Strongly Coupled Anisotropic Plasma in AdS/CFT

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arXiv:1202.4436 JHEP
hep-th, hep-ph
and ongoing work.

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Short talk, therefore non conventional presentation:

- **Motivation**
- **Brief Results**
- **Theory and Results**

Why are we looking at these anisotropic theories?

The motivation:

- The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Properties of the supergravity solutions, that are dual to the anisotropic theories. for example, see talks of J. Erdmenger
- There exist several results for observables in weakly coupled anisotropic plasmas. Do their predictions carry on in the strongly coupled limit models?

  [Dumitru, Martinez, Rebhan, Romatschke, Strickland,...]

The main question we answer accurately here is:
How the inclusion of anisotropy modifies the results on several observables in our dual QGP compared to the isotropic theory?

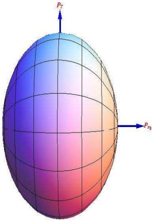
How does the anisotropic Quark Gluon Plasma looks?

Answer: Momentum and Pressure anisotropy.
Anisotropic parameters can be introduced

$$\xi := \frac{\langle p_\perp^2 \rangle}{2\langle p_\parallel^2 \rangle} - 1 \,, \qquad \Delta := \frac{P_\perp}{P_\parallel} - 1 \,.$$

[Romatschke, Strickland,2003]

where $p_{\parallel,\perp}$ are the average longitudinal, transverse momenta of the particles with respect to the anisotropic direction. $P_{\parallel,\perp}$ refers to pressures.
ie: Anisotropy in the momentum space:



What have we calculated?

Answer: Observables using Heavy Quark Probes in an anisotropic holographic plasma.

- The **static potential** and **static force**.
- The **drag force**.
- The **jet quenching**.

•We study the observables dependence on the anisotropy.
•We compare the results along the different directions(obtaining dimensionless quantities) and the isotropic theory.
•We answer the question if there is a possibility for more "quantitative" predictions for a QGP using this set up.

What have we found?

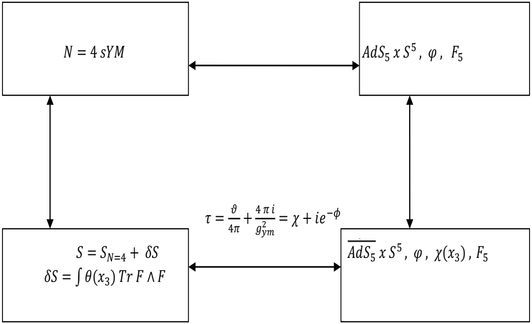
- All the results depend on the particular spatial direction considered and the anisotropic parameter.
- Decrease of static potential and force compared to isotropic theory.
- Increase of the drag force above a critical velocity.
- Existence of three different jet quenching parameters, all enhanced compared to the isotropic theory.

How does the anisotropy is introduced?

- Introduction of additional branes. [Azeyanagi, Li, Takayanagi, 2009]

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following deformation diagram.



How does the background looks?

The metric in string frame [Mateos, Trancanelli, 2011]

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\, dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H} d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\, d\Omega_{S^5}^2 \,.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy. The anisotropic parameter is α with units of inverse length. In sufficiently high temperatures, $T \gg \alpha$, and imposed boundary conditions the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24 u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

The isotropic limit $\alpha \to 0$ reproduce the well know result of the isotropic D3-brane solution (dual to $\mathcal{N} = 4$ finite sYM solution).

The metric can be expressed in α, T parameters through

$$u_h = \frac{1}{\pi T} + \alpha^2 \frac{5 log 2 - 2}{48\pi^3 T^3} \ .$$

The pressures can be found from the expectation value of the stress tensor, where the elements $\langle T_{11} \rangle = \langle T_{22} \rangle = P_{x_1 x_2}$ denote the pressure along the x_1 and x_2 directions and $\langle T_{33} \rangle = P_{x_3}$ is the pressure along the anisotropic direction. The analytic expression read

$$P_{x_1 x_2} = \frac{\pi^2 N_c^2 T^4}{8} + \alpha^2 \frac{N_c^2 T^2}{32}.$$

$$P_{x_3} = \frac{\pi^2 N_c^2 T^4}{8} - \alpha^2 \frac{N_c^2 T^2}{32}.$$

$$P_{x_3} < P_{x_1 x_2}$$

resembling the plasma pressure anisotropies.

Reminder of the notation:

- $Q_{\parallel} := Q_{x_3} = Q_{anisotropic}$, anisotropic, parallel, longitudinal, direction of collision.
- $Q_{\perp} := Q_{x_1 \ or \ x_2}$, transverse direction.
- Q_{iso} , isotropic theory, $\alpha = 0, \mathcal{N} = 4sYM$.

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{Q_{\parallel}}{Q_{\perp}} = ?, \quad \frac{Q_{\parallel}}{Q_{iso}} = ?, \ldots$$

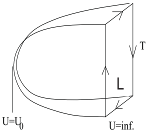
Static Potential in the anisotropic background.

- We consider a string world-sheet (τ, σ) of the following form.

String Configuration

$$x_0 = \tau, \qquad x_p = \sigma, \qquad u = u(\sigma) \ .$$

The x_p is the direction where the pair is aligned:
$x_p = x_2 =: x_\perp$ pair along transverse direction,
$x_p = x_3 =: x_\parallel$ pair along parallel direction to anisotropy.



In general the length of the two endpoints of the string on the boundary is given by

$$L = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu} c_0^2}{(g_{00} g_{pp} + c_0^2) g_{pp}}} \; .$$

Which should be inverted as $u_0(L)$. The normalized energy of the string is

$$2\pi\alpha' V = c_0 L + 2\left[\int_{u_0}^{\infty} du \sqrt{-g_{uu} g_{00}} \left(\sqrt{1 + \frac{c_0^2}{g_{pp} g_{00}}} - 1\right) - \int_{u_h}^{u_0} du \sqrt{-g_{00} g_{uu}}\right] \; .$$

[Sonnenschein 2000;...; D.G...]

In the anisotropic case we get:

• $V_\parallel < V_\perp < V_{iso}$ when the comparison is done with LT keeping α, T fixed.

• $\alpha_1 < \alpha_2 \Rightarrow V_{\parallel_1} > V_{\parallel_2}$. Increase of anisotropy, leads to decrease of the static potential.

• The critical length of the string beyond the quarks are not bounded is decreased in presence of anisotropy as $L_{c\parallel} < L_{c\perp} < L_{c\ iso}$.

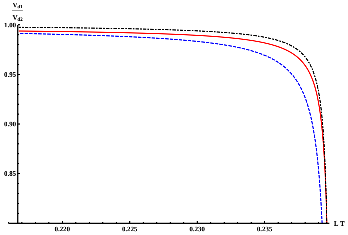


Figure: $V_\parallel / V_\perp$, $V_\parallel / V_{iso}$, $V_\perp / V_{iso}$ vs LT and $T = 3$, $\alpha = 0.35T$.

Reminder

The static force need to be calculated and has been. Static potential has a non-physical constant.

$$F_{Q\bar{Q},anisotropic} < F_{Q\bar{Q},isotropic}$$

Drag Force.

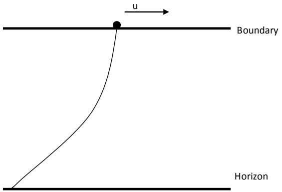
In AdS/CFT the drag force of a single quark moving in the anisotropic plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole. [Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser, 2006]

In radial gauge the trailing string motion along the $x_p := x_{\parallel,\perp}$ directions described by:

String Configuration

$$x_0 = \tau, \qquad u = \sigma, \qquad x_p = v\tau + \xi(u) \ ,$$

where v is the constant speed of the quark in the boundary along the x_p direction.



Calculating the momentum flowing from the boundary to the bulk we can find the drag force for any background to be

$$F_d = -\Pi_u^1 = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)}\Big|_{u=u_0}$$

where here u_0 is given by

$$\left(g_{uu}(g_{00} + g_{pp}v^2)\right) \big|_{u=u_0} = 0 \ .$$

[...]

Drag Force results

The qualitative behavior is

- $F_{\parallel} > F_{iso}$
- $F_{\perp} > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_{\perp} < F_{iso}$.
- $$\frac{F_{\parallel}}{F_{\perp}} = 1 + \alpha^2 \frac{\left(2 - v^2\right) \text{Log}\left[1 + \sqrt{1 - v^2}\right]}{8\pi^2 T^2 \left(1 - v^2\right)} \ .$$

For any velocity: $F_{drag,\parallel} > F_{drag,\perp}$

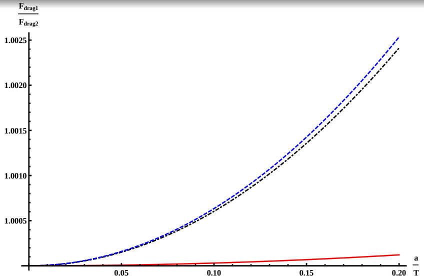


Figure: $F_{drag,\parallel}/F_{drag,\perp}$, $F_{drag,\parallel}/F_{drag,iso}$, $F_{drag,\perp}/F_{drag,iso}$, vs α/T, $v \simeq 0.98$ and $T = 1$.

The jet Quenching.

In the gravity dual description the jet quenching can be calculated from the minimal surface of a world-sheet which ends on an orthogonal Wilson loop lying along the light-like lines.

$$\langle W(\mathcal{C})\rangle = \exp^{-\frac{1}{4\sqrt{2}}\hat{q}L_\perp^2 L_-}$$

[Liu,Rajagopal,Wiedermann,2006]

- The parameter is a measure of energy loss of the quark.

• We go to the light-cone coordinates as $\sqrt{2}x^{\pm} = x_0 \pm x_p$ where $i, p, k = 1, 2, 3$

String configuration

$$x_- = \tau, \quad x_k = \sigma, \quad u = u(\sigma)$$
$$x_+, \; x_{p \neq k} \quad \text{are constant} \; ,$$

The indices k, p denote a chosen direction.
By calculating the on-shell action, canceling the divergences and applying approximations we obtain

$$\hat{q}_{p\ (k)} = \frac{\sqrt{2}}{\pi \alpha'} \left(\int_0^{u_h} \frac{1}{g_{kk}} \sqrt{\frac{g_{uu}}{g_{--}}} \right)^{-1} .$$

Three different parameters.

$\hat{q}$	x_p	x_k	Energetic parton along	Momentum broadening along
$\hat{q}_{\perp(\parallel)}$	$x_\perp$	$x_\parallel$	$x_\perp$	$x_\parallel$
$\hat{q}_{\parallel(\perp)}$	$x_\parallel$	$x_\perp$	$x_\parallel$	$x_\perp$
$\hat{q}_{\perp(\perp)}$	$x_{\perp,1}$	$x_{\perp,2}$	$x_{\perp,1}$	$x_{\perp,2}$

- $\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.
- Enhancement of the jet quenching in presence of anisotropy!

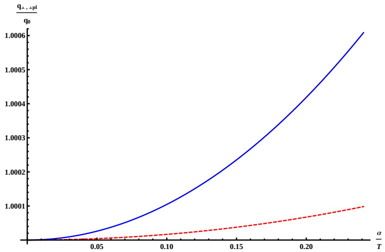


Figure: $\hat{q}_{\perp(\parallel)}$, $\hat{q}_{\perp(\perp)}$ vs α/T. $T = 5$.

Generic Remark

The static potential, the drag force and the jet quenching have known formulas for any generic background! for example [1202.4436, D.G.]
No need to calculate each time the quantities for new backgrounds- just need to apply the formulas.

Other Extensions.

- Extensions of the jet quenching and the drag force calculations to generic directions and larger anisotropies have been done. [Chernicoff, Fernandez, Mateos, Trancanelli, 2012a,b,c].
- Results agree for small anisotropies. Their results change for larger anisotropies but also the inequality of pressures does differ.

Attempt for "quantitative" predictions.

The anisotropic parameter α in supergravity model and the parameter ξ measuring the anisotropy in weakly coupled plasmas can be related by

$$T \gg \alpha \Rightarrow \xi \ll 1 \Rightarrow \xi \backsimeq \frac{5\alpha^2}{8\pi^2 T^2} .$$

Using any comparison normalization scheme (direct or fixed energy or entropy density scheme)

$$\xi_{aSYM} \gtrsim \xi .$$

In our model $\xi \ll 1$; so for $\xi \simeq 1$ values that correspond to the QCD anisotropic plasma, our set up is not valid for a quantitative approach.

Only qualitative results in presence of anisotropy. **Still Very interesting!**

Conclusions.

We have calculated several observables using a IIB supergravity solution in the dual anisotropic finite temperature $\mathcal{N} = 4$ sYM plasma.

- The Static Potential and Force.
- The Drag Force.
- The Jet Quenching.

We have found clear qualitative results for the observables in the anisotropic plasma.

Work in progress:

- Anisotropic holographic baryon.
  eg. extension of [Lozano, Picos, Siampos, DG 2012]
- k-strings.
- Inclusion of flavors$\rightarrow$ many interesting applications.
  eg. [Erdmenger, Evans, Kirsch, Threlfall 2007]
- ...

Anisotropic momentum distribution function in weakly coupled plasmas

The anisotropic distribution function that can be written as

$$f_{aniso} = c_{norm}(\xi) f_{iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

where [Romatschke, Strickland, 2003]

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

and **n** the unit vector along the anisotropic direction.

To relate ξ and α we use the pressures

$$\Delta := \frac{P_T}{P_L} - 1 = \frac{P_{x_1 x_2}}{P_{x_3}} - 1 \ .$$

Using the anisotropic distribution function: [Martinez, Strickland, 2009]

$$\Delta = \frac{1}{2}(\xi - 3) + \xi \left((1+\xi) \frac{\arctan \sqrt{\xi}}{\sqrt{\xi}} - 1 \right)^{-1}$$

Using the supergravity model

$$\Delta = \frac{\alpha^2}{2\pi^2 T^2} \ .$$

For

$$T \gg \alpha \Rightarrow \xi \ll 1 \Rightarrow \xi \backsimeq \frac{5\alpha^2}{8\pi^2 T^2} \ .$$

Supposing we trust the estimation of the anisotropic parameter $\xi \simeq 1$ obtained from

$$\xi = \frac{10\eta}{T\tau s} \ .$$

and using any comparison normalization scheme (direct or fixed energy or entropy density scheme)

$$\xi_{aSYM} \gtrsim \xi \ .$$

In our model $\xi \ll 1$ so for $\xi \simeq 1$ values that correspond to the QCD anisotropic plasma, our approximations are not valid.

Partial List of Results

We have calculated several observables using a IIB supergravity solution in the dual anisotropic finite temperature $\mathcal{N} = 4$ sYM plasma.

- The static potential:
  - $V_{\parallel} < V_{\perp} < V_{iso}$.
  - $\alpha_1 < \alpha_2 \Rightarrow V_{\parallel_1} > V_{\parallel_2}$.
- The drag Force:
  - $F_{\parallel} > F_{iso}$ and $F_{\parallel} > F_{\perp}$.
  - $F_{\perp} > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_{\perp} < F_{iso}$.
- The jet quenching:
  - $\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.
  - In weak coupling has been observed enhancement of the jet quenching as $\hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)}$ in agreement with our results. [Dumitru, Nara, Schenke, Strickland; Baier, Mehtar-Tani, 2008,..].

$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

$$\mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d\,(\mathcal{R})} \left\langle \mathrm{Tr} \left[W_\mathcal{R}^\dagger[0, x_\perp] \, W_\mathcal{R}[0, 0] \right] \right\rangle$$

$$W_\mathcal{R}\left[x^+, x_\perp\right] \equiv P \left\{ \exp\left[ig \int_0^{L^-} dx^- \, A_\mathcal{R}^+(x^+, x^-, x_\perp) \right] \right\}$$

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{L} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$$