Theory of Electroweak Interactions

Giuseppe Degrassi
Universita' di Roma Tre, I.N.F.N. Sezione Roma Tre

Corfu Summer Institute
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History

- Fermi theory of ß-decay (34):

contact interactions between two currents (prototype of modern effective theories)

$$\mathcal{L}_F = -\frac{G_\beta}{\sqrt{2}} J^\dagger_{(h)\,\mu} J^\mu_{(l)} + h.c. = -\frac{G_\beta}{\sqrt{2}} [\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu(x)] + h.c.$$

- Parity nonconservation (56-57); V-A law (58); CVC hypothesis ($G_ß \sim G_\mu$) (58)

$$\mathcal{L}_F = -\frac{G_\beta}{\sqrt{2}} [\bar{p}(x)\gamma_\mu (1-\lambda\gamma_5) n(x)][\bar{e}(x)(\gamma^\mu (1-\gamma_5)\nu(x)] + h.c. \qquad (\lambda \simeq 1.27)$$

- Quark hypothesis (60); Cabibbo theory (63);

$$\begin{aligned}
\mathcal{L}_{eff} &= -\frac{G_\mu}{\sqrt{2}} J^\dagger_\lambda J^\lambda \\
J^\lambda &= J^\lambda_{(h)} + J^\lambda_{(l)} \\
J^\lambda_{(l)} &= \bar{\nu}_e \gamma^\lambda (1-\gamma_5) e + \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5) \mu \\
J^\lambda_{(h)} &= \cos\theta_c\, \bar{u}\gamma^\lambda (1-\gamma_5) d + \sin\theta_c\, \bar{u}\gamma^\lambda (1-\gamma_5) s
\end{aligned}$$

Θ_c : Cabibbo angle

$$G_\beta = G_\mu \cos\theta_c \simeq 0.98$$

Determination of G_μ from $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

$$\begin{aligned}
\mathcal{M} &= -i\frac{G_\mu}{\sqrt{2}}\bar{u}(e)\gamma^\lambda(1-\gamma_5)v(\nu_e)u(\nu_\mu)\gamma_\lambda(1-\gamma_5)u(\mu) \\
\frac{1}{\tau} &= \frac{G_\mu m_\mu^5}{192\pi^3}
\end{aligned}$$

Today:

$$\frac{1}{\tau} = \frac{G_\mu m_\mu^5}{192\pi^3}F[x]\left(1+\frac{\alpha(m_\mu)}{\pi}H_1[x]+\frac{\alpha(m_\mu)^2}{\pi^2}H_2[x]\right) \qquad x \equiv m_e^2/m_\mu^2$$

$$\begin{aligned}
F[x] &= 1-8x+8x^3-x^4-12x^2\ln x = 0.99981295 \\
H_1[x] &= \frac{25}{8}-\frac{\pi^2}{2}+\mathcal{O}(x) = 1.80793 \qquad (1959) \\
H_2[x] &= .... = 6.64 \qquad (2000) \\
\alpha(m_\mu)^{-1} &= \alpha^{-1}+\frac{1}{3\pi}\ln x+\frac{1}{6\pi} = 135.901
\end{aligned}$$

$$\begin{aligned}
m_\mu &= 105.6583715 \pm 0.0000035\,\mathrm{MeV} \\
m_e &= 0.510998928 \pm 0.000000011\,\mathrm{MeV} \\
\tau &= (2.1969811 \pm 0.0000022)\times 10^{-6}s
\end{aligned}$$

→

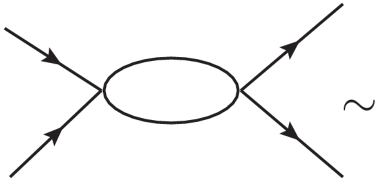
$$G_\mu = 1.1663787(6)\times 10^{-5}\mathrm{GeV}^{-2}$$

Particle data book value

Fermi theory (or any effective):

- Not renormalizable

$$[\mathcal{L}] = 4,\ [\psi] = 3/2 \Rightarrow [G_\mu] = -2$$



$\sim \int \frac{d^4k}{k^2}$

- Violate unitarity :
  Ex.: $\nu_\mu(k_1) + e^-(p_1) \rightarrow \nu_e(p_2) + \mu^-(k_2)$

$$\mathcal{M} = -i\frac{G_\mu}{\sqrt{2}}\bar{u}(\mu)\gamma^\lambda(1-\gamma_5)u(\nu_\mu)\bar{u}(\nu_e)\gamma_\lambda(1-\gamma_5)u(e)$$

$$|\bar{\mathcal{M}}|^2 = \frac{G_\mu^2}{2}\mathrm{Tr}\,[\not{k}_2\gamma_\mu(1-\gamma_5)\not{k}_1\gamma_\nu(1-\gamma_5)]\,\frac{1}{2}\mathrm{Tr}\,[\not{p}_2\gamma^\mu(1-\gamma_5)\not{p}_1\gamma^\nu(1-\gamma_5)] = \frac{G_\mu^2}{2}32s^2$$

$$d\sigma = |\bar{\mathcal{M}}|^2\,\frac{1}{4s}\,\frac{1}{(4\pi)^2}d\Omega$$

$$\sigma = \frac{G_\mu^2\ s}{\pi}$$

But optical theorem tells us the total cross section is related to the amplitude for elastic scattering in the forward direction

$$\sigma_T(\nu_\mu, e^- \rightarrow anything) = \frac{1}{s}\mathrm{Im}\mathcal{A}(s, J, \theta = 0)$$

$$\mathcal{A}(s, l, \theta) = 16\pi\sum_{l=o}^{\infty}(2l+1)P_l(\cos\theta)e^{i\delta_l}\sin\delta_l$$

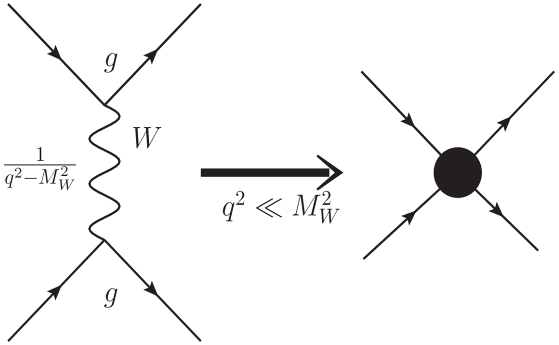
Spinless particle

$$\sigma \leq \sigma_T \leq \frac{O(\pi)}{s} \Rightarrow s \leq \frac{O(\pi)}{G_\mu}$$

Intermediate Vector Boson theory (IVB)

The contact interaction between currents is the result of the exchange of a heavy charged vector boson

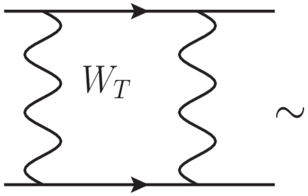
$$\mathcal{L}_{int} = -g J_-^\mu W_\mu^+ + h.c. \qquad J_-^\mu \equiv J_{(l)}^\mu + J_{(h)}^\mu$$



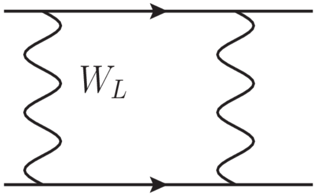
$$G_\mu/\sqrt{2} = g^2/M_W^2$$

[g]=0 but theory not renormalizable; problem stays in the longitudinal part of the vector boson propagator

$$i\Delta_W^{\mu\nu}(k) = -i\frac{g^{\mu\nu} - k^\mu k^\nu/M_W^2}{k^2 - M_W^2 + i\epsilon}$$



$$\sim \int \frac{d^4k}{k^6}$$



$$\sim \int \frac{d^4k}{k^2}$$

Similarly we expect unitarity problem in processes with longitudinal W's like

$$e^+ + e^- \to W^+ + W^-$$

The Standard Electroweak Theory

Promote the IVB to be the carrier of a gauge interaction as described by a gauge Lagrangian $\mathcal{L}_g$

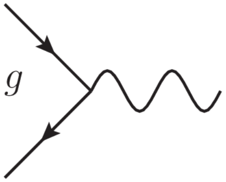
To any vector boson V^A_μ there is an associated generator T^A of the gauge group G forming a closed algebra

$$[T^A, T^B] = i f^{ABC} T^C, \qquad f^{ABC}$$ Structure constants of G

$$\begin{aligned}
\mathcal{L}_g &= -\frac{1}{4}\sum_{A=1}^{N} F^A_{\mu\nu} F^{A\,\mu\nu} + i\bar{\Psi}\not{D}\Psi + |D_\mu\phi|^2 \\
F^A_{\mu\nu} &= \partial_\mu V^A_\nu - \partial_\nu V^A_\mu + g f^{ABC} V^B_\mu V^C_\nu \\
D_\mu &= \partial_\mu - ig\sum_{A=1}^{N} V^A_\mu T^A
\end{aligned}$$

Gauge symmetry dictates the Interactions of V^A_μ

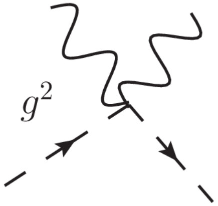
V^A_μ interact with matter fields via currents $\sum_{A=1}^{N} J^A_\mu V^{A\mu}$

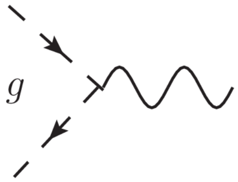


$$J^A_\mu(\Psi) = \bar{\Psi}\gamma_\mu T^A \Psi, \; J^A_\mu(\phi) = \phi^\dagger T^A \partial_\mu\phi - \partial_\mu\phi^\dagger T^A \phi$$

For scalars there Is also a “sea-gull” term

$$\sum_{A,B=1}^{N} \phi^\dagger T^A T^B \phi V^{A\dagger}_\mu V^{B\mu}$$





Fermions and scalars are arranged in representation of G. For massless fermions the l.h. and r.h. components can be given different transformation properties under the Symmetry

$$\bar{\Psi} i \not{D} \Psi = \bar{\Psi}_L i \not{D} \Psi_L + \bar{\Psi}_R i \not{D} \Psi_R$$

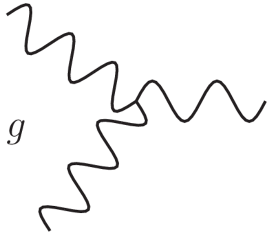
$$\Psi_L = \frac{1-\gamma_5}{2}\Psi \qquad \bar{\Psi}_L = \Psi_L^\dagger \gamma_0 = \bar{\Psi}\frac{1+\gamma_5}{2}$$
$$\Psi_R = \frac{1+\gamma_5}{2}\Psi \qquad \bar{\Psi}_R = \Psi_R^\dagger \gamma_0 = \bar{\Psi}\frac{1-\gamma_5}{2}$$

(Ψ Dirac field)

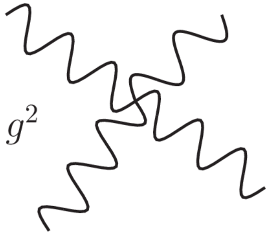
Mass terms break the symmetry if l.h. and r.h. fermions have different symmetry transformations

$$m\bar{\Psi}\Psi = m\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L$$

Non Abelian group: N generators, $f^{ABC} \neq 0$
Gauge symmetry gives trilinear and quadrilinear self- interactions of V^A_μ



derivative



contact

Abelian group: U(1) (N=1, f^{ABC} =0)
QED: T^1 = Q, g =e, no self-interactions between photons

Gauge symmetry does not allow an explicit mass term m V^A_μ $V^{A\mu}$

Getting the electroweak group (Glashow 61)

l.h. fermions enters into the weak charged current interactions.
l.h. and r.h. fermions enter into the e.m. Interactions.
Fermi charged current can be rewritten as a gauge current of an SU(2) group

$$J^F_\mu = \bar{\nu}\gamma_\mu(1-\gamma_5)e + \bar{u}\gamma_\mu(1-\gamma_5)e \Rightarrow J^-_\mu = \bar{l}_L\gamma_\mu\tau^- l_L + \bar{q}_L\gamma_\mu\tau^- q_L$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}; \quad l_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \quad \tau^\pm = \tau_1 \pm i\tau_2, \quad \tau_i$$ Pauli matrices

Algebra of SU{2)

$$T^A = \frac{\tau^A}{2} \quad (A=1,2,3), \quad \left[T^A, T^B\right] = i\epsilon^{ABC}T^C \to \left[T^1, T^2\right] = iT^3 = i\frac{\tau_3}{2}$$ Neutral current

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}}\left(J^-_\mu W^{+\mu} + J^+_\mu W^{-\mu}\right) + gJ^3_\mu W^3_\mu$$

$$J^+_\mu = (J^-_\mu)^\dagger, \quad W^\pm = \frac{W^1 \pm iW^2}{\sqrt{2}}, \quad J^3_\mu = \bar{q}_L\gamma_\mu\frac{\tau_3}{2}q_L + \bar{l}_L\gamma_\mu\frac{\tau_3}{2}l_L$$

Q cannot identified with T_3 but Q-T_3 has the same value on the members of the SU(2) doublets

$[Q- T_3, T_i] =0$

Electroweak Group: SU(2)x $U(1)_Y$ where Y = Q - T_3

Fermions quantum numbers (one generation)

ψ	u_L	d_L	u_R	d_R	ν_L	e_L	e_R	ν_R
T_3	1/2	-1/2	0	0	1/2	-1/2	0	0
Y	1/6	1/6	2/3	-1/3	-1/2	-1/2	-1	0

l.h. fermions are in SU(2) doublets, r.h. fermions in SU(2) singlets

Electroweak Lagrangian (gauge part, no mass terms)

$$
\begin{aligned}
\mathcal{L}_{symm} &= -\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\bar{\psi}_L \not{D}\psi_L + i\bar{\psi}_R \not{D}\psi_R \\
F^A_{\mu\nu} &= \partial_\mu W^A_\nu - \partial_\nu W^A_\mu + g\epsilon^{ABC} W^B_\mu W^C_\nu \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
D_\mu \psi_L &= \left[\partial_\mu - i\frac{g}{\sqrt{2}}\left(\tau^- W^+ - \tau^+ W^-\right) - igT^3 W^3_\mu - ig' Y B_\mu\right]\psi_L \\
D_\mu \psi_R &= \left[\partial_\mu - ig' Y B_\mu\right]\psi_R
\end{aligned}
$$

Neutral currents

$$\begin{aligned} g T^3 W^3_\mu + g' Y B_\mu \qquad & l.h. \\ g' Y B_\mu \qquad & r.h. \end{aligned}$$

Rotate the W^3, B field to obtain a new field with vectorial couplings

$$\begin{aligned} W^3_\mu &= \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{aligned}$$

A_μ couplings:

$$\underbrace{g \sin\theta_W}_{e} T^3 + \underbrace{g' \cos\theta_W}_{e} Y = e(T^3 + Y) = eQ; \qquad \tan\theta_W = \frac{g'}{g}$$

$$e\bar{\psi}_L \gamma_\mu \left[T^3 + Y\right] \psi_L + e\bar{\psi}_R \gamma_\mu Y \psi_R = e\bar{\psi}\gamma_\mu Q \psi$$

Vectorial current

Z_μ couplings:

$$g \cos\theta_W T^3 - g' \sin\theta_W Y = g \cos\theta_W T^3 - g' \sin\theta_W (Q - T^3) = \frac{g}{\cos\theta_W} \left(T^3 - Q \sin^2\theta_W\right)$$

$$\frac{g}{\cos\theta_W} \bar{\psi}_L \gamma_\mu \left[T^3 - Q\sin^2\theta_W\right] \psi_L + \underbrace{g \tan\theta_W}_{g'} \sin\theta_W \bar{\psi}_R \gamma_\mu Y \psi_R = \frac{g}{\cos\theta_W} \bar{\psi} \gamma_\mu \left[T^3 \frac{1-\gamma_5}{2} - Q \sin^2\theta_W\right] \psi$$

Effective 4-fermion interactions at low energy:

Charged current:

$$\mathcal{L}^{cc}_{eff} = \frac{-g^2}{2M_W^2} J^+_\mu J^{-\mu} \Rightarrow \frac{g^2}{8M_W^2} = \frac{G_\mu}{\sqrt{2}}$$

Neutral current:

$$\mathcal{L}^{nc}_{eff} = \frac{-g^2}{2\cos^2_W M_Z^2} J^Z_\mu J^{Z\mu} = 4\frac{G_\mu}{\sqrt{2}}\frac{M_W^2}{M_Z^2 \cos^2\theta} J^Z_\mu J^{Z\mu} \equiv 4\frac{G_\mu}{\sqrt{2}}\rho J^Z_\mu J^{Z\mu}$$

Symmetry factor for 2 identical currents

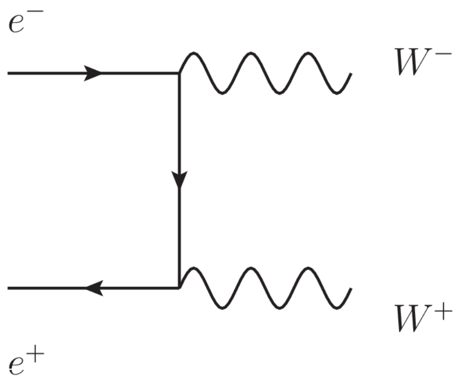
$$\frac{1}{4}\mathcal{L}_{eff} = \frac{1}{4}\left(\mathcal{L}^{CC}_{eff} + \mathcal{L}^{NC}_{eff}\right) = \frac{G_\mu}{\sqrt{2}}\left(J^+_\mu J^{-\mu} + \rho J^Z_\mu J^{Z\mu}\right) \qquad \rho \equiv \frac{M_W^2}{M_Z^2 \cos\theta_W}$$

Note: if I know sin θ$_W$, for example from N.C. experiments, I can predict M$_W$

$$M_W^2 = \frac{\sqrt{2}g^2}{8G_\mu} = \frac{\sqrt{2}\,e^2}{8G_\mu \sin^2\theta_W} = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{\sin^2\theta_W} \equiv \frac{A^2}{\sin^2\theta_W}$$

$$A = \left(\frac{\pi\alpha}{\sqrt{2}G_\mu}\right)^{1/2} = 37.28039(1)\ \mathrm{GeV}$$

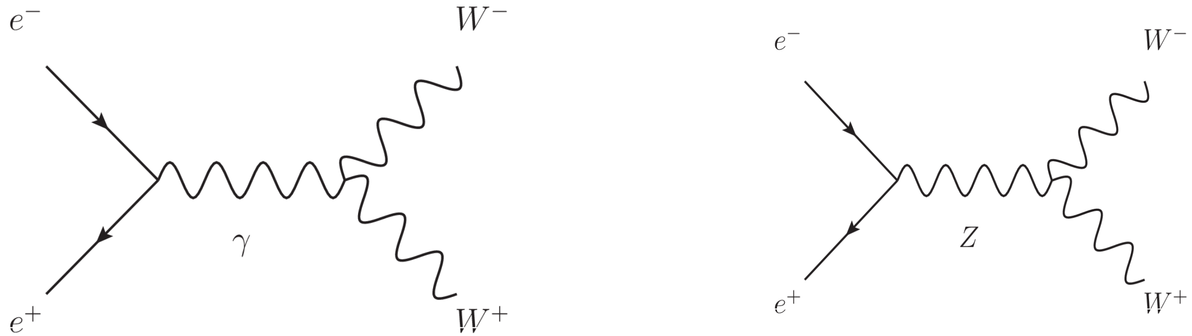
In the IVB we expected the $e^+ + e^- \rightarrow W^+ + W^-$ cross-section to raise with s (the C.M. energy) when the W's are longitudinally polarized



IVB

But in our gauge theory we have two extra contributions from

$$-\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\,\mu\nu} \Rightarrow 2\frac{1}{2}g\epsilon^{ABC}\partial_\mu W^A_\nu W^B_\mu W^C_\nu \Rightarrow g\epsilon^{123}\underbrace{\partial_\mu W^1_\nu W^2_\mu}_{W^\pm}\underbrace{W^3_\nu}_{\gamma\; Z}$$



These two diagrams cancel the bad high energy behavior of the neutrino exchange diagram

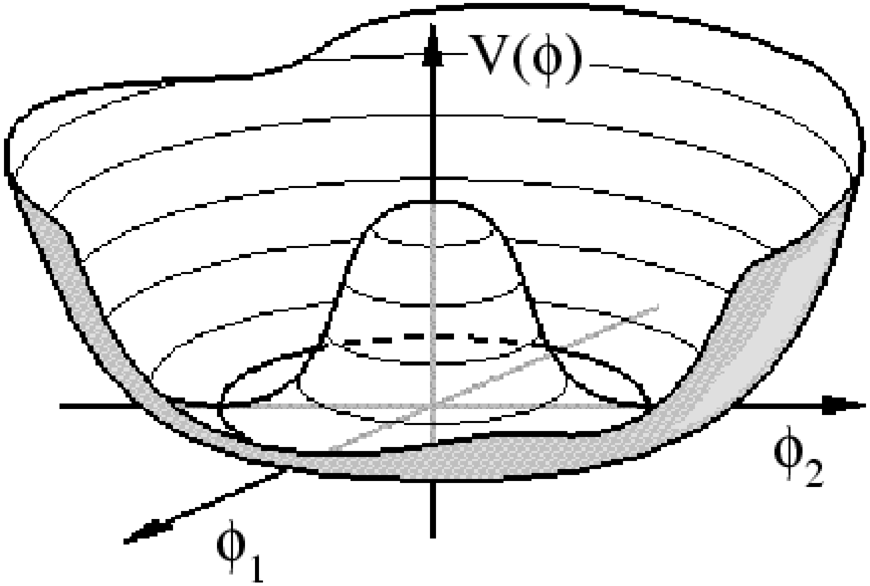
Getting the masses via spontaneous symmetry breaking

The Lagrangian of the theory respects a symmetry, but the vacuum state breaks it

Consider a single complex scalar with a “mexican hat” potential *(Goldstone model)*

$$\phi \equiv \frac{1}{\sqrt{2}}(\phi_1 + i\,\phi_2)\,, \quad \mathcal{L} \;=\; \partial^\mu \phi^* \,\partial_\mu \phi - V(\phi)\,, \quad V(\phi) \;=\; m^2\,|\phi|^2 + \lambda\,|\phi|^4$$

$$m^2 < 0\,, \quad \lambda > 0$$



The potential has an infinite number of equivalent minima for $|\phi|^2 \;=\; -\frac{m^2}{2\,\lambda}$

The system will choose one specific minimum, breaking the global rotational symmetry

We can expand the scalar field around a *real* vacuum expectation value (vev)

$$\phi \equiv \frac{1}{\sqrt{2}}\,[v + H(x) + i\,G(x)]\ , \qquad v = \sqrt{-\frac{m^2}{\lambda}}$$

At the minimum of the scalar potential (= the vacuum state) we have $\langle\phi\rangle = \frac{v}{\sqrt{2}}$

Up to an irrelevant constant, the scalar potential becomes

$$\begin{aligned} V \;&=\; (m^2 v + \lambda v^3)\,H + \frac{1}{2}\,(m^2 + 3\lambda v^2)\,H^2 + \frac{1}{2}\,(m^2 + \lambda v^2)\,G^2 \\ &\quad + \lambda v\,H(H^2 + G^2) + \frac{\lambda}{4}\,(H^2 + G^2)^2 \end{aligned}$$

Inserting the value of v the linear term vanishes, and the masses of the scalars become

$$m_H^2 = -2\,m^2 = 2\,\lambda\,v^2\,, \qquad m_G^2 = 0$$

G is the *Goldstone boson* associated with the spontaneous breaking of the global symmetry

In general: the number of Goldstone boson is related to the number of broken generators of the symmetry

Broken generator: it does not annihilate the vacuum

The Higgs mechanism

Simplest U(1) model

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |\underbrace{(\partial_\mu - ieA_\mu)}_{D_\mu}\phi|^2 - (-m^2)\,|\phi|^2 - \frac{\lambda}{4}\,|\phi|^4 \qquad\qquad m^2 > 0$$

Invariant under:

$$\phi \quad \rightarrow \quad \phi' = \phi\, exp[-i\epsilon(x)]$$

$$A_\mu \quad \rightarrow \quad A'_\mu = A_\mu + \frac{1}{e}\,\partial_\mu\epsilon(x)$$

if $<\phi>= \frac{v}{\sqrt{2}}$

I shift the field Φ and write it in polar coordinates: $\phi(x) = \frac{1}{\sqrt{2}}\,(\rho(x) + v)\, exp[+i\frac{\chi(x)}{v}]$

via a gauge transformation I can eliminate χ $\left(\epsilon(x) = \frac{\chi(x)}{v}\right)$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}e^2v^2A^\mu A_\mu + \frac{1}{2}e^2\rho^2A^\mu A_\mu + e^2\rho v A^\mu A_\mu + \mathcal{L}(\rho)$$

No χ, A_μ massive (3 d.o..f.); χ eaten by A_μ

SU(2)xU(1): (Weinberg 67, Salam 68)

SSB via an Higgs doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{1/2}$

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{symm} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk} \\ \mathcal{L}_{Higgs} &= (D_\mu\Phi)^\dagger (D^\mu\Phi) - \underbrace{\left[(-m^2)\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2\right]}_{V(\Phi^\dagger\Phi)} \end{aligned}$$

Renormalizable interaction

$$\mathcal{L}_{Yuk} = -Y_d\bar{\psi}_L\Phi\psi_R^d - Y_u\bar{\psi}_L\tilde{\Phi}\psi_R^u + h.c., \quad \psi_L = \begin{pmatrix} \psi^u \\ \psi^d \end{pmatrix}_L, \quad \tilde{\Phi} = i\tau_2\Phi^*$$

if $< \Phi >= \begin{pmatrix} o \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

Shift **Φ** and write it in terms of 4 real fields, ρ, χ_1,χ_2,χ_3 as

$$\Phi(x) = exp[i\tau \cdot \chi(x)/v] \begin{pmatrix} 0 \\ \frac{\rho(x)+v}{\sqrt{2}} \end{pmatrix}$$

via gauge transformation I can eliminate χ
(unitary gauge)

Q = T_3 +Y annihilates the vacuum $Q\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = 0$

3 broken generators, 3 χ's eaten: 3 massive vector boson, one massless:
$SU(2)xU(1) \rightarrow U(1)_{em}$

Gauge boson masses:

$$D_\mu \Phi = \left[\partial_\mu - i \frac{g}{\sqrt{2}} \left(\tau^- W^+ - \tau^+ W^- \right) - i g T^3 W^3_\mu - i g' Y B_\mu \right] \Phi, \quad \Phi \rightarrow < \Phi >$$

$$\begin{aligned}
M_W^2 W_\mu^\dagger W^\mu &= \frac{g^2}{2} \left| \tau_+ < \phi > \right|^2 W_\mu^\dagger W^\mu = \frac{g^2}{2} < \Phi > \tau_- \tau_+ < \Phi > W_\mu^\dagger W^\mu = \frac{g^2 v^2}{4} W_\mu^\dagger W^\mu \\
\frac{1}{2} M_Z^2 Z^\mu Z_\mu &= \left| \left(g \cos\theta_W T^3 - g' \sin\theta_W (\overbrace{Q - T^3}^{Y}) \right) < \Phi > \right|^2 Z^\mu Z_\mu, \qquad (Q < \phi >= 0) \\
&= \left(g \cos_W + g' \sin\theta_W \right)^2 \left| T^3 < \phi > \right|^2 Z^\mu Z_\mu = \frac{g^2}{\cos^2\theta_W} \left| T^3 < \phi > \right|^2 Z^\mu Z_\mu = \frac{1}{2} \frac{g^2 v^2}{4 \cos^2\theta_W} Z^\mu Z_\mu
\end{aligned}$$

$$M_W^2 = \frac{g^2 v^2}{4}; \quad M_Z^2 = \frac{g^2 v^2}{4 \cos^2\theta_W} \Rightarrow \rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

Only if the Higgs fields are singlets or doublets

If there are several Higgses in generic representation (T, T_3)

$$\rho = \frac{\sum_{\Phi_a} \frac{1}{2} < T^+ T^- + T^- T^+ >|_{<\Phi_a>} v^2_{\Phi_a}}{\sum_{\Phi_a} 2 < (T^3)^2 >|_{<\Phi_a>} v^2_{\Phi_a}} == \frac{\sum_{\Phi_a} \left[T(T+1) - (T^3)^2 \right]_{\Phi_a} v^2_{\Phi_a}}{\sum_{\Phi_a} 2 \left[(T^3)^2 \right]_{\Phi_a} v^2_{\Phi_a}}$$

We must have at least one Higgs doublet to give mass to the fermions:

doublet, doublet, singlet

$$\begin{aligned}\mathcal{L}_{Yuk} &= -Y_d\bar{\psi}_L\Phi\psi_R^d - Y_u\bar{\psi}_L\tilde{\Phi}\Psi_R^u + h.c. \\ & -Y_d\frac{v}{\sqrt{2}}\left(\bar{\psi}_L^d\psi_R^d + \bar{\psi}_R^d\psi_L^d\right) - Y_u\frac{v}{\sqrt{2}}\left(\bar{\psi}_L^u\psi_R^u + \bar{\psi}_R^u\psi_L^u\right) \qquad \Phi \Rightarrow < \Phi > \\ \Rightarrow & \quad m_u = \frac{Y_u v}{\sqrt{2}}, \qquad m_d = \frac{Y_d v}{\sqrt{2}}\end{aligned}$$

The ρ parameter fixes the relative strength of the charged-and neutral current interactions. Its experimental value is very close to 1, but the exact value depends on the experiments one is considering (radiative corrections enters into the game) . The value of ρ extracted in neutrino-hadron scattering is (slightly) different from that of neutrino-electron scattering.

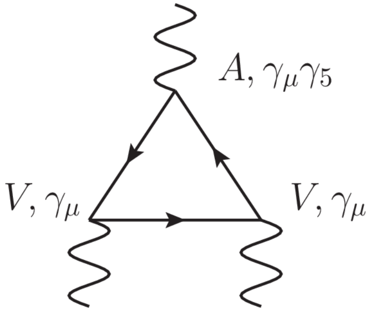
$$\rho = 1 \Rightarrow \cos^2\theta_W = \frac{M_W^2}{M_Z^2}$$

If I know M_Z I can predict M_W

$$M_W^2 = \frac{A^2}{\sin^2\theta_W} = \frac{A^2}{1 - M_W^2/M_Z^2} \Rightarrow M_W^2 = \frac{M_Z^2}{2}\left\{1 + \left[1 - \frac{4A^2}{M_Z^2}\right]^{1/2}\right\}$$

The S.M. Is a renormalizable theory ('t Hooft, Veltman 71-72)

The theory is anomaly free



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$$Tr(T^3Q^2) = \underbrace{3 \cdot \frac{1}{2} \cdot \frac{4}{9}}_{u} + \underbrace{3 \cdot (-\frac{1}{2}) \cdot \frac{1}{9}}_{d} + \underbrace{\frac{1}{2} \cdot 0}_{\nu} + \underbrace{(-\frac{1}{2}) \cdot 1}_{e} = 0$$

However in the unitary gauge where only physical fields are present (the would be G.B. are eliminated) the propagator of the massive V.B. has a bad high energy behavior

$$i\Delta_W^{\mu\nu}(k) = -i\frac{g^{\mu\nu} - k^\mu k^\nu/M_W^2}{k^2 - M_W^2 + i\epsilon}$$

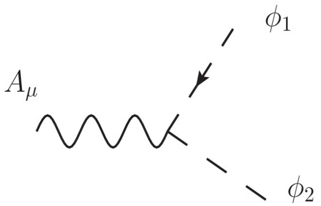
and the theory seems to be not renormalizable by power-counting arguments.
However, it is possible to choose a smart gauge (R_ξ gauge)
where the V.B. propagator has a good high energy behavior

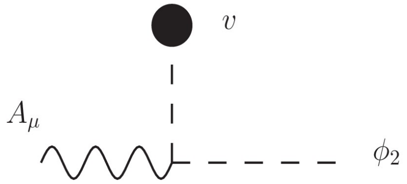
$$i\Delta_W^{\mu\nu}(k) = \frac{-i}{k^2 - M_W^2 + i\epsilon}\left[g^{\mu\nu} + (\xi - 1)\frac{k^\mu k^\nu}{k^2 - \xi M_W^2}\right]$$

U(1): $\phi = \phi_1 + \phi_2; \qquad <\phi_1>= v, <\phi_2>= 0$

unbroken

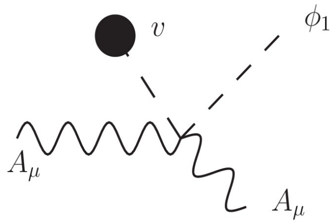
broken



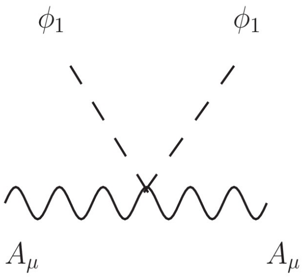


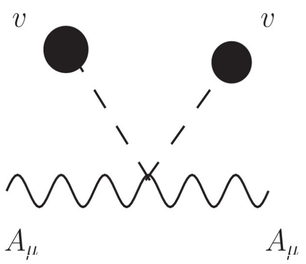
mixing eliminated via

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi}(\partial^\mu A_\mu + \xi \underbrace{e\,v}_{M_A} \phi_2)^2$$



interaction





mass

Renormalization of the S.M.

In the gauge sector there are 3 parameters: g , g', v → 3 renormalization conditions. Whatever renormalization scheme we use we want to express our results in terms of the 3 best known parameters:

$$\begin{aligned} \alpha &= 1/137.035999174(35)\ [0.25 ppb] \quad (a_e) \\ G_\mu &= 1.1663787(6) \times 10^{-5}\ \ \mathrm{GeV}^{-2} \\ M_Z &= 91.1876(21)\ \ \mathrm{GeV}/c^2 \quad (\text{peak of } \sigma(e^+e^- \to all)) \end{aligned}$$

$$a_e(QED) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)}$$

Computed up to n=5

We need the radiative corrected relations between our renormalized parameters and α, G_μ and M_Z

Bare relations:

$$e_0 = g_0 \sin\theta_{W_0}, \quad g'_0 = g_0 \tan\theta_{W_0}, \quad M_{W_0} = \frac{g_0 v_0}{2}, \quad \rho_0 = \frac{M_{W_0}^2}{M_{Z_0}^2 \cos^2\theta_{W_0}} = 1$$

Natural to identify:

$$g = \frac{e}{\sin\theta_W} \qquad e = \sqrt{4\pi\alpha}$$

fine structure constant

renormalized

Using the physical (pole) masses of the W and Z we have two possibility:

$$\cos^2\theta_W = \frac{M_W^2}{M_Z^2} \qquad \rho = 1 \qquad (1)$$

$$\cos^2\theta_W \neq \frac{M_W^2}{M_Z^2} \qquad \rho = 1 + \mathcal{O}(\alpha) \qquad (2)$$

Notation:

$$c_0^2 \equiv \cos^2\theta_{W_0}, \quad s_0^2 \equiv \sin^2\theta_{W_0}$$
$$c_0^2 = c^2 - \delta c^2, \quad s_0^2 = s^2 - \delta s^2$$

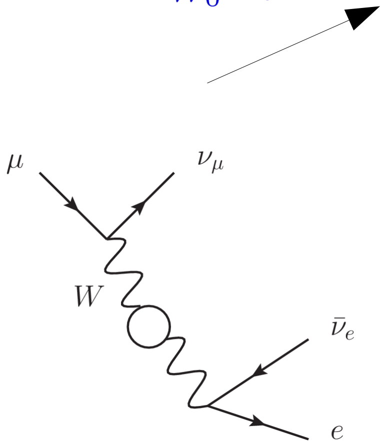
On-Shell scheme

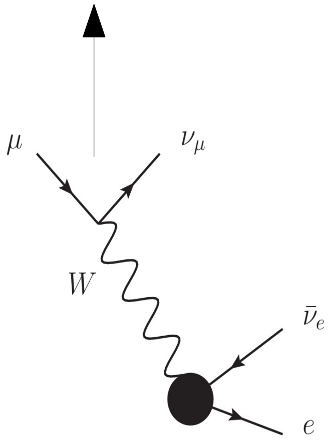
$$(1) \qquad \frac{\delta c^2}{c^2} \simeq \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} = \frac{Re\, A_{WW}(M_W^2)}{M_W^2} - \frac{Re\, A_{ZZ}(M_Z^2)}{M_Z^2}$$

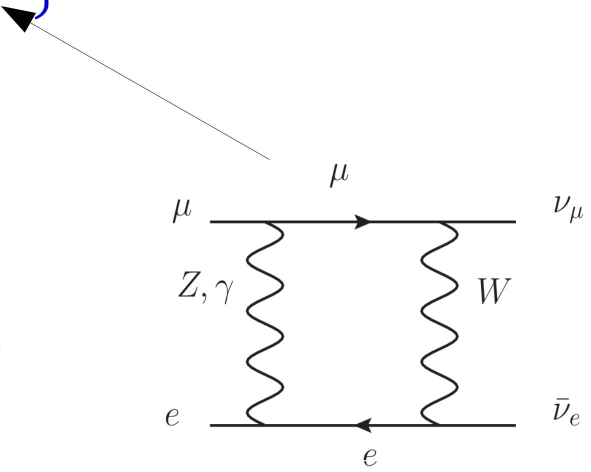
$A_{WW}(q^2)$ Transverse part of the WW self-energy at momentum q^2

Relation with G_μ

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8M_{W_0}^2}\left\{1 - \frac{A_{WW}(0)}{M_W^2} + V_W + M_W^2 B_W\right\}$$







separate e.m.

$$\begin{aligned}\frac{G_\mu}{\sqrt{2}} &= \frac{g_0^2}{8M_{W_0}^2}\left\{1-\frac{A_{WW}(0)}{M_W^2}+V_W+M_W^2 B_W\right\}\\ &= \frac{e^2}{8\,s^2\,M_W^2}\left\{1+\frac{\delta M_W^2}{M_W^2}-\frac{A_{WW}(0)}{M_W^2}+V_W+M_W^2 B_W-\frac{\delta e^2}{e^2}+\frac{\delta s^2}{s^2}\right\}\\ &= \frac{e^2}{8\,s^2\,M_W^2}\left\{1+\frac{Re\,A_{WW}(M_W^2)}{M_W^2}-\frac{A_{WW}(0)}{M_W^2}+V_W+M_W^2 B_W\right.\\ &\qquad\left.+\Pi_{\gamma\gamma}(0)+\frac{c^2}{s^2}\left[\frac{Re\,A_{ZZ}(M_Z^2)}{M_Z^2}-\frac{Re\,A_{WW}(M_W^2)}{M_W^2}\right]\right\}\\ &= \frac{e^2}{8\,s^2\,M_W^2}\{1+\Delta r\}\end{aligned}$$

$$\Pi_{\gamma\gamma}(q^2)=\frac{A_{\gamma\gamma}(q^2)}{-q^2}$$

$$M_W^2=\frac{M_Z^2}{2}\left\{1+\left[1-\frac{4A^2}{M_Z^2(1-\Delta r)}\right]^{1/2}\right\}$$

Radiatively corrected

Large contributions in Δr: $\Pi_{\gamma\gamma}(0),\quad \frac{c^2}{s^2}\left[\frac{Re\,A_{ZZ}(M_Z^2)}{M_Z^2}-\frac{Re\,A_{WW}(M_W^2)}{M_W^2}\right]$

$\Pi_{\gamma\gamma}(0)$

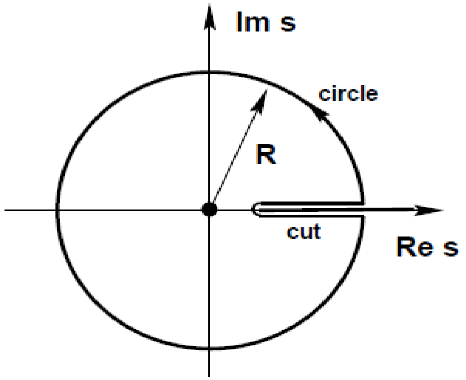
Contains hadronic contributions at low energy that cannot be computed in perturbation theory

$$\Pi^h_{\gamma\gamma}(0) = \underbrace{\Pi^h_{\gamma\gamma}(0) - Re\,\Pi^h_{\gamma\gamma}(M_Z^2)} + Re\,\Pi^h_{\gamma\gamma}(M_Z^2) \longleftarrow$$

perturbative

$$= -\frac{M_Z^2}{\pi} Re \int_{4m_\pi^2}^{\infty} ds \frac{Im\Pi^h_{\gamma\gamma}(s)}{s(s-M_Z^2-i\epsilon)}$$

Dispersion relation (Cauchy)



but, from optical theorem

$$Im\,\Pi^h_{\gamma\gamma} = -\frac{s}{e^2}\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)(s)$$

using

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma_{tot}(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

$$\begin{aligned}\Delta\alpha^{(5)}_{hadrons} &\equiv Re\,\Pi^h_{\gamma\gamma}(M_Z^2) - \Pi^h_{\gamma\gamma}(0) = -\frac{\alpha M_Z^2}{3\pi} Re \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\epsilon)} \\ &= -\frac{\alpha M_Z^2}{3\pi} Re \int_{4m_\pi^2}^{s_0} ds \frac{R(s)}{s(s-M_Z^2-i\epsilon)} - \frac{\alpha M_Z^2}{3\pi} Re \int_{s_0}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\epsilon)}\end{aligned}$$

Using experimental data

Using perturbative QCD

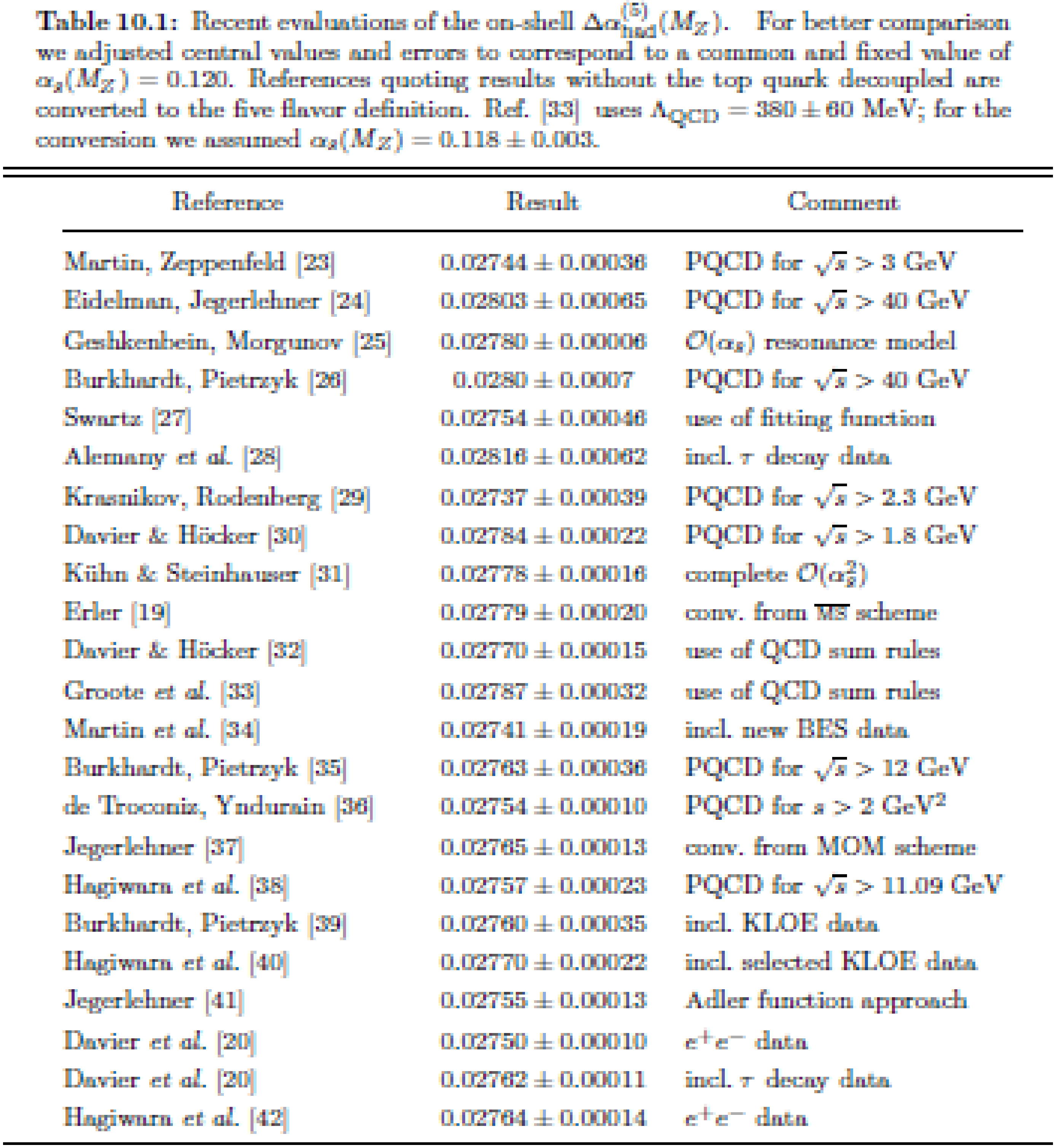


Table 10.1: Recent evaluations of the on-shell $\Delta\alpha^{(5)}_{\rm had}(M_Z)$. For better comparison we adjusted central values and errors to correspond to a common and fixed value of $\alpha_s(M_Z) = 0.120$. References quoting results without the top quark decoupled are converted to the five flavor definition. Ref. [33] uses $\Lambda_{\rm QCD} = 380 \pm 60$ MeV; for the conversion we assumed $\alpha_s(M_Z) = 0.118 \pm 0.003$.

Reference	Result	Comment
Martin, Zeppenfeld [23]	0.02744 ± 0.00036	PQCD for $\sqrt{s} > 3$ GeV
Eidelman, Jegerlehner [24]	0.02803 ± 0.00065	PQCD for $\sqrt{s} > 40$ GeV
Geshkenbein, Morgunov [25]	0.02780 ± 0.00006	$\mathcal{O}(\alpha_s)$ resonance model
Burkhardt, Pietrzyk [26]	0.0280 ± 0.0007	PQCD for $\sqrt{s} > 40$ GeV
Swartz [27]	0.02754 ± 0.00046	use of fitting function
Alemany *et al.* [28]	0.02816 ± 0.00062	incl. τ decay data
Krasnikov, Rodenberg [29]	0.02737 ± 0.00039	PQCD for $\sqrt{s} > 2.3$ GeV
Davier & Höcker [30]	0.02784 ± 0.00022	PQCD for $\sqrt{s} > 1.8$ GeV
Kühn & Steinhauser [31]	0.02778 ± 0.00016	complete $\mathcal{O}(\alpha_s^2)$
Erler [19]	0.02779 ± 0.00020	conv. from $\overline{\rm MS}$ scheme
Davier & Höcker [32]	0.02770 ± 0.00015	use of QCD sum rules
Groote *et al.* [33]	0.02787 ± 0.00032	use of QCD sum rules
Martin *et al.* [34]	0.02741 ± 0.00019	incl. new BES data
Burkhardt, Pietrzyk [35]	0.02763 ± 0.00036	PQCD for $\sqrt{s} > 12$ GeV
de Troconiz, Yndurain [36]	0.02754 ± 0.00010	PQCD for $s > 2$ GeV^2
Jegerlehner [37]	0.02765 ± 0.00013	conv. from MOM scheme
Hagiwara *et al.* [38]	0.02757 ± 0.00023	PQCD for $\sqrt{s} > 11.09$ GeV
Burkhardt, Pietrzyk [39]	0.02760 ± 0.00035	incl. KLOE data
Hagiwara *et al.* [40]	0.02770 ± 0.00022	incl. selected KLOE data
Jegerlehner [41]	0.02755 ± 0.00013	Adler function approach
Davier *et al.* [20]	0.02750 ± 0.00010	e^+e^- data
Davier *et al.* [20]	0.02762 ± 0.00011	incl. τ decay data
Hagiwara *et al.* [42]	0.02764 ± 0.00014	e^+e^- data

Differences:

- Treatment of data and errors
- Integration
- Threshold for PQCD

$$\Delta\alpha^{(l)} = 0.031421$$

$$\Delta\alpha = \Delta\alpha^{(l)} + \Delta\alpha^{(h)} \simeq 0.06$$

$$\alpha(M_Z^2) = \frac{\alpha}{1-\Delta\alpha} \simeq \frac{1}{128}$$

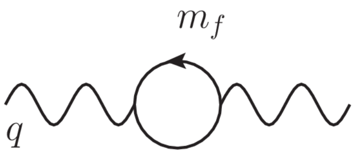
The other “large” term

$$\frac{c^2}{s^2}\left[\frac{Re\,A_{ZZ}(M_Z^2)}{M_Z^2}-\frac{Re\,A_{WW}(M_W^2)}{M_W^2}\right]\underset{\uparrow}{\simeq}-\frac{c^2}{s^2}\delta\rho$$

heavy particles ($M>>M_Z$)

Heavy particles contribute to the W-Z mass difference correction in the same way as in the ρ parameter (relative strength between NC and CC interactions)

$$\rho=\frac{\left(1-\frac{A_{ZZ}(0)}{M_Z^2}+V_Z+M_Z^2B_Z\right)}{\left(1-\frac{A_{WW}(0)}{M_W^2}+V_W+M_W^2B_W\right)}\simeq 1+\frac{A_{WW}(0)}{M_W^2}-\frac{A_{ZZ}(0)}{M_Z^2}+\cdots=1+\delta\rho+\ldots$$



$$\sim q^2,\quad m_f^2$$

Quadratic terms in m_f are going to survive in the difference?

M_W and M_Z are degenerate in the limit g' → 0.

Higgs potential is a function of $\Phi^\dagger\Phi = (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$
custodial symmetry $SU(2)_L$ x $SU(2)_R$

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}, \qquad \tilde{\Phi} = i\tau_2 \Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi_- \end{pmatrix}; \quad H = (\tilde{\Phi}, \Phi) = \begin{pmatrix} \phi_0^* & \phi_+ \\ -\phi_- & \phi_0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{Higgs} &= \frac{1}{2} Tr(D_\mu H)^\dagger (D^\mu H) - V(H) \\ D_\mu H &= \partial_\mu H + \frac{i}{2} g \vec{W}_\mu \cdot \vec{\tau} H - \frac{i}{2} g' B_\mu H \tau_3 \\ V(H) &= \lambda \left(\frac{1}{2} Tr(H^\dagger H) - \frac{v^2}{2} \right)^2 \end{aligned}$$

If g'=0 $\mathcal{L}_{Higgs}$ has global $SU(2)_L$ x $SU(2)_R$ invariance
$(U_{L,R} = exp[i\vec{\tau} \cdot \vec{\epsilon}_{L,R}])$

$$\begin{aligned} H &\rightarrow U_L H U_R^\dagger \\ \vec{W}_\mu \cdot \vec{\tau} &\rightarrow U_L (\vec{W}_\mu \cdot \vec{\tau}) U_L^\dagger \end{aligned}$$

$SU(2)_L$ survives when $< H >= \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow M_{W_3} = M_W$

$$\begin{aligned} H &\rightarrow U_L H U_L^\dagger \\ \vec{W}_\mu \cdot \vec{\tau} &\rightarrow U_L (\vec{W}_\mu \cdot \vec{\tau}) U_L^\dagger \end{aligned}$$

$$\begin{aligned}\mathcal{L}_{Yuk} &= -Y_d\bar{\psi}_L\Phi\psi_R^d - Y_u\bar{\psi}_L\tilde{\Phi}\psi_R^u + h.c.\\ &= Y\bar{\psi}_L H\psi_R + \Delta Y\bar{\psi}_L H\tau_3\psi_R + h.c. \qquad \psi_R = \begin{pmatrix}\psi_R^u\\ \psi_R^d\end{pmatrix}\end{aligned}$$

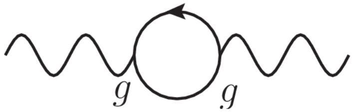
First term invariant under: $H \rightarrow U_L H U_L,\ \psi_L \rightarrow U_L\psi_L,\ \psi_R \rightarrow U_L\psi_R$

Corrections to the W-Z mass difference are due to hypercharge effects (g' ≠ 0) or to mass splitting within isospin multiplets

$$\begin{aligned}\delta\rho^{top} &= Nc\frac{G_\mu m_t^2}{8\sqrt{2}\,\pi^2} \qquad (m_b = 0)\\ \delta\rho^{Higgs} &\simeq -\frac{3G_\mu M_W^2}{8\sqrt{2}\,\pi^2}\tan\theta_W \ln\frac{M_H^2}{M_W^2} \qquad (M_H \gg M_W)\end{aligned}$$

Corrections proportional to M_H^2 appear at two-loop but are too small to be important

Heavy particles do not decouple in δρ. In a diagram if couplings do not grow with mass heavy particles decouple, running of α or α_s not affected by heavy quarks.



$$\delta\rho = \frac{A_{WW}(0)}{M_W^2} - \frac{A_{ZZ}(0)}{M_Z^2} \neq 0 \quad g,\, g' \to 0$$

$$M_W^2 = \frac{g^2 v^2}{4}$$

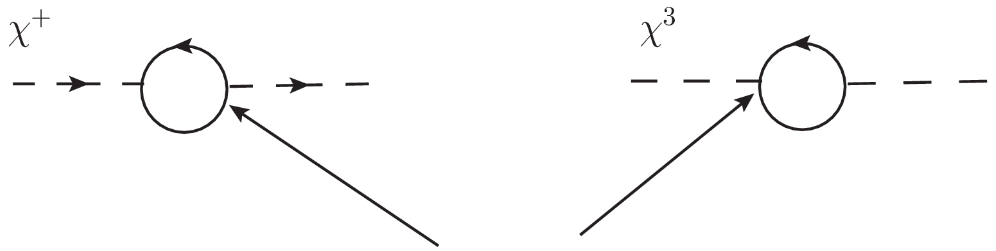
Corrections to δρ can be computed in the gauge-less limit of the SM, a Yukawa theory with gauge boson as external non propagating fields.

$$\mathcal{L}_{g.l.} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) \; - (Y_t \bar{\psi}_L \tilde{\Phi} t_R + h.c.)$$

W, Z (no kinetic term)

$$\mathcal{L}_{kin} = Z_2^+ \left|\partial_\mu \chi^+ - \frac{g\,v}{2} W^+\right|^2 + \frac{1}{2} Z_2^0 \left(\partial_\mu \chi^3 - \frac{g\,v}{2\cos\theta_W} Z\right)^2 \Rightarrow \rho = \frac{Z_2^+}{Z_2^0}$$

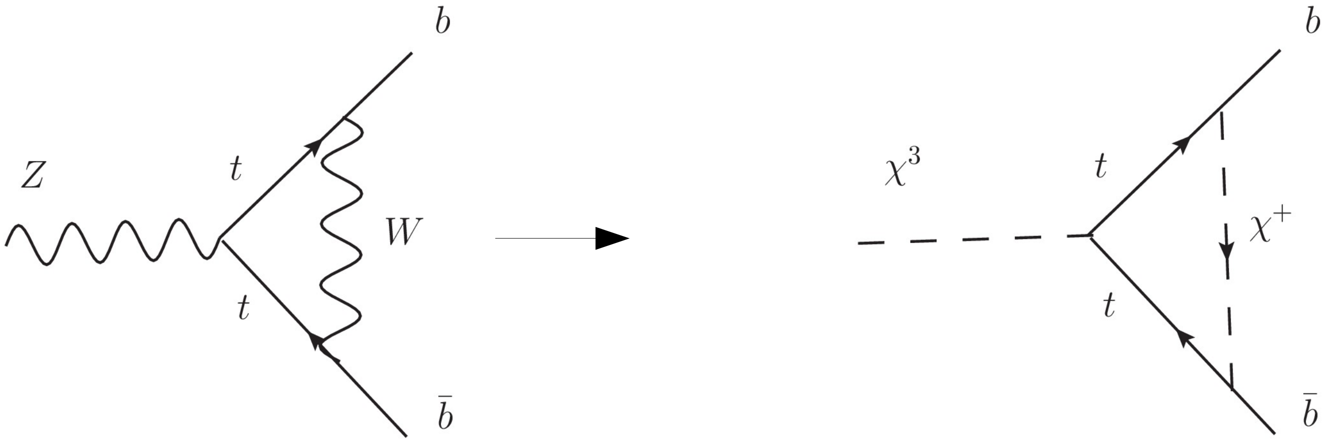
w.f.r, of the Goldstone bosons, Z_2= 1 + O(α)



grows with the mass

X's are the longitudinal modes of the W,Z

Other non decoupling effect in Z→ b b



$\simeq G_\mu m_t^2$

Large counterterm contribution associated with δ s^2 can be eliminated using a $\overline{MS}$

definition of θ_w $\hat{s}^2 \equiv \sin^2 \hat{\theta}_W(z)$

$$M_W^2 = \frac{\hat{\rho} M_Z^2}{2}\left\{1 + \left[1 - \frac{4A^2}{\hat{\rho}\, M_Z^2(1-\Delta\hat{r}_W)}\right]^{1/2}\right\}$$

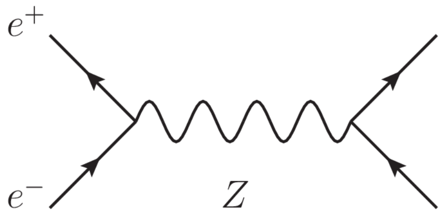
$$\hat{\rho} = \frac{M_W^2}{M_Z^2 \hat{c}^2} \simeq 1 + \delta\rho$$

$$\hat{s}^2 = \frac{1}{2}\left\{1 + \left[1 - \frac{4A^2}{\hat{\rho} M_Z^2(1-\Delta\hat{r})_W}\right]^{1/2}\right\}$$

no m_t^2

Z-pole physics:

Dominant contribution is the resonant Z exchange diagram



The bulk of the corrections can be absorbed into effective couplings

$$\frac{g}{2c} J_Z^\mu = \frac{g}{2c} \bar{\psi}(\gamma_\mu(g_v - g_a\gamma_5)\psi$$

$$\begin{aligned} g_v^f &= T_f^3 - 2Q_f \sin^2\theta_W \Rightarrow \sqrt{\rho}(T_f^3 - 2Q_f \sin^2\theta_{eff}^f) \\ g_a^f &= T^3 \Rightarrow \sqrt{\rho}(T_f^3) \end{aligned}$$

$$\sin^2\theta_{eff}^f = \hat{k}(M_Z^2)\,\sin^2\theta_W(M_Z) \simeq \sin^2\theta_W(M_Z) \qquad \hat{k}(M_Z^2) = 1 + ...$$

$\sin^2\theta^f_{eff}$ can be obtained from asymmetries

$$\frac{d\sigma_{f\bar f}}{d\cos\theta} = \frac{3}{8}\sigma^{tot}_{f\bar f}\left[(1-P_e A_e)(1+\cos^2\theta)+2(A_e-P_e)A_f\cos\theta\right]$$

$$A_f \equiv= \frac{2g_v^f g_a^f}{(g_v^f)^2+(g_a^f)^2} = \frac{2x_f}{1+x_f^2},$$

$$x_f \equiv \frac{g_v^f}{g_a^f} = 1-4|Q_f|\sin^2\theta^f_{eff}$$

e^- polarization

Forward-Backward

$$A^f_{FB}(e^+e^-\to f\bar f) = \frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B} \simeq \frac{3}{4}A_eA_f$$

f = l one measures $A_e^2 \simeq 4x_f^2$ $(g_v^e \ll g_a^e)$ $A_e \sim 0.15$

f =q one measures A_eA_q but $g_v^q \sim g_a^q$ one measures mainly A_e

τ polarization

$$A^\tau_{pol}(e^+e^-\to\tau^+\tau^-) = \frac{\sigma(\tau_L)-\sigma(\tau_R)}{\sigma(\tau_L)-+\sigma(\tau_R)} \sim -A_\tau$$

Left-Right

$$A^f_{LR}(e^+e^-\to f\bar f) = \frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R}\frac{1}{<|P_e|>} \simeq A_e \qquad \sigma_L \equiv \sigma(e^-\ l.h.)$$

Left-Right Forward-Backward

$$A^f_{LRFB}(e^+e^-\to f\bar f) = \frac{(\sigma_F-\sigma_B)_L-(\sigma_F-\sigma_B)_R}{(\sigma_F-\sigma_B)_L+(\sigma_F-\sigma_B)_R}\frac{1}{<|P_e|>} \simeq \frac{3}{4}A_f$$

$$a_\mu = \frac{(g-2)_\mu}{2}$$

$$a_\mu(exp) = 116\,592\,089(63) \times 10^{-11} \quad [0.5ppm]$$

$$a_\mu(th.) = a_\mu(QED) + a_\mu(hadronic) + a_\mu(weak)$$

$$a_\mu(QED) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)}$$

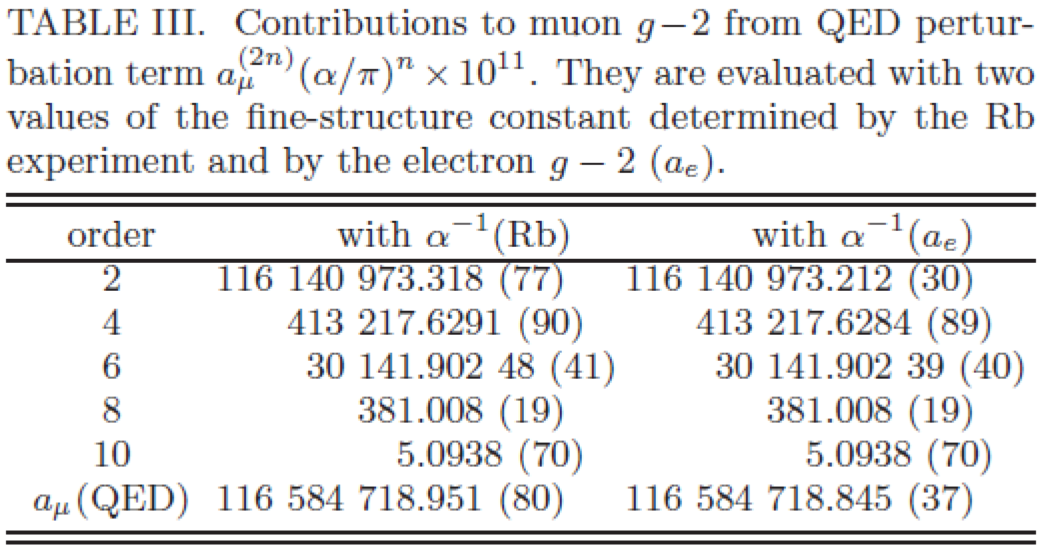


TABLE III. Contributions to muon $g-2$ from QED perturbation term $a_\mu^{(2n)}(\alpha/\pi)^n \times 10^{11}$. They are evaluated with two values of the fine-structure constant determined by the Rb experiment and by the electron $g-2$ (a_e).

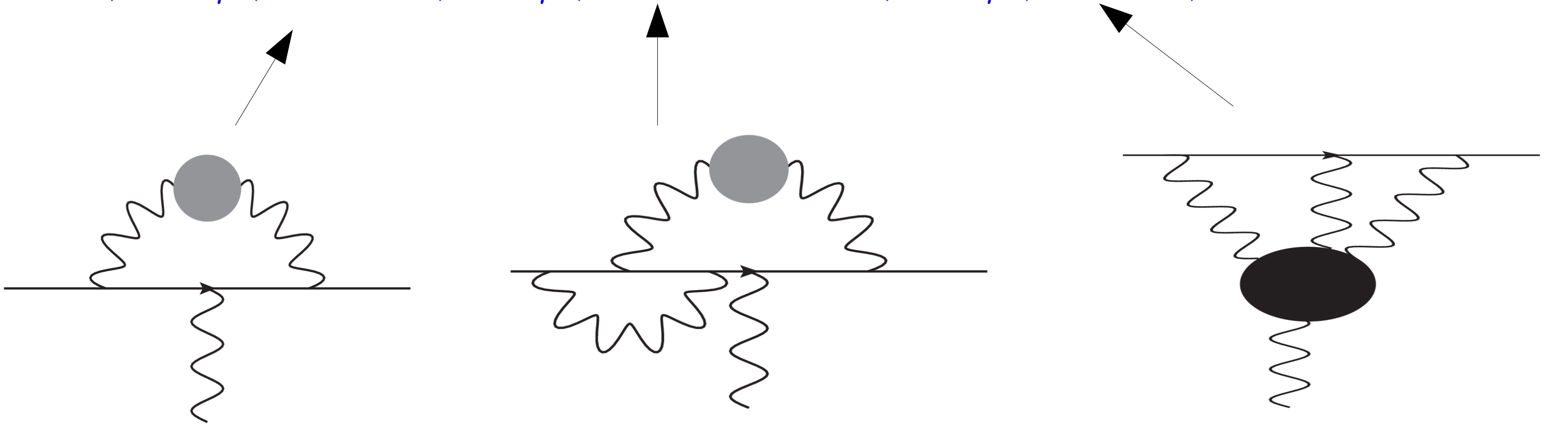
order	with α^{-1}(Rb)	with $\alpha^{-1}(a_e)$
2	116 140 973.318 (77)	116 140 973.212 (30)
4	413 217.6291 (90)	413 217.6284 (89)
6	30 141.902 48 (41)	30 141.902 39 (40)
8	381.008 (19)	381.008 (19)
10	5.0938 (70)	5.0938 (70)
a_μ(QED)	116 584 718.951 (80)	116 584 718.845 (37)

$$\alpha^2 = \frac{2R_\infty}{c} \frac{m_{\rm Rb}}{m_e} \frac{h}{m_{\rm Rb}},$$

$$\alpha(Rb) = 1/137.035999049(90) \; [0.66ppb]$$

$$a_\mu(weak) = 154(2) \times 10^{-11}$$

$$a_\mu(hadronic) = a_\mu(had\,.v.p.) + a_\mu(had.\,NLO\,v.p.) + a_\mu(had.\,l.l.)$$



$$a_\mu(had.\,v.p.) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}K(s)R(s) = 6949.1(37.2)_{exp}(21.0)_{rad} \times 10^{-11}$$

K(s) monotonically decreasing function for increasing s

$$a_\mu(had.\,NLO\,v.p.) = -98.4(06)_{exp}(0.4)_{rad} \times 10^{-11}$$

$$a_\mu(had.\,l.l) = 116(40) \times 10^{-11}$$

Models of low-energy hadronic interactions with e.m. current

$$a_\mu(th) = 116\,592\,840(59) \times 10^{-11}$$

$$a_\mu(exp) - a_\mu(th.) = 249(87) \times 10^{-11}$$

The main source of error in a_μ(th.) comes from a_μ(hadronic) where in the dispersion relation enters the same experimental data that are employed in the calculation of $\Delta\alpha^{(5)}_{had.}$

If I change a_μ(hadronic) I get a too light Higgs.

New Physics explanations:

One needs a relatively light not colored particle with couplings to the down fermion enhanced with respect to the SM

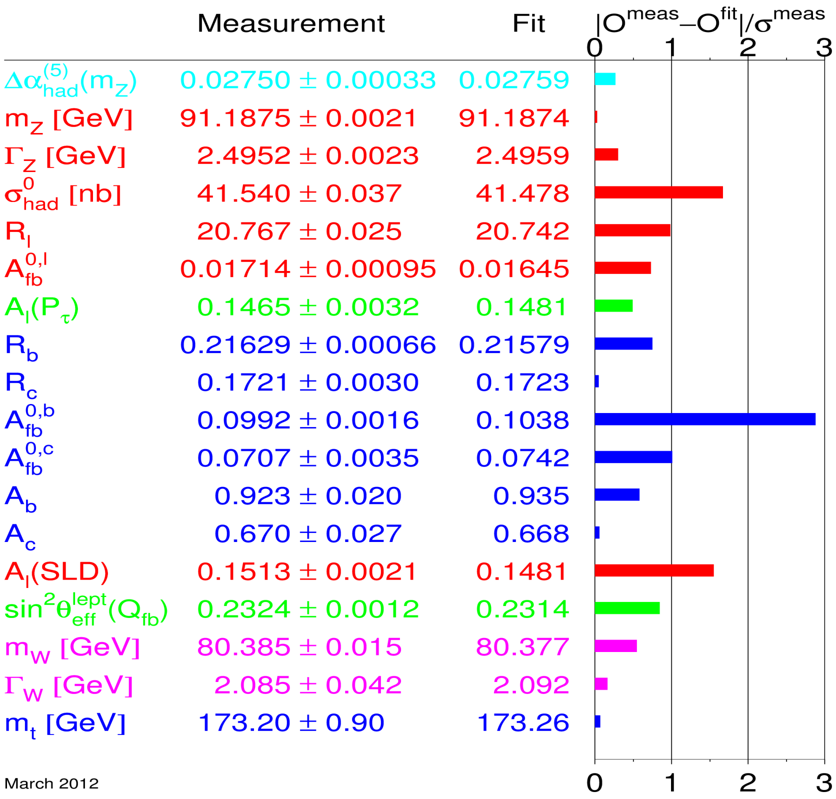
SUSY

$$a_\mu^{\rm SUSY} \simeq ({\rm sgn}\,\mu) \times (130 \times 10^{-11}) \left(\frac{100\,{\rm GeV}}{M_{\rm SUSY}}\right)^2 \tan\beta$$

SM Fit

One can make a global fit including “all” possible measurements and using the radiatively corrected predictions for the various observable. The latter, besides α, G_{μ}, M_Z and lepton masses depend upon:

$$m_t,\ \Delta\alpha^{(5)}_{had},\ \alpha_s(M_Z),\ M_H$$

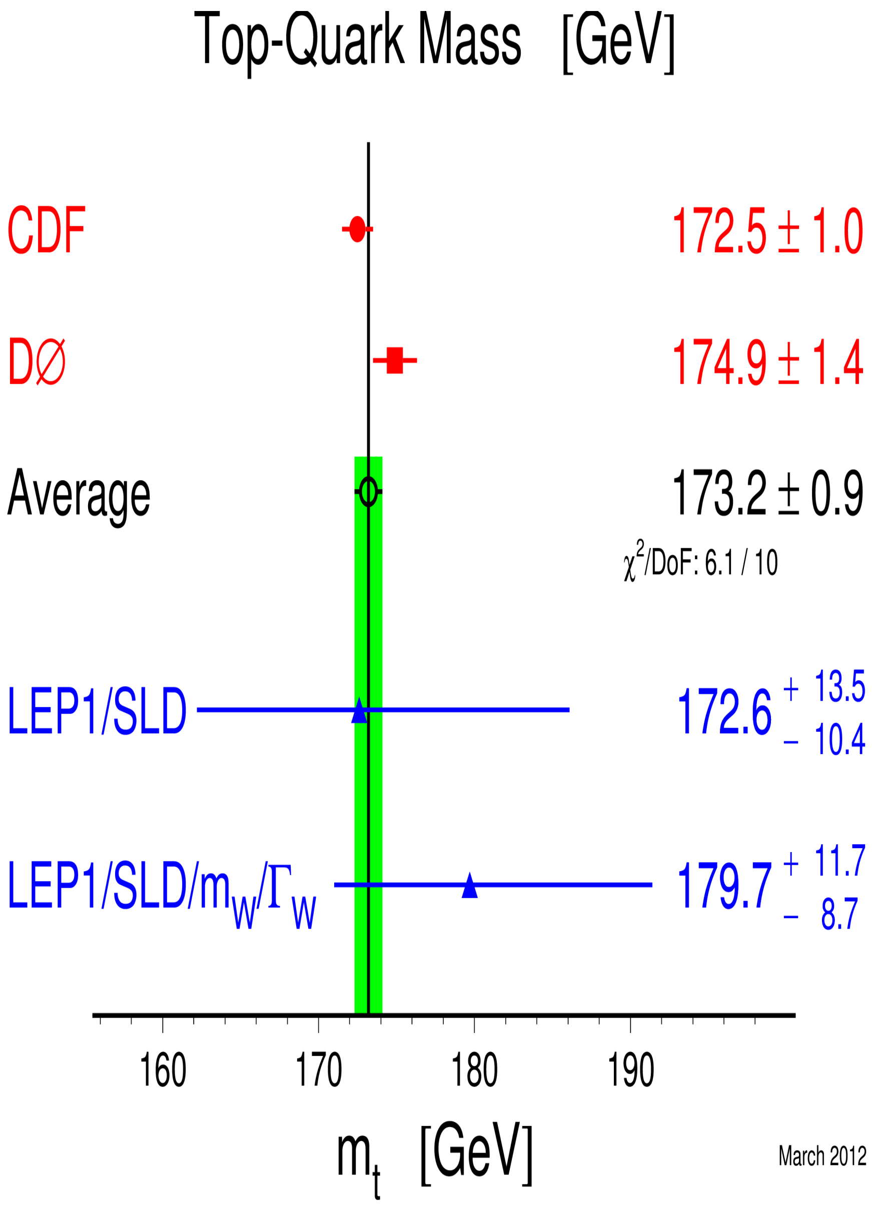


	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26

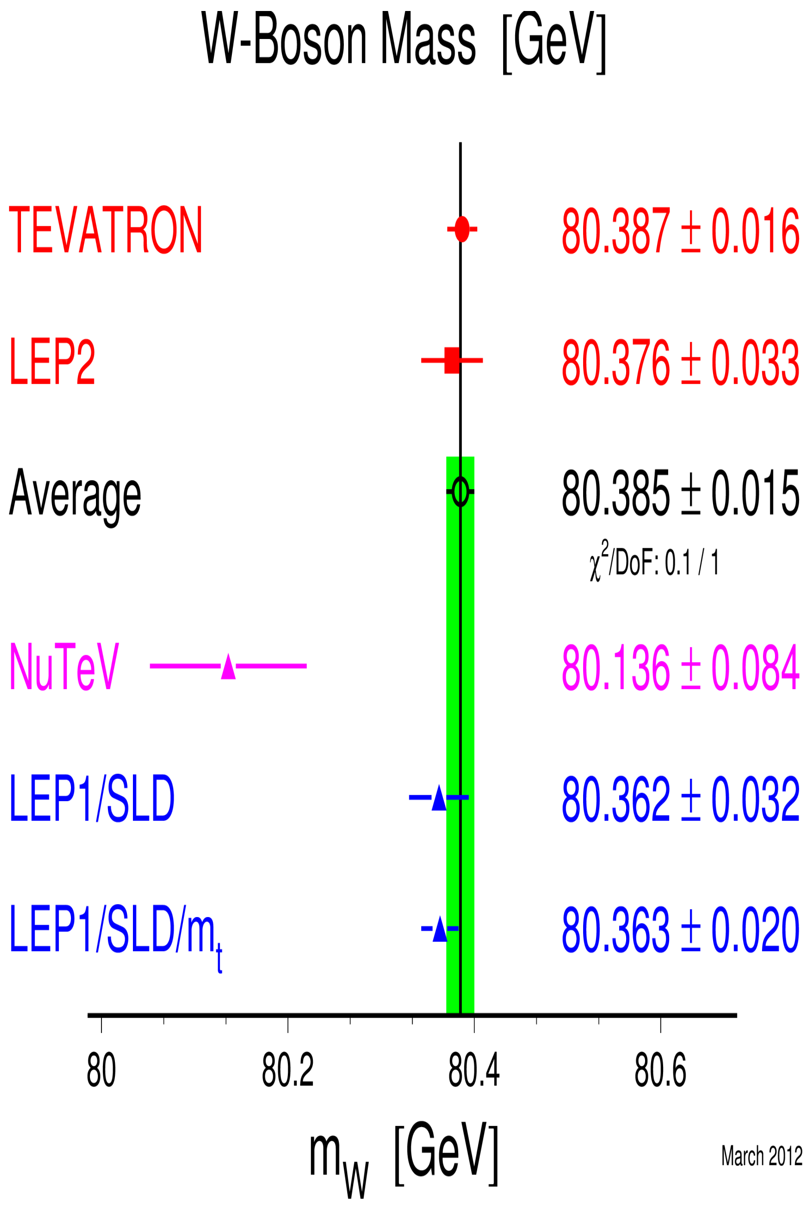
Predictions for m_t, M_W, M_H

Very weak sensitivity to M_H, without the Value of m_T we cannot predict it.

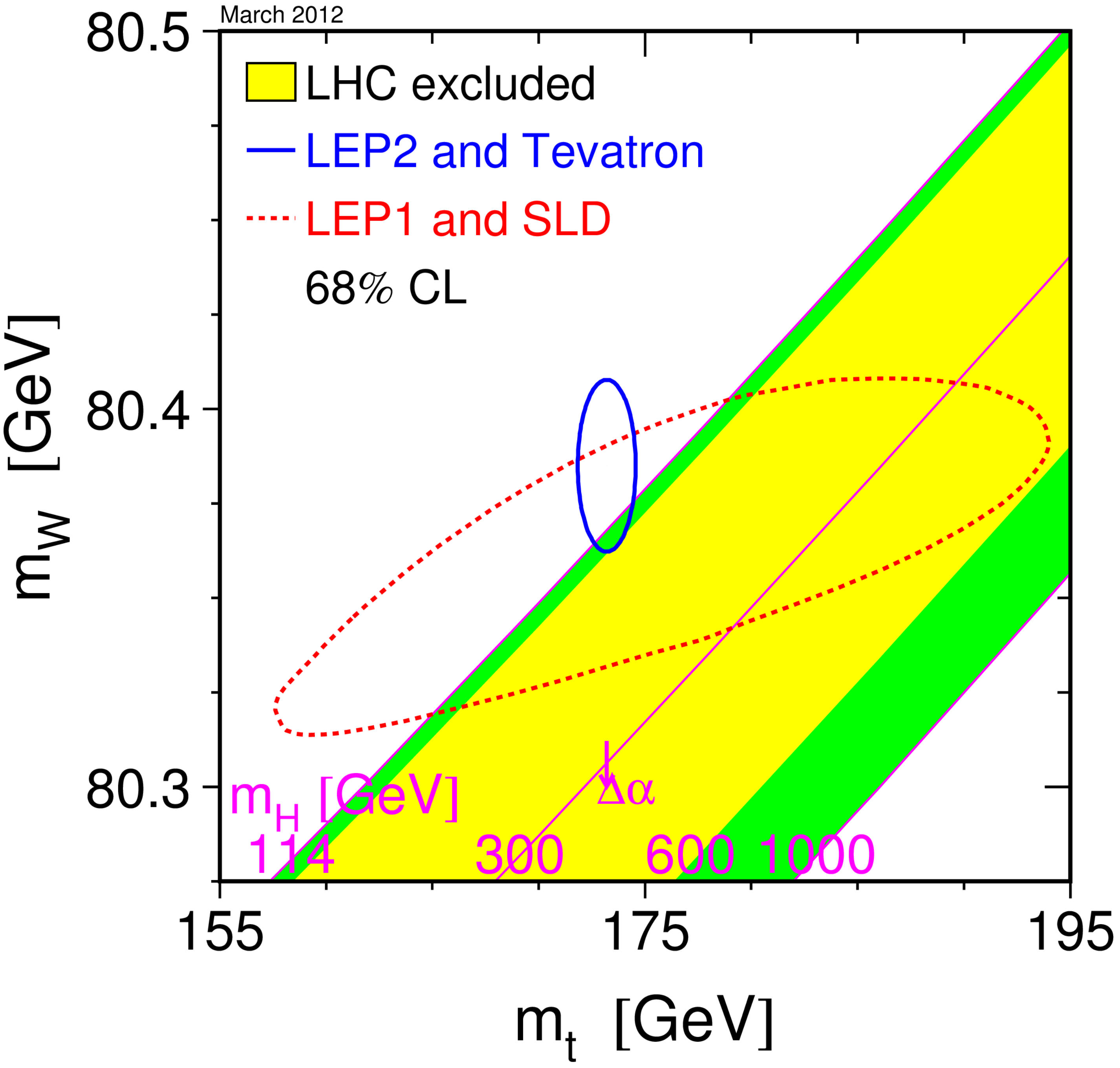
$$R_l = \frac{\Gamma_{had}}{\Gamma_l}, \quad R_q = \frac{\Gamma_q}{\Gamma_{had}}$$



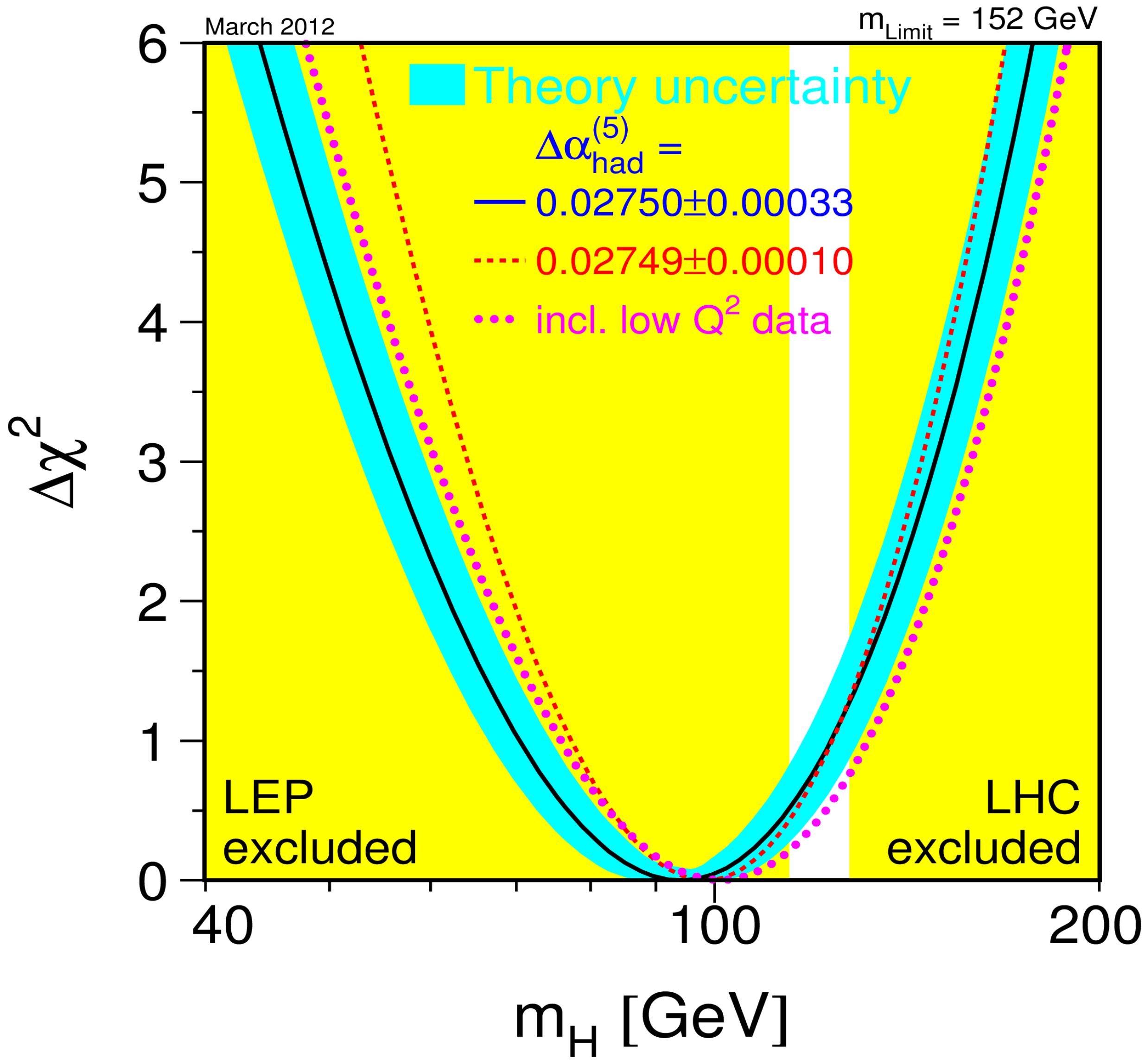
Top-Quark Mass [GeV]



W-Boson Mass [GeV]


March 2012
LHC excluded
LEP2 and Tevatron
LEP1 and SLD
68% CL
m_H [GeV]
114
300
600
1000
$\Delta\alpha$
80.5
80.4
80.3
m_W [GeV]
155
175
195
m_t [GeV]

Global fit to M_H



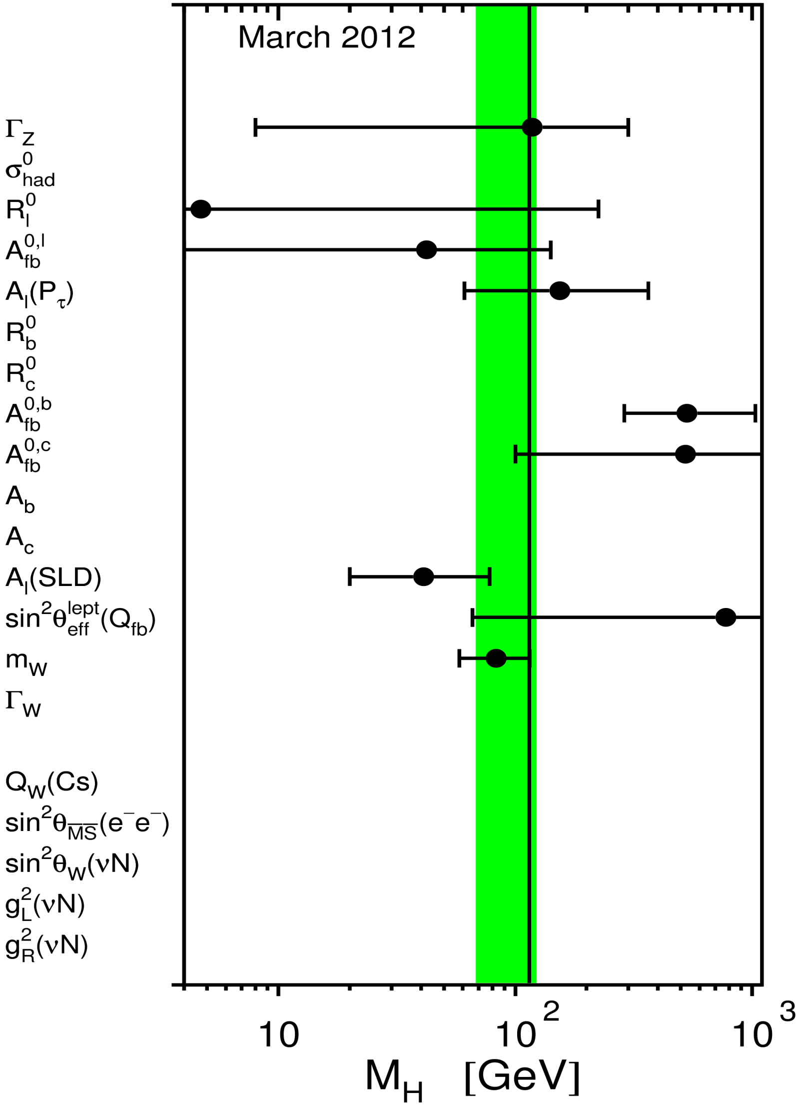
Alternative approach: I want to get a probability density function for M_H in the SM using all the available information, from precision physics and from direct searches (obviously excluding LHC results) to see if the particle that has been discovered at LHC has a mass compatible with the SM prediction (p.d.f $\neq$ 0)

Few observables are really sensitive to the Higgs

Simplified analysis using

$M_W, \sin^2\theta_{eff}^{lept.}$

asymmetries



- Parametrization:

$$\sin^2\theta^{lept}_{eff} = (\sin^2\theta^{lept}_{eff})^\circ + c_1\,A_1 + c_5\,A_1^2 + c_2\,A_2 - c_3\,A_3 + c_4\,A_4,$$

$$M_W = M_W^\circ - d_1\,A_1 - d_5\,A_1^2 - d_2\,A_2 + d_3\,A_3 - d_4\,A_4,$$

where

$$A_1 = \ln\frac{M_H}{100\ \mathrm{GeV}}, \qquad A_2 = \frac{\Delta\alpha^{(5)}_{had}}{0.02761} - 1,$$
$$A_3 = \left(\frac{m_t}{175\ \mathrm{GeV}}\right)^2 - 1, \quad A_4 = \frac{\alpha_s(M_Z)}{0.118} - 1,$$

c_i, d_i > 0 theoretical coefficients (depend upon the RS)

- Two quantities normally distrubuted

$$\begin{aligned} W &= \sin^2\theta^{lept}_{eff} - (\sin^2\theta^{lept}_{eff})^\circ - c_2 A_2 + c_3 A_3 - c_4 A_4, \\ Y &= M_W^\circ - M_W - d_2 A_2 + d_3 A_3 - d_4 A_4 \end{aligned}$$

- Likelihood of our indirect measurements Θ ={W, Y} is a two-dimensional correlated normal

$$f(\underline{\theta}\,|\,\ln(m_H)) \propto e^{-\chi^2/2}$$

$$\chi^2 = \underline{\Delta}^T\mathbf{V}^{-1}\underline{\Delta}, \quad V_{ij} = \sum_l \frac{\partial\Theta_i}{\partial X_l}\cdot\frac{\partial\Theta_j}{\partial X_l}\cdot\sigma^2(X_l), \quad \underline{\Delta} = \begin{pmatrix} a_1 - c_1\ln(M_H/100) - c_5\ln^2(M_H/100) \\ y - d_1\ln(M_H/100) - d_5\ln^2(M_H/100) \end{pmatrix}$$

- Using Bayes' theorem the likelihood is turned into a p.d.f. of M_H via a uniform prior in ln (M_H)

$$f(M_H \mid ind.) = \frac{M_H^{-1}\, e^{-(\chi^2/2)}}{\int_0^\infty M_H^{-1}\, e^{-(\chi^2/2)}\, \mathrm{d}M_H}.$$

Likelihood

Bayes' theorem: $$f(\mu|x) = \frac{f(x|\mu)\cdot f(\mu)}{\int f(x|\mu)\cdot f(\mu)\, d\mu}$$

prior

How f(M_H | *ind*) is going to be modified by the results of the direct search experiments?

Ideal experiment (sharp kinematical limit, M_K) with outcome no candidate:

- f(M_H) must vanish below M_K (we did not observe)
- Above M_K the relative probabilities cannot change (experiment is not sensitive there)

$$f(M_H \mid dir.\,\&\, ind.) = \begin{cases} 0 & M_H < M_K \\ \frac{f(M_H \mid ind.)}{\int_{M_K}^\infty f(M_H \mid ind.)\, \mathrm{d}M_H} & M_H \geq M_K\,, \end{cases}$$

Just Bayes theorem:

$$f(M_H \,|\, dir. \,\&\, ind.) \propto f(dir. \,|\, M_H) \cdot f(M_H \,|\, ind.)$$

Likelihood for the ideal experiment:

$$f(dir. \,|\, M_H) = f(\text{“zero cand.”} \,|\, M_H) = \begin{cases} 0 & M_H < M_K \\ 1 & M_H \geq M_K \,. \end{cases}$$

Step function

Real life:
no sharp kinematical limit, step function should be replace by a smooth curve that goes to zero for low masses and to 1 for $M_H \to M_{Keff}$
Normalize the likelihood to the no signal case (pure background)
(Constant factors do not play any role in Bayes' theorem)

$$\mathcal{R}(M_H) = \frac{L(M_H)}{L(M_H \to \infty)}$$

Likelihood ratio
(should be providwed by the experiments)

$$f(M_H \,|\, dir.\,\&\, ind.) = \frac{\mathcal{R}(M_H)\, f(M_H \,|\, ind.)}{\int_0^\infty \mathcal{R}(M_H)\, f(M_H \,|\, ind.)\, \mathrm{d}M_H}$$

Role of $\mathcal{R}(M_H)$

$\mathcal{R} = 1$

Region where the experiment is not sensitive;
shape of f(M_H | *ind*) does not change

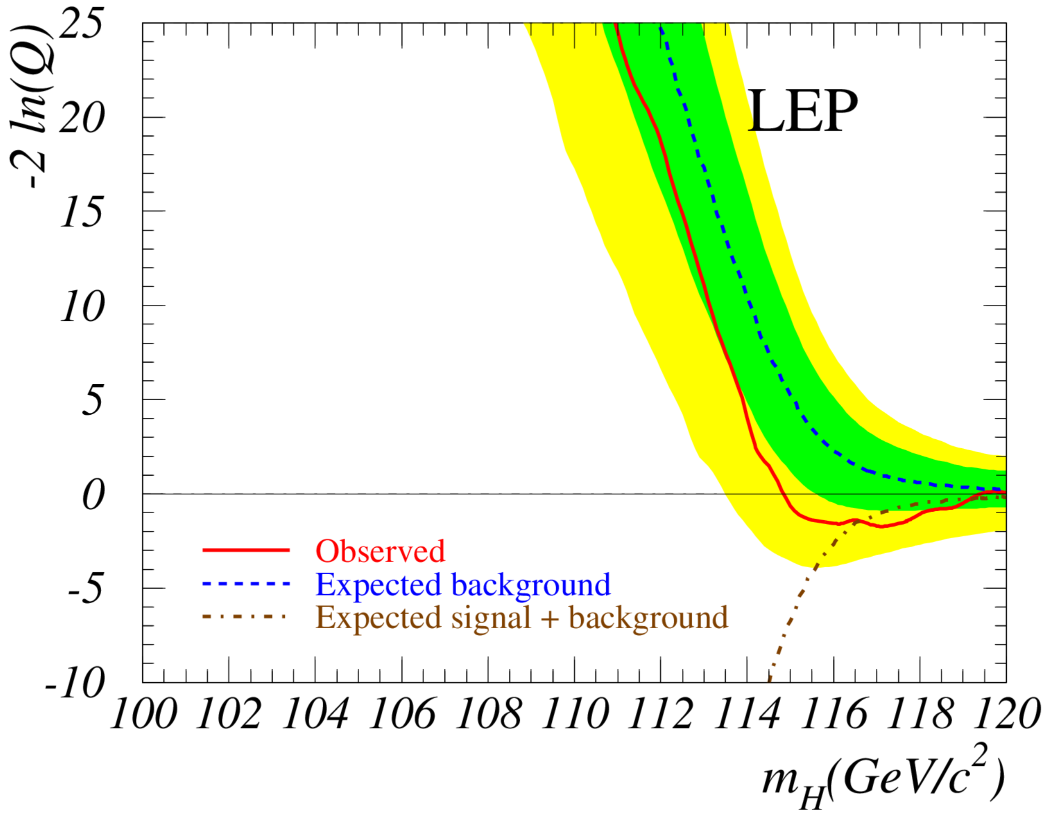
$\mathcal{R} < 1$

Probability is decreased,
p.d.f. is pushed above M_K
$\mathcal{R}(M_H) \to 0$ cuts the region

$\mathcal{R} > 1$

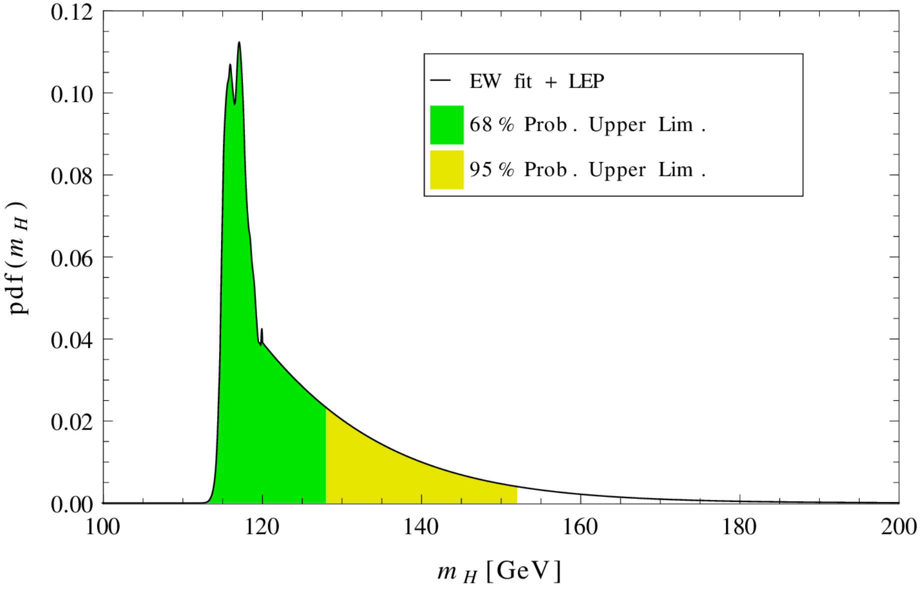
Probability is increased,
p.d.f is streched below $M_{K,}$
very large $\mathcal{R}(M_H)$ prompt a discovery

$\mathcal{Q} = \mathcal{R}$

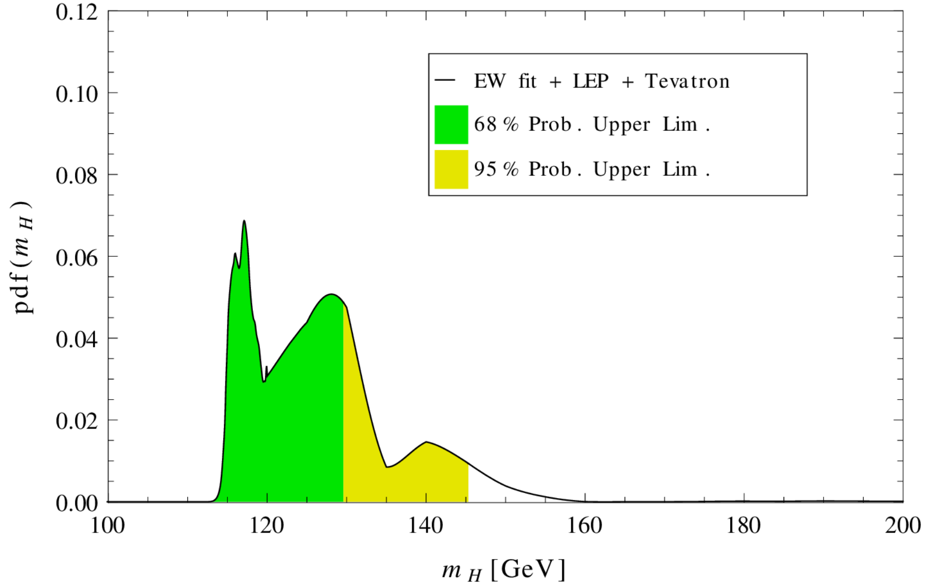


Combining direct and indirect information:

LEP

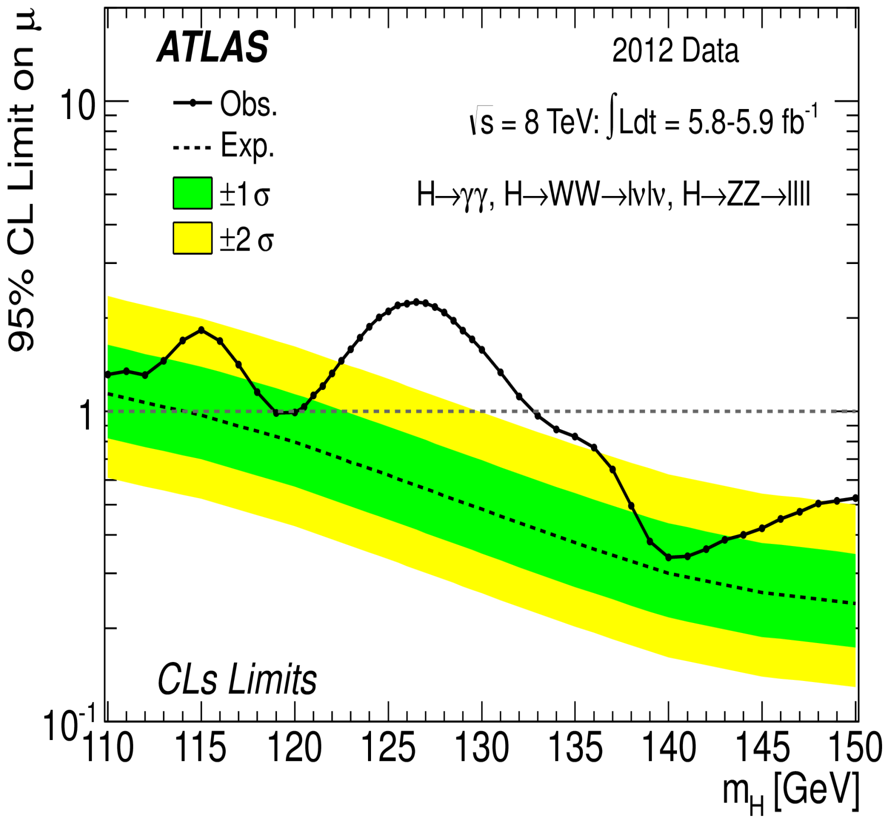


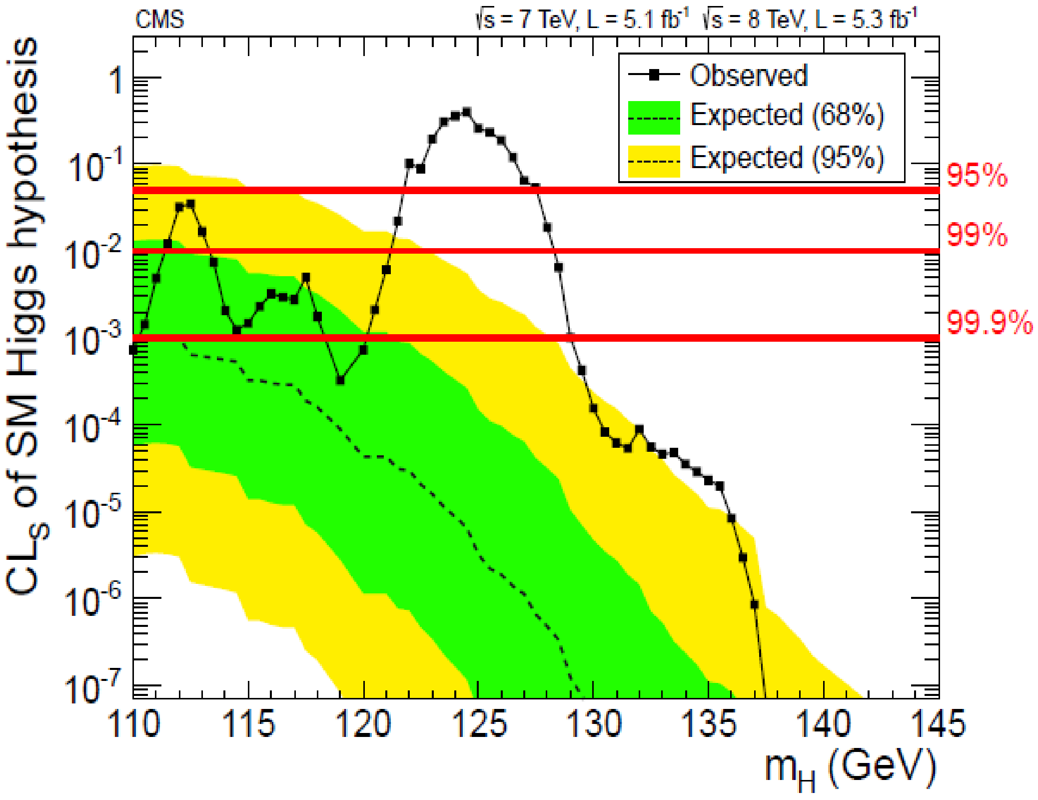
LEP+ TEVATRON



SM: M_H between 114 and 160 GeV with 95% probability below 145 GeV

It is where the SM predicts it should be

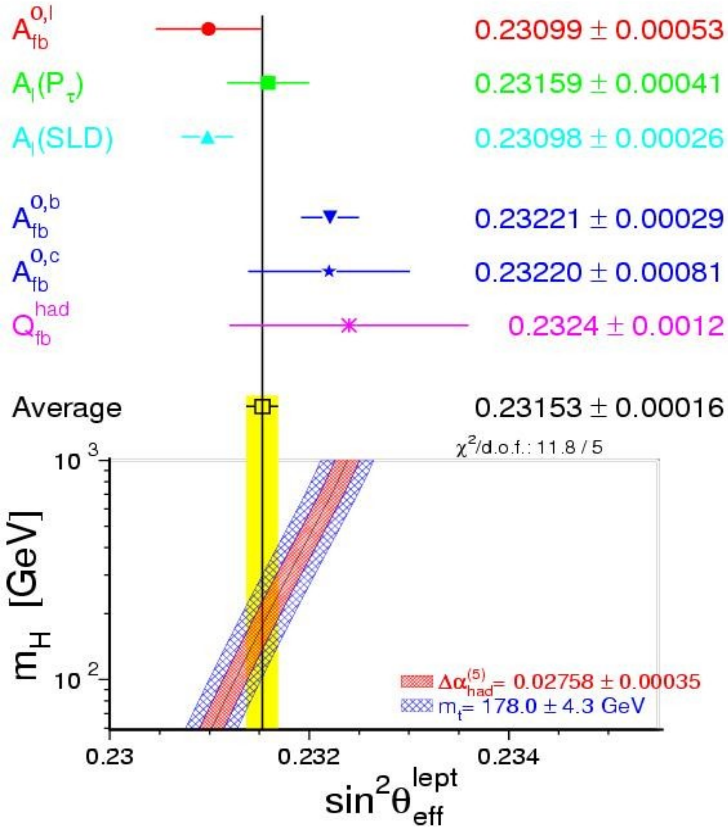


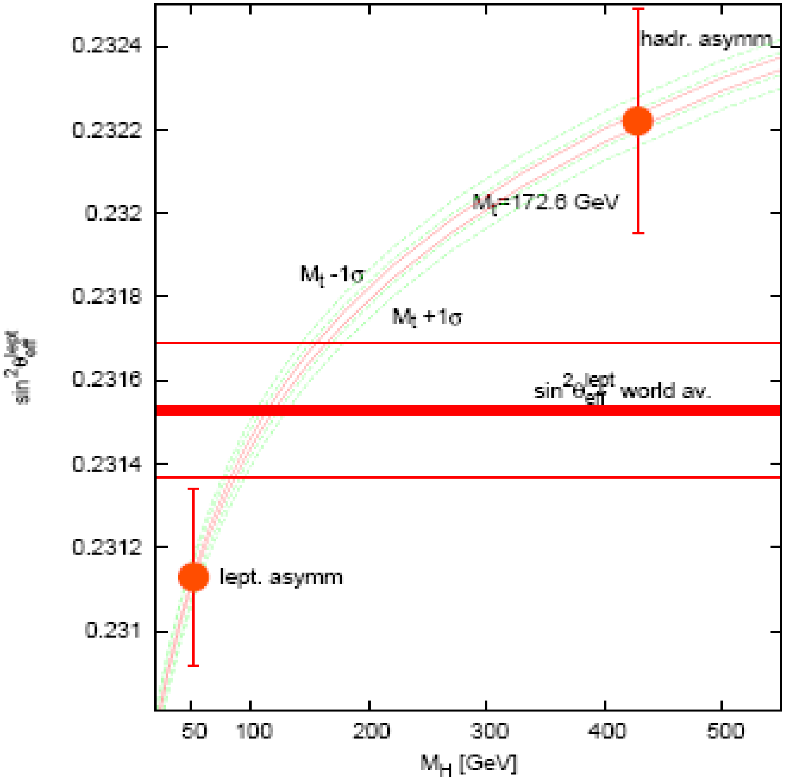


$$M_H = 126 \pm 0.4\,(stat.) \pm 0.4\,(syst.)\,\text{GeV}$$

$$M_H = 125.3 \pm 0.4\,(stat.) \pm 0.5\,(syst.)\,\text{GeV}$$

Playing with experimental results not always works fine.





$$A^l(SLD) - A^{0,b}_{FB} \sim 3\sigma$$

New Physics effects, where they could be?

New particles are going to contribute to the W,Z self-energies (process-independent contributions) and to vertices (for specific processes). With $M_{NP} >> M_Z$ where and what kind of "large" effects can we expect?

Self-energy: 3 types of NP contributions

★ $$\alpha(M_Z)T \equiv \frac{A_{WW}(0)}{M_W^2} - \frac{A_{ZZ}(0)}{M_Z^2} \propto \Pi_{11}(0) - \Pi_{33}(0)$$

isospin violation

Isosplitted particles: effects grow as the difference in the mass squared between partners of multiplet. Top contributes quadratically, Higgs logarithmically

★ $$\frac{\alpha(M_Z)}{4s^2c^2}S \equiv \frac{1}{M_Z^2}\left\{A_{ZZ}(M_Z^2) - A_{ZZ}(0) - \frac{c^2-s^2}{c\,s}\left[A_{\gamma Z}(M_Z^2) - A_{\gamma Z}(0)\right] - A_{\gamma\gamma}(M_Z^2)\right\}$$
$$\propto \Pi_{3Y}(M_Z^2) - \Pi_{3Y}(0)$$

No-effects that grow quadratically with the masses, but constant terms possible (≠ 0 , $M_{NP} \to \infty$)

Top and Higgs logaritmically

★
$$\begin{aligned}\frac{\alpha(M_Z)}{4s^2c^2}U &\equiv \frac{A_{WW}(M_W^2)-A_{WW}(0)}{M_W^2} - c^2\frac{A_{ZZ}(M_Z^2)-A_{ZZ}(0)}{M_Z^2}\\ &- \frac{1}{M_Z^2}\left[2cs\left(A_{\gamma Z}(M_Z^2)-A_{\gamma Z}(0)\right) - s^2A_{\gamma\gamma}(M_Z^2)\right]\\ &\propto \frac{1}{M_W^2}\left[\Pi_{11}(M_W^2)-\Pi_{11}(0)\right] - \frac{1}{M_Z^2}\left[\Pi_{33}(M_Z^2)-\Pi_{33}(0)\right]\end{aligned}$$

Isospin violation in the derivatives

U in many models is usually very small
U=0

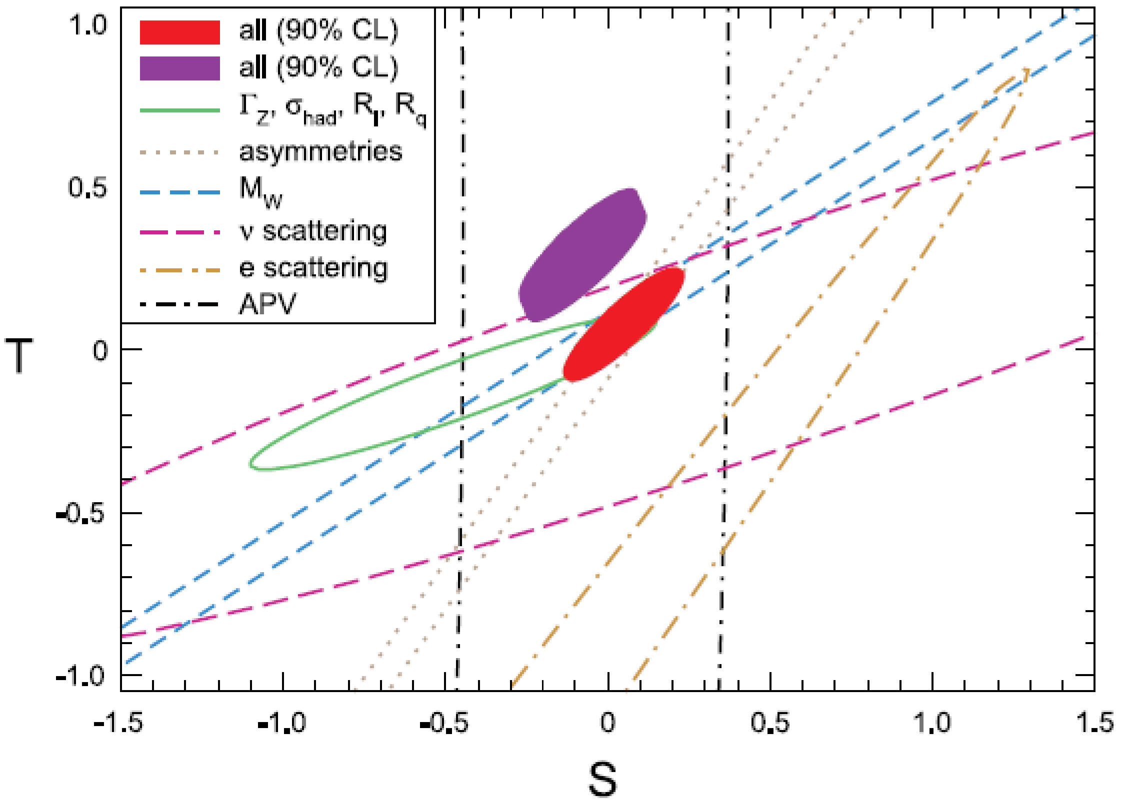
Two parameters fit:

🟥

$115.5\,\mathrm{GeV} < M_H < 127\,\mathrm{GeV}$

🟪

$600\,\mathrm{GeV} < M_H < 1\,\mathrm{TeV}$



Before the discovery of the Higgs one could envisage a situation in which NP contributions were going to mask the effect of a heavy Higgs (“conspiracy”).

Simple explanation:

$$\begin{aligned}\hat{\rho} &= \rho_0 + \delta\rho \, (\rho_0^{\rm SM} = 1, \delta\rho \leftrightarrow T) \\ \Delta\hat{r}_W &\leftrightarrow S\end{aligned}$$

$$\begin{aligned}\sin^2\theta_{eff}^{lept} &\sim \frac{1}{2}\left\{1 - \left[1 - \frac{4A^2}{M_Z^2\,\hat{\rho}\,(1 - \Delta\hat{r}_W)}\right]^{1/2}\right\} \\ &\sim (\sin^2\theta_{eff}^{lept})^\circ + c_1 \ln\left(\frac{M_H}{M_H^\circ}\right) + c_2\left[\frac{(\Delta\alpha)_h}{(\Delta\alpha)_h^\circ} - 1\right] - c_3\left[\left(\frac{m_t}{m_t^\circ}\right)^2 - 1\right] + \ldots\end{aligned}$$

$c_i > 0$

To increase the fitted M_H :

$$\begin{cases} \hat{\rho} > 1 \rightarrow & \begin{cases} \rho_0 > 1 \\ \delta\rho > 0 \end{cases} \\ \Delta\hat{r}_W < 0 & \end{cases}$$

NP better to be of the decoupling type

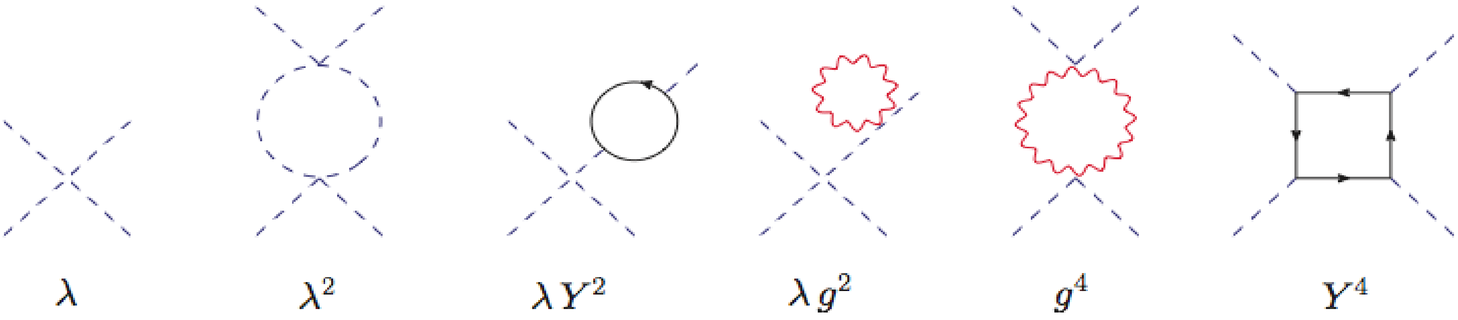
Theoretical bounds on the Higgs mass in the SM

$$V_{\text{Higgs}} \sim \lambda\phi^4 \to \left[\lambda + \gamma \ln\left(\frac{\phi^2}{\Lambda^2}\right)\right]\phi^4 \to \lambda(\Lambda)\phi'^4(\Lambda)$$

$$\phi' = \phi\, exp \int^t \gamma(t')dt', \quad t = \ln(\Lambda/v)$$

$$\text{V}_{\text{Higgs}} > 0 \to \lambda(\Lambda) > 0$$

λ runs



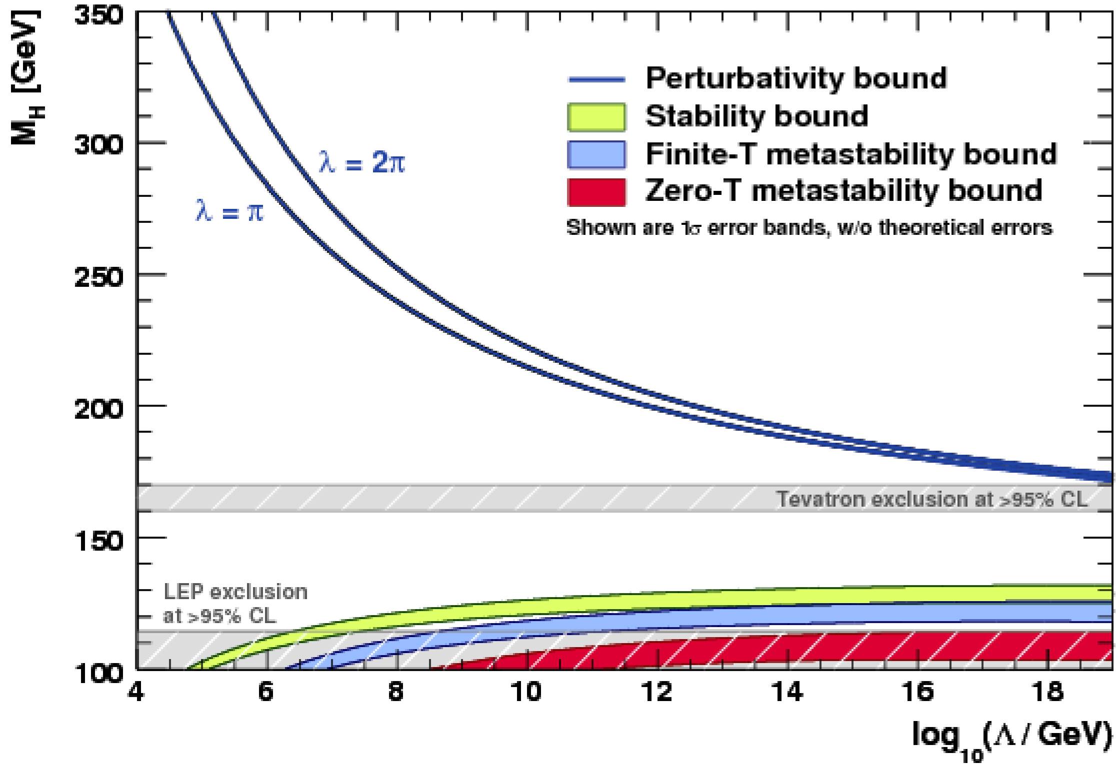
$$\frac{d\lambda}{d\log Q} = \frac{1}{16\pi^2}\Big\{24\,\lambda^2 + \lambda\,[12\,Y_t^2 + 12\,Y_b^2 + 4\,Y_\tau^2 - 9\,g^2 - 3g'^2]$$
$$+\frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2\,g'^2 - 6\,Y_t^4 - 6\,Y_b^4 - 2\,Y_\tau^4\Big\}$$

M_H large: λ^2 wins
non-perturbative regime

$$\lambda(m_t) \to \lambda(\Lambda) \gg 1$$

M_H small: $-Y_t^4$ wins
vacuum (meta)stability

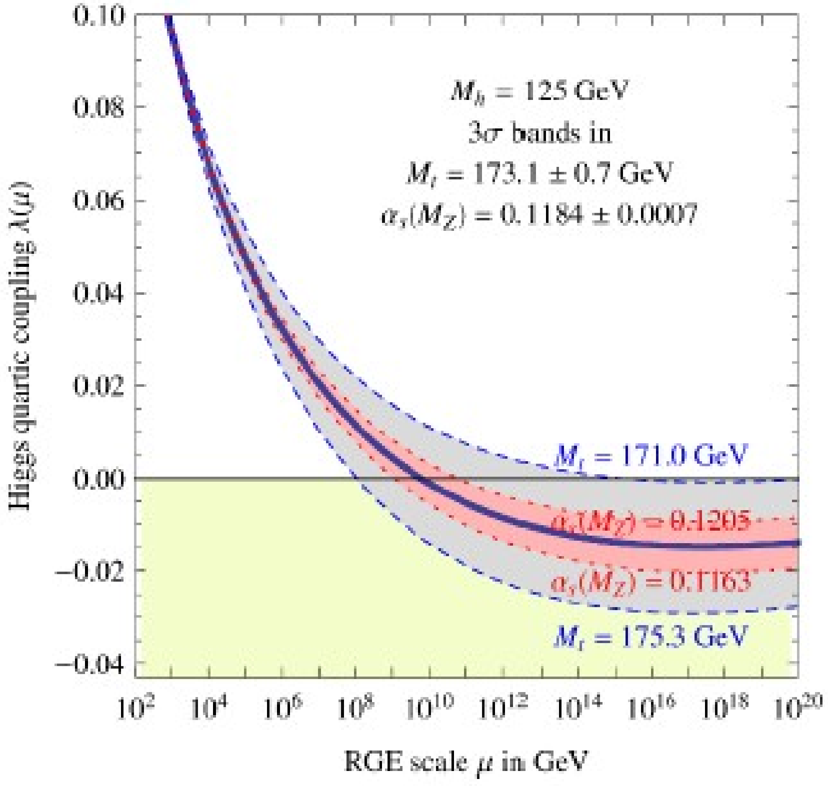
$$\lambda(m_t) \to \lambda(\Lambda) \ll 1$$

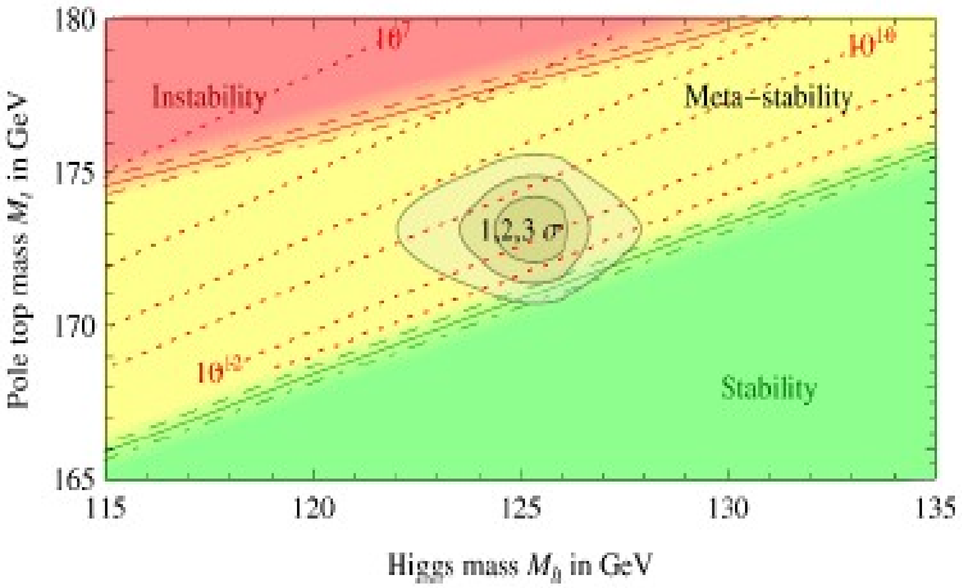


Running depends on

$$m_t,\ \alpha_s, \ldots$$

MH ~ 126 GeV: no problem with the Landau pole





$$M_h\,[\mathrm{GeV}] > 129.4 + 1.4\left(\frac{M_t\,[\mathrm{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\mathrm{th}}$$

Full stability is at the border. Universe becomes metastable at Λ ~ 10^{11} GeV.
λ never becomes too negative, small probability of quantum tunneling.
Lifetime of the EW vacuum longer than the age of the Universe.
SM ok up to Planck mass.