Geometrical CP violation

and non-renormalisable Higgs potentials

David EMMANUEL-COSTA

(CFTP, Instituto Superior Técnico, Lisboa)

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with Ivo de Medeiros Varzielas and Philipp Leser, Phys. Lett. B 716 (2012) 193
Ivo de Medeiros Varzielas, Phys.Rev. D 84 (2011) 117901

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Outline

The Kobayshi-Maskawa mechanism (with at least three generations of SM fermion Multiplets) incorporates successfully CP violation observations, but one still does not have a deep origin of CP Violation...

- Spontaneous CP Violation
- Geometric CP Violation
- (Non-)Renormalising Higgs Potentials
- Conclusions

Spontaneous CP Violation

[T.D. Lee]

One starts form a CP conserving Lagrangian, *i.e.* $V(\phi_i)$ has real parameters so that CP symmetry is broken by complex VEVs

$$\langle \phi_i \rangle = v_i e^{i\varphi_i}$$

One must then verify that there is no transformation as,

$$\phi_i \to \phi_i' = U_{ij}\, \phi_j$$

that leaves the full Lagrangian invariant and

$$U_{ij} \langle \phi_j \rangle^* = \langle \phi_i \rangle$$

The SCPV phases are in general function of arbitrary parameters

Spontaneous CP Violation Features

- Some Remarkable Features from SCPV constructions
  - it reduces the amount of the independent phases, since the phases of the parameters in the Lagrangian are generated from φ_i
  - an appealing solution to the strong CP problem [G.C. Branco's talk]
  - it can soften the SUSY CP Problem [Abel, Khalil, Lebedev]
  - in perturbative string constructions CPV may arise from complex VEVs of moduli and matter fields [Strominger, Witten]

- Geometric Spontaneous CP Violating Phases:
  Could one find a discrete symmetry where the SCPV phases are independent of real parameters in the scalar potential?
  If such symmetry exists SCPV phases are protect to all order of radiative corrections [Weinberg; Georgi, Pais]

These geometric phases become then calculable and stable!!

Geometric CP Violation

[G.B. Branco, J.M. Gérard, W. Grimus]

A simple geometric SCPV exercise within the two Higgs doublet model:

$$V(\phi_1, \phi_2) = \cdots + (\phi_1^\dagger \phi_2)\left[\lambda_1(\phi_1^\dagger \phi_1) + \lambda_2(\phi_2^\dagger \phi_2)\right] + \lambda_3(\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_2) + \text{H.c.}$$

Parametrising the Higgs VEVs as $\langle \phi_1 \rangle = v_1$ and $\langle \phi_2 \rangle = v_2 e^{i\,\theta}$

$$\frac{\partial V}{\partial \theta} = 0 \quad \Longrightarrow \quad \cos\theta = -\,(4\lambda_3 v_1 v_2)^{-1}(\lambda_1 v_1^2 + \lambda_2 v_2^2)$$

If $\lambda_1 = \lambda_2 = 0$ then $\theta = \frac{\pi}{2}$ is geometric and realisable by a Z_2 symmetry

$$\begin{cases} \phi_1 \longrightarrow \phi_1 \\ \phi_2 \longrightarrow -\phi_2 \end{cases} \quad \Longrightarrow \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 e^{i\,\theta} \end{pmatrix}^* = \begin{pmatrix} v_1 \\ v_2 e^{i\,\theta} \end{pmatrix}$$

The phase θ is not SCP violating

One needs more than two Higgs Doublets and non-Abelian groups

Higgs Potential for three Doublets

$$
\begin{aligned}
V(\phi) = &\sum_i \left[-\lambda_i \phi_i^\dagger \phi_i + A_i (\phi_i^\dagger \phi_i)^2\right] \\
&+ \sum_{i<j} \Bigg[\frac{\gamma_i}{2}(\phi_i^\dagger \phi_j + \text{H.c.}) + C_i (\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) \\
&+ \bar{C}_i \left|\phi_i^\dagger \phi_j\right|^2 + \frac{D_i}{2}\left((\phi_i^\dagger \phi_j)^2 + \text{H.c.}\right)\Bigg] \\
&+ \frac{1}{2}\sum_{i\neq j}\left[E_{1ij}(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \text{H.c.}\right] \\
&+ \frac{1}{2}\sum_{\substack{i\neq j\neq k \\ j<k}} \Big[E_{2i}(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + E_{3i}(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) \\
&+ E_{4i}(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) \; + \text{H.c.}\Big]
\end{aligned}
$$

The constants λ_i, A_i, γ_i, C_i, $\bar{C}_i$, D_i, E_{2i}, E_{3i}, E_{4i}, and E_{1ij} , $\forall_{i,j}$, $i, j = 1, 2, 3$ are taken real since CP invariance is imposed at the Lagrangian level

Higgs Potential under S_3 (only with phase independent terms)

[E. Derman]

$$V(\phi) = \sum_i \left[-\lambda\, \phi_i^\dagger \phi_i + A\, (\phi_i^\dagger \phi_i)^2 \right]$$

$$+ \sum_{i<j} \left[\frac{\gamma}{2} (\phi_i^\dagger \phi_j + \text{H.c.}) + C\, (\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) \right.$$

$$\left. + \bar{C} \left| \phi_i^\dagger \phi_j \right|^2 + \frac{D}{2} \left((\phi_i^\dagger \phi_j)^2 + \text{H.c.} \right) \right]$$

$$+ \frac{1}{2} \sum_{i \neq j} \left[E_1\, (\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \text{H.c.} \right]$$

$$+ \frac{1}{2} \sum_{\substack{i \neq j \neq k \\ j<k}} \left[E_2\, (\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + E_3\, (\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) \right.$$

$$\left. + E_4\, (\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) \; + \; \text{H.c.} \right]$$

Which are the terms that would lead to geometric SCPV?

Higgs Potential under $\Delta(27)$ or $\Delta(54)$

[G.B. Branco, J.M. Gérard, W. Grimmus]

[I. de Medeiros Varzielas, D. E.-C.]

$$V(\phi) = \cdots + \frac{1}{2} \sum_{\substack{i\neq j\neq k \\ j<k}} \left[E_4 \, (\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) \, + \, \text{H.c.} \right]$$

The groups found are $\Delta(27) \equiv (Z_3 \times Z_3) \ltimes Z_3$ and $\Delta(54) \equiv (Z_3 \times Z_3) \ltimes S_3$

Although $\Delta(27), \Delta(54)$ lead to the same scalar potential, which is not generally the case for the full Lagrangian

The E_4 phase dependence is $-2\varphi_i + \varphi_j + \varphi_k$ with minima ($\omega \equiv e^{i\frac{2\pi}{3}}$)

$$\langle\phi\rangle^\mathsf{T} = \frac{v}{\sqrt{3}} (1, \omega, \omega^2) \quad \text{for } E_4 < 0 \quad \text{(CP invariant)}$$

$$\langle\phi\rangle^\mathsf{T} = \frac{v}{\sqrt{3}} (\omega^2, 1, 1) \quad \text{for } E_4 > 0 \quad \text{(CP violating)}$$

The fields $\phi_i \backsim \mathbf{3}$ of $\Delta(3n^2)$ or $\Delta(6n^2)$, provide n is multiple of 3

Fermion Masses under $\Delta(27)$ or $\Delta(54)$

[I. de Medeiros Varzielas, D. E.-C.]

$$\mathbf{Q}\,\phi\,\mathbf{u}^c \text{ and } \mathbf{Q}\,\tilde{\phi}\,\mathbf{d}^c$$

$\Delta(27)$ case

- Q triplet: 1 sector $3_{0i} \times 3_{0i} \times 3_{0i}$ not viable
- Q singlets: both sectors $3_{01} \times 3_{02} \times 1_{rs}$
  - rank-1 mass matrices
  - one generation decoupled
  - diagonal matrices

$\Delta(54)$ case

- Q triplet: 1 sector $3_1^a \times 3_1^a \times 3_1^a$: two degenerate masses not viable
- Q singlets: rank-1 mass matrices
- Q doublet and singlet: degenerate masses

Potentially interesting for leptonic sector since it leads to a leading order structure with rank-1 charged lepton plus two degenerate neutrinos

[I. de Medeiros Varzielas, D. E.-C., P. Leser]

Phase-independent parameters: Is it possible to preserve the VEV structure $(1,1,1)$?

$$\left.\begin{aligned} v_1^n + v_2^n + v_3^n \\ v_1^n v_2^m + v_2^n v_3^m + v_3^n v_1^m \end{aligned}\right\} \quad \text{appear in the renormalisable case}$$

$$v_1^n v_2^m v_3^l + v_2^n v_3^m v_1^l + v_3^n v_1^m v_2^l \quad \text{appears only in the non-renormalisable case}$$

<u>The groups $\Delta(27)$ and $\Delta(54)$ leads to the same non-renormalisable scalar Potential</u>

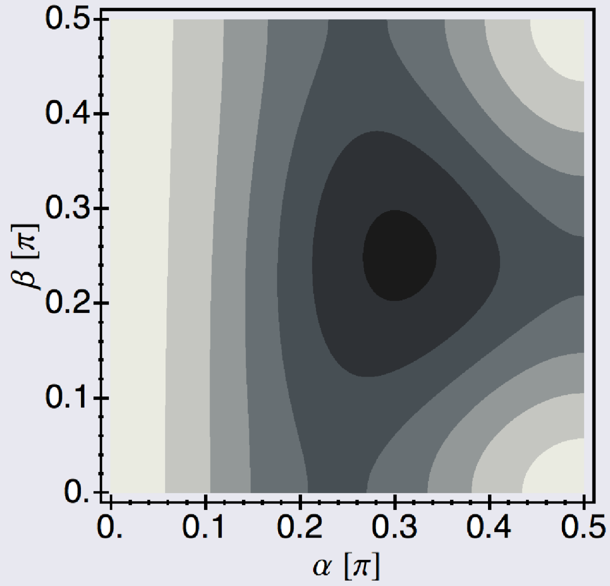
Preferred VEVs according to the sign of their coefficient

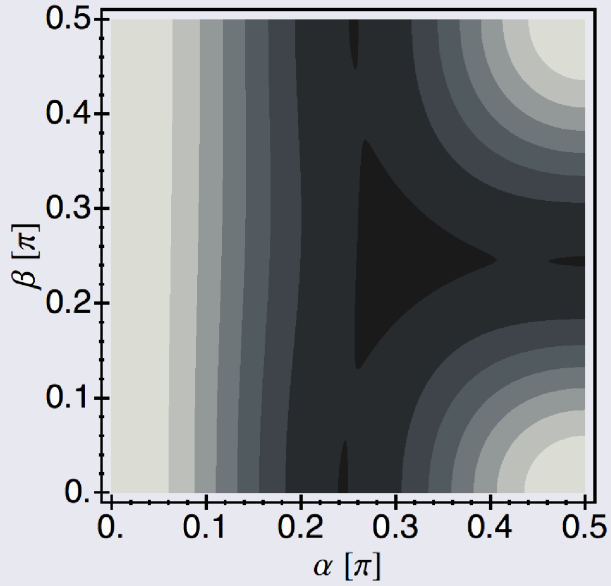
	$+$	$-$
v_i^n	$(1,1,1)$	$(0,0,1)$
$v_i^m v_j^n$	$(0,0,1)$	$(0,1,1)$
$v_1^l v_2^m v_3^n$	$(0,0,1)\,/\,(0,1,1)$	$(1,1,1)$

Non-renormalisable Scalar Potential

[I. de Medeiros Varzielas, D. E.-C., P. Leser]

- Phase-independent terms





$$(|v_1|, |v_2|, |v_3|) = (\sin(\alpha \cdot \pi)\cos(\beta \cdot \pi)\,,\ sin(\alpha \cdot \pi)\sin(\beta \cdot \pi)\,,\ \cos(\alpha \cdot \pi))$$

One can rely on $(1, 1, 1)$

- Phase-dependent terms

$$\left(\phi_i^2 \left(\phi_j \phi_k\right)^\dagger\right)^n \ \rightarrow\ 2n\varphi_i + n\varphi_j + n\varphi_k$$

$$\left(\phi_i^\dagger \phi_j\right)^{3n} \ \rightarrow\ 3n\varphi_i - 3n\varphi_j + 0\varphi_k$$

$$i \neq j \neq k$$

Conclusions

- $\Delta(27)$ and $\Delta(54)$ are the smallest groups that lead to geometrical complex VEVs, that violate CP symmetry spontaneously, with phases that are calculable and are stable against radiative corrections with the minimum number of three Higgs $SU(2)$ doublets

- It is possible to exactly preserve both the $(1,1,1)$ type of VEV together with calculable phases to an arbitrarily high order if one is willing to choose the appropriate signs of the respective combined coefficients

- At the renormalisable level it is not possible to fully incorporate all the observed quantities in the fermionic, although it is possible to have rank-1 mass matrices that may then be properly ajusted through non-renormalisable scalar-fermion interactions (work in progress)