Gravity is the unique field theory of interacting massless spin-2 particles around a given spacetime background that mediates the gravitational force

Problem: Gravity is perturbative non-renormalizable

$$\mathcal{L} \sim R + a\left(R_{\mu\nu}{}^{ab}\right)^2 + b\,(R_{\mu\nu})^2 + c\,R^2 \;:$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no (bulk) massless spin 2 !

$\Rightarrow$ "New Massive Gravity"

Hohm, Townsend + E.B. (2009)

Massive Spin-2 by Explicit Mass Term

- Massive Gravity is an IR modification of Einstein gravity that describes a massive spin-2 particle via an explicit mass term

- modified gravitational force

$$V(r) \sim \frac{1}{r} \quad \rightarrow \quad V(r) \sim \frac{e^{-mr}}{r}$$

  - characteristic length scale $r = \frac{1}{m}$

- Cosmological Constant Problem

The vDVZ Discontinuity

Proca : $$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + A_\mu J^\mu$$

- limit $m \to 0$: 3 $\to$ 2 ?
- field redefinition: $A_\mu \to A_\mu + \frac{1}{m}\partial_\mu \phi$
- coupling $\phi\,\partial_\mu J^\mu$ vanishes if J^μ is conserved

Spin 2

- limit $m \to 0$: 5 $\to$ 2 ?
- field redefinitions: $h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{m}\partial_{(\mu}A_{\nu)}$ and $A_\mu \to A_\mu + \frac{1}{m}\partial_\mu \phi$
- couplings $A_\mu \partial_\nu T^{\mu\nu}$ and $\phi\,\partial_\mu\partial_\nu T^{\mu\nu}$ vanish if $T^{\mu\nu}$ is conserved but a coupling $\phi\,\eta_{\mu\nu}T^{\mu\nu}$ survives ! (due to $h_{\mu\nu} \to h_{\mu\nu} + \eta_{\mu\nu}\phi$)

The Boulware-Deser Ghost

counting d.o.f. massless gravity

$$6 + 6\ (g_{ij}, \pi^{ij}; i = 1, 2, 3) - 4 - 4\ (N, N^i) = 2 + 2\ \text{: massless spin-2}$$

counting d.o.f. massive gravity

$$6 + 6\ (g_{ij}, \pi^{ij}) = 5 + 5\ \text{(massive spin-2)}\ + 1 + 1\ \text{(BD ghost)}\ - 1 - 1$$

4D : Gabadadze, de Rham, Tolley (GdRT) (2010); Chamseddine, Mukhanov (2010)

Outline

Free Fierz-Pauli

- $\left(\Box - m^2\right) \tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,, \quad \partial^\mu \tilde{h}_{\mu\nu} = 0$

- $\mathcal{L}_{\text{FP}} = \frac{1}{2}\tilde{h}^{\mu\nu} G^{\text{lin}}_{\mu\nu}(\tilde{h}) + \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu}\tilde{h}_{\mu\nu} - \tilde{h}^2\right)\,, \qquad \tilde{h} \equiv \eta^{\mu\nu}\tilde{h}_{\mu\nu}$

no obvious non-linear extension !

number of propagating modes is $\frac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

A Trick

3D Proca : $\partial^\mu A_\mu = 0 \quad \Rightarrow \quad A_\mu = \epsilon_\mu{}^{\nu\rho}\,\partial_\nu V_\rho$

gauge theory

- warning : this trick does not work for Proca !

Higher-Derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \epsilon_\nu{}^{\gamma\delta} \partial_\alpha \partial_\gamma h_{\beta\delta} \equiv G^{\text{lin}}_{\mu\nu}(h)$$

$$\left(\Box - m^2\right) \, G^{\text{lin}}_{\mu\nu}(h) = 0 \,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g}\left[-R - \frac{1}{2m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right]$$

"New Massive Gravity" : unitary !

What We Now Know

- NMG is (most likely) non-renormalizable

- NMG plus c.c. Λ : massive gravitons $\Leftrightarrow$ black holes

Outline

The GdRT Model

Gabadadze, de Rham, Tolley (2010), Hinterbichler, Rosen (2012)

$$I_{\text{GdRT}}[e] = M_P \int d^3x \left\{ eR(e) - \frac{1}{16} m^2 \varepsilon^{\mu\nu\rho} \varepsilon_{abc} (e_\mu^a + \delta_\mu^a)(e_\nu^b - \delta_\nu^b)(e_\rho^c - \delta_\rho^c) + \right.$$

$$\left. + \alpha\, m^2 \varepsilon^{\mu\nu\rho} \varepsilon_{abc} (e_\mu^a - \delta_\mu^a)(e_\nu^b - \delta_\nu^b)(e_\rho^c - \delta_\rho^c) \right\}$$

- α is a dimensionless parameter

$$e_\mu{}^a = \delta_\mu{}^a + h_\mu{}^a \quad \Rightarrow \quad \text{Fierz-Pauli}$$

3D NMG and 3D massive gravity are different limits

of a 3D bi-metric gravity model

Hassan and Rosen (2012), Paulos and Tolley (2012)

3D Bi-metric Gravity

Hassan, Schmidt-May and von Strauss (2012)

3D : Banados, Theisen (2009), Afshar, Alishahiha, Naseh (2009), Zinoviev (2012)

$$I\,[e,f] \;=\; \int d^3x \,\Big\{\, \sigma M_e\, eR(e) + M_f\, fR(f) \;- \qquad \sigma = \pm 1$$

$$-\, \frac{1}{16} M m^2\, \varepsilon^{\mu\nu\rho}\, \varepsilon_{abc}\,(e_\mu^a + f_\mu^a)(e_\nu^b - f_\nu^b)(e_\rho^c - f_\rho^c) \;+$$

$$+\, \alpha\, M\, m^2 \varepsilon^{\mu\nu\rho}\, \varepsilon_{abc}\,(e_\mu^a - f_\mu^a)(e_\nu^b - f_\nu^b)(e_\rho^c - f_\rho^c)\Big\}$$

- ${e_\mu}^a$ and ${f_\mu}^a$ are two Dreibeins
- $M_e, M_f, M = \frac{M_e M_f}{M_e + M_f}$ and m are (positive) mass parameters

The GdRT limit $(\sigma = +1)$

$$f_\mu{}^a = \delta_\mu{}^a + M_f^{-1/2}\delta f_\mu^a\,, \qquad M_f \to \infty\,, M_e = M = M_P$$

$$I_{\mathrm{GdRT}}[e] = M_P \int d^3x \left\{ eR(e) - \frac{1}{16} m^2\, \varepsilon^{\mu\nu\rho}\, \varepsilon_{abc}\, (e_\mu^a + \delta_\mu^a)(e_\nu^b - \delta_\nu^b)(e_\rho^c - \delta_\rho^c) \; + \right.$$

$$\left. + \alpha\, m^2 \varepsilon^{\mu\nu\rho}\, \varepsilon_{abc}\, (e_\mu^a - \delta_\mu^a)(e_\nu^b - \delta_\nu^b)(e_\rho^c - \delta_\rho^c) \right\}$$

The NMG limit $(\sigma = -1)$

$$f_\mu{}^a = e_\mu{}^a + \lambda\, q_\mu^a\,, \qquad \lambda \to 0\,, M_f \to \infty\,, M_e - M_f = \lambda M_f = M_P$$

$$I_{\rm NMG}\,[e,q] = M_P \int d^3x\, \left\{ -eR(e) + G^{\mu\nu}(e) q_{\mu\nu} - m^2 (q^{\mu\nu} q_{\nu\mu} - q^2)\right\},$$

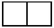
- $q_{\mu\nu}$ is an auxiliary field

Outline

What did we learn?

- two theories can be equivalent at the linearized level (FP and boosted FP) but only one of them allows for a simple and unique non-linear extension i.e. interactions

- we need massive spin 2 whose massless limit describes 0 d.o.f.

Example :  in 3D

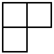
- what about 4D?

Generalized spin-2 FP

standard spin-2 :

$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \qquad m \neq 0 \\ 2 \quad \text{d.o.f.} \qquad m = 0 \end{cases}$$

generalized spin-2 :



$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \qquad m \neq 0 \\ 0 \quad \text{d.o.f.} \qquad m = 0 \end{cases}$$

Curtright (1980)

Connection-metric Duality

- Use first-order form with independent fields ${e_\mu}^a$ and ${\omega_\mu}^{ab}$
- linearize around Minkowski: ${e_\mu}^a = {\delta_\mu}^a + {h_\mu}^a$
  and add a FP mass term $-m^2(h^{\mu\nu}h_{\nu\mu} - h^2)$
- solve for ω $\rightarrow$ standard spin-2 FP
- solve for $h_{\mu\nu}$ $\rightarrow$ generalized spin-2 FP

Present Status

- 4D NMG exists at the quadratic level

- Interactions?

Bekaert, Boulanger, Cnockaert (2005)

- compare to Eddington-Schrödinger theory

$$\mathcal{L}'_{\text{ES}} = \sqrt{-\det g}\,[g^{\mu\nu} R_{\mu\nu}(\Gamma) - 2\Lambda] \quad \Leftrightarrow \quad \mathcal{L}_{\text{ES}} = \sqrt{|\det R_{(\mu\nu)}(\Gamma)|}$$

$$g_{\mu\nu} = \frac{(D-2)}{2\Lambda} R_{(\mu\nu)}(\Gamma)$$

Outline

Summary

- 3D "New" massive gravity and 3D massive gravity are two different ways to decribe massive gravitons

- in 3D both models can be viewed as different limits of 3D bi-metric gravity

Open Issues

- constructing a 4D NMG model including interactions
- supersymmetry ?
- higher spins ?