A Zip-Code for Quarks, Leptons and Higgs Bosons

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Based on ArXiV:1209.xxxx
In collaboration with H. P. Nilles and P.-K. Oehlmann

XVIII European Workshop on String Theory

Corfu, Greece, September 20, 2012

Outline:

1. Motivation and Introduction
2. The $\mathbb{Z}_{6-\mathrm{II}}$ Mini-Landscape
3. The $\mathbb{Z}_2 \times \mathbb{Z}_4$ Orbifold
4. Conclusion

Motivation and Outlook

- Heterotic Orbifolds are promising candidates for a stringy completion of the Standard Model.
- A vast amount of promising models has been identified within the $\mathbb{Z}_{6-\mathrm{II}}$ orbifold.

[Kobayashi *et.al.*'05, Buchmuller *et.al.*'06, Lebedev *et. al.*'08]

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2. Are these results special for the $\mathbb{Z}_{6-\mathrm{II}}$ or do they provide a general pattern?

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   $\rightarrow$ Construct the new $\mathbb{Z}_2 \times \mathbb{Z}_4$ Orbifold.

- Can we confirm similar benchmarks in $\mathbb{Z}_2 \times \mathbb{Z}_4$?

The $\mathbb{Z}_{6-\mathrm{II}}$ Mini-Landscape

- Starting Point: The ten-dimensional $E_8 \times E_8$ Heterotic String.

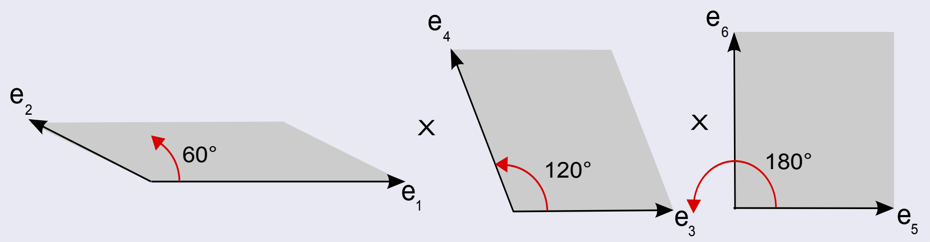
[Gross *et. al.*'85]

- Compactify the Extra Dimensions:

$$\mathcal{M}_{9,1} = \mathcal{M}_{3,1} \times \mathbb{O}_6$$

- Take $\mathbb{O}_6$ as the torus of $G_2 \times SU(3) \times SU(2)^2$ after modding out the isometry $\theta = \mathrm{e}^{2\pi \mathrm{i} v}$, $v = (\frac{1}{6}, \frac{1}{3}, -\frac{1}{2})$, and its powers.

[Kobayashi, Raby, Zhang'04]



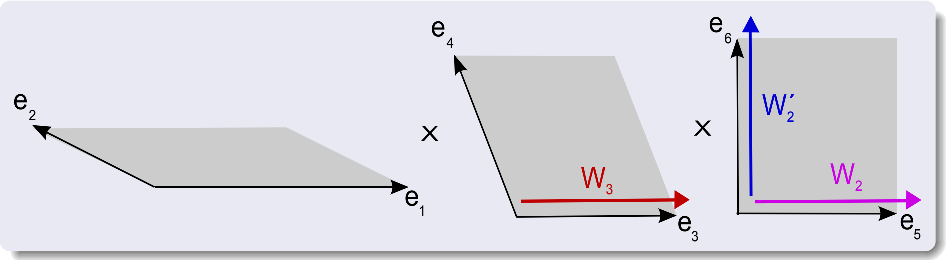
Gauge Symmetries

- Orbifolding breaks the $E_8 \times E_8$:
  - Consistency with modular invariance requires the “orbifolding” to act also in gauge space

    [Dixon *et. al.*'86, Ibañez, Nilles, Quevedo'87]

  - Simplest Alternative: Sifts and Wilson Lines

$$\theta \hookrightarrow V, \text{ such that } 6V \text{ belongs to the gauge lattice}$$



An example

[Lebedev *et. al.*'08]

1. Consider the shift

$$V = \left(\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0, 0\right)\left(\frac{1}{2}, -\frac{1}{6}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right),$$

it leads to the breaking

$$E_8 \times E_8 \xrightarrow{V} [SO(10) \times SU(2) \times SU(2) \times U(1)] \times [SO(14) \times U(1)]$$

with the following spectrum

U	T_2	T_3	T_4	T_5
$(\mathbf{10},\mathbf{2},\mathbf{2},\mathbf{1})_{0,0}$	$3(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})_{-\frac{28}{3},-\frac{2}{3}}$	$4+4(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})_{12,-2}$	$3+3(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})_{\frac{28}{3},\frac{2}{3}}$	$12\,(\overline{\mathbf{16}},\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{14}{3},\frac{1}{3}}$
$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})_{4,6}$	$3+3(\mathbf{10},\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{28}{3},-\frac{2}{3}}$	$4(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{14})_{0,0}$	$3(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{14})_{-\frac{20}{3},-\frac{10}{3}}$	$12(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})_{\frac{20}{3},\frac{10}{3}}$
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$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{64})_{6,-1}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{16}{3},\frac{16}{3}}$		$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{16}{3},-\frac{16}{3}}$	
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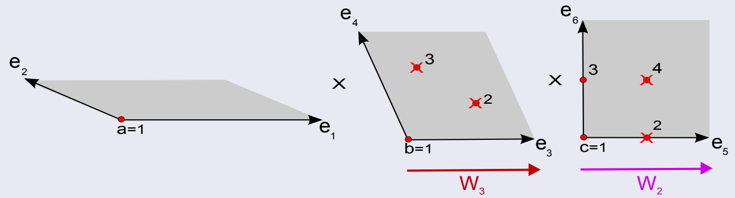
$$E_8 \times E_8 \xrightarrow{V} [SO(10) \times SU(2) \times SU(2) \times U(1)] \times [SO(14) \times U(1)]$$

2. Turn on W_2 and W_3 to break the $SO(10)$ down to G_{SM}.



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Two families from the T_5 sector.

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→ A purely untwisted trilinear coupling.

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$\rightarrow$ A purely untwisted trilinear coupling.

- The coupling $(\mathbf{10},\mathbf{2},\mathbf{2},\mathbf{1})_{0,0} \cdot (\overline{\mathbf{16}},\mathbf{1},\mathbf{2},\mathbf{1})_{-2,-3} \cdot (\overline{\mathbf{16}},\mathbf{2},\mathbf{1},\mathbf{1})_{2,3}$ is allowed by all symmetries.
- Specific choice of WLs permits the splitting

$$(\mathbf{10},\mathbf{2},\mathbf{2},\mathbf{1})_{0,0} \rightarrow \overbrace{(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{1/2,\dots}}^{H_u} + \overbrace{(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{-1/2,\dots}}^{H_d}$$

$$(\overline{\mathbf{16}},\mathbf{1},\mathbf{2},\mathbf{1})_{-2,-3} \rightarrow \underbrace{(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})_{-2/3,\dots}}_{\overline{U}} + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{0,\dots} + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{0,\dots}$$

$$(\overline{\mathbf{16}},\mathbf{2},\mathbf{1},\mathbf{1})_{2,3} \rightarrow \underbrace{(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1})_{1/6,\dots}}_{Q} + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{0,\dots}\ .$$

An example

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- Surviving pieces ensure a top-Yukawa

$$(\mathbf{10}, \mathbf{2}, \mathbf{2}, \mathbf{1}) \cdot (\overline{\mathbf{16}}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \cdot (\overline{\mathbf{16}}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \xrightarrow{W_2, W_3} Q H_u \overline{U}$$

- BONUS: $H_u \cdot H_d \subset (\mathbf{10}, \mathbf{2}, \mathbf{2}, \mathbf{1})^2$ is a neutral monomial under all selection rules, including R-symmetries!

Lessons from the Mini-Landscape

1. The Higgs System

- Compactifications provide plenty of Higgs candidates
- Neutral Higgs bilinear from the untwisted sector occurs very often in the models
  - Preference for gauge-Higgs unification.
  - A miraculous solution to the μ problem:

[Casas, Muñoz'93]

$\rightarrow$ R-symmetries are remnants of the Lorentz group in compact space

$\rightarrow$ $\mu H_u H_d \not\subset \mathcal{W}$ thanks to the R-symmetries

$\rightarrow$ R-symmetry breaking scale $\sim$ SUSY breaking scale

[Lee *et. al.*'11]

Lessons from the Mini-Landscape

2. The top-quark

- Top quark mass of the order of the weak scale
  Stringy top-Yukawa at trilinear order
- Usually, this coupling exists if $\overline{U}_3$, Q_3 and H_u are in the bulk

  $\rightarrow$ Top family is a patchwork of fields sitting at different positions in the extra dimensions

  $\rightarrow$ Gauge-Higgs-Top Unification

Lessons from the Mini-Landscape

3. The Light Families

- Two complete families as multiplets of an underlying GUT (e.g. $SO(10)$ or E_6)
- No trilinear couplings $\rightarrow$ Masses are suppressed
- Light families are a doublet of a D_4 flavor symmetry.

  $\rightarrow$ ameliorates the flavor problem

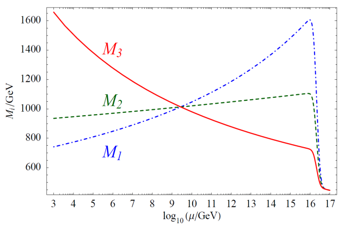
Lessons from the Mini-Landscape

4. The SUSY Breaking Pattern

- Hidden sector gaugino condensation is favored

[Lebedev *et. al.*'07]

- Dilaton stabilized at a realistic GUT value $\Rightarrow m_{3/2}$ in the multi-TeV range.
  $\rightarrow$ Mirage mediation



[Lebedev, Nilles, Ratz'06]

- Fields sitting at F.P. feel only $\mathcal{N}=1$ SUSY
  $\rightarrow$ Scalar masses $\sim m_{3/2}$
- Bulk and F.T. fields feel remnants of $\mathcal{N}=4,2$
  $\rightarrow$ Superpartner masses suppresed by $\log(M_{\mathrm{Pl}}/m_{3/2})$

$\rightarrow$ Natural SUSY

[Krippendorf *et. al.*'12]

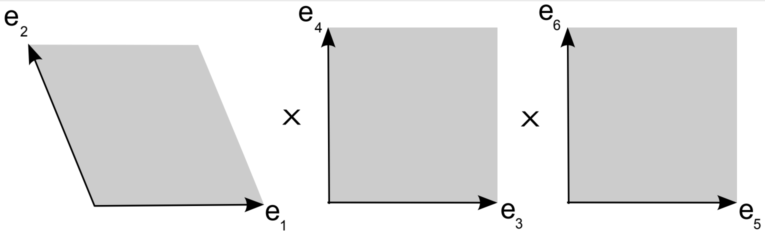
The $\mathbb{Z}_2 \times \mathbb{Z}_4$ orbifold

- Point group generators

$$\begin{array}{lll} \mathbb{Z}_2 : & \omega & v_2 = (\frac{1}{2}, -\frac{1}{2}, 0) \\ \mathbb{Z}_4 : & \theta & v_4 = (0, \frac{1}{4}, -\frac{1}{4}) \end{array}$$

- Compactification lattice

$SU(2)^2 \times SO(4) \times SO(4)$



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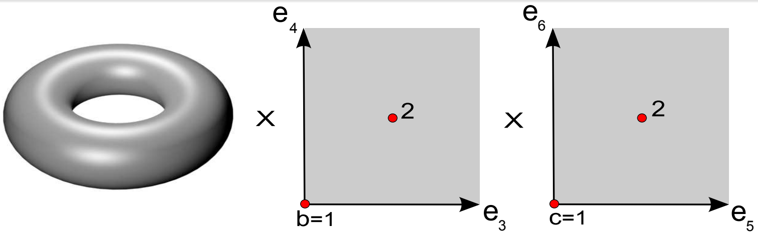
- Twisted sectors

Twisted Sector $T_{(0,1)}$ ($T_{(0,3)}$): 4 fixed tori (5-branes)



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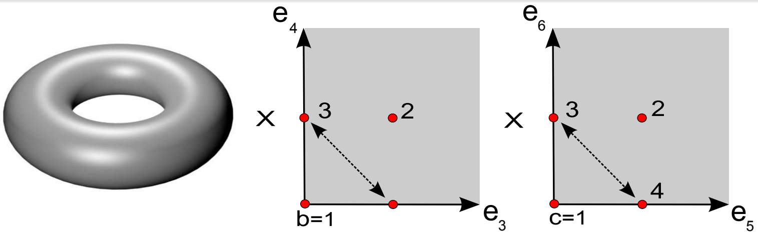
- Twisted sectors

Twisted Sector $T_{(0,2)}$: 10 fixed tori (5-branes)



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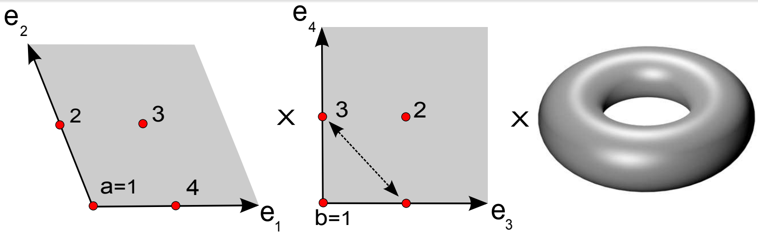
Twisted Sector $T_{(1,0)}$: 12 fixed tori (5-branes)



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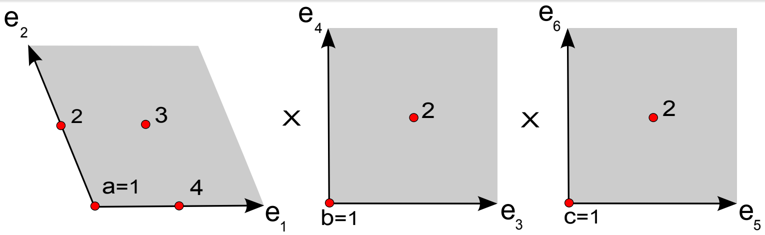
Twisted Sector $T_{(1,1)}$ ($T_{(1,3)}$): 16 fixed points (3-branes)



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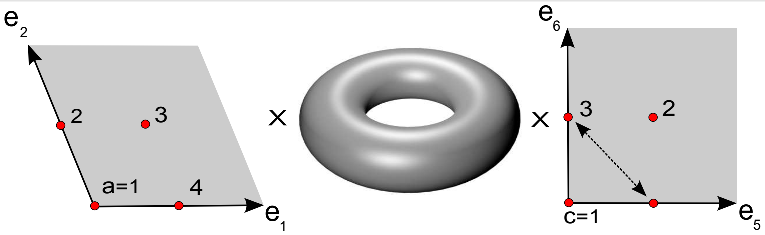
Twisted Sector $T_{(1,2)}$: 12 fixed tori (5-branes)



Gauge Embedding

- Two shifts V_2 and V_4 required to embed the point group
- Equivalences up to lattice vectors and lattice automorphisms lead to a finite amount of physical theories
- Modular invariance constraints

$$\gcd(N_i, N_j)(V_i \cdot V_g - v_i \cdot v_j) = 0 \bmod 2\,, \quad i,j = 1,2$$

[Ploger *et. al.*'07]

have to be satisfied.

- 144 inequivalent embeddings for (V_2, V_4). Two inequivalent models per embedding.
  1. 35 embeddings lead to the presence an SO(10) factor in the gauge group.
  2. 26 include E_6.
  3. 25 include SU(5).

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Model	Untwisted	(0,1)	(0,2)		(0,3)	(1,0)		(1,1)	(1,2)		(1,3)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
67	2(**10**), 2(**16**), 2($\overline{\mathbf{16}}$)					**16**	**10**,**16**				
						10	**10**,**16**				
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Multiplicities		4	4	6	4	8	4	16	8	4	16

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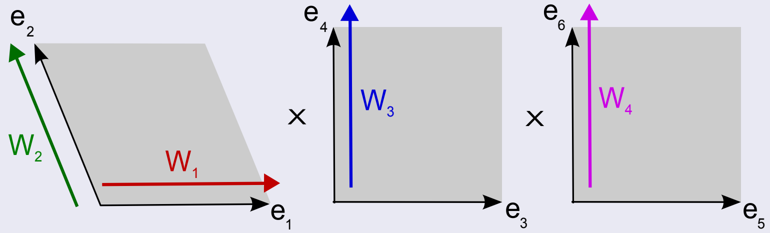
  $\Rightarrow$ The matter content at protected and split fixed points descends directly from the spectrum at the GUT level.

- Assume renormalizable top-Yukawa coupling does not involve unshielded states

  $\Rightarrow$ Search for locations and models which make the coupling possible upon a certain choice of WLs.

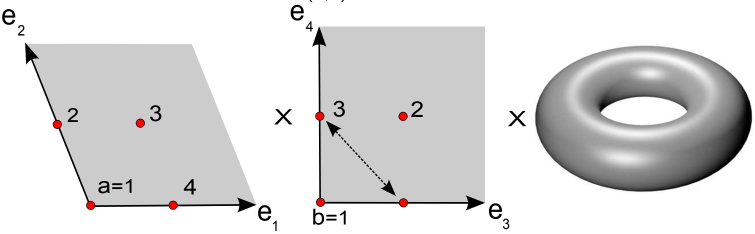
Wilson Lines and Gauge Topographies

- Four different posibilites for Wilson lines, all WL have to be of order two.



Wilson Lines and Gauge Topographies

- Four different posibilites for Wilson lines, all WL have to be of order two. 16 possible configurations!
- How do the WLs affect the gauge topographies? Consider for instance the $T_{(1,0)}$ sector:



Wilson Lines and Gauge Topographies

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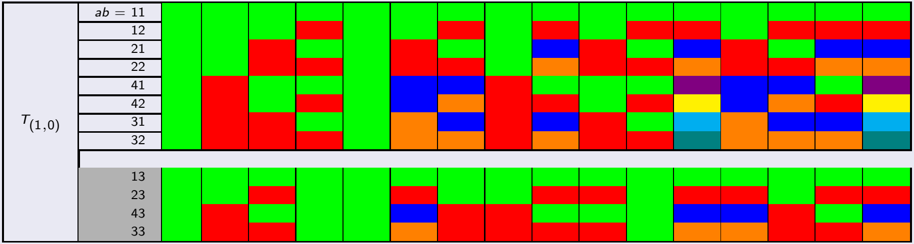
Config.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
W_1		✓				✓	✓	✓				✓	✓	✓		✓
W_2			✓			✓			✓	✓		✓	✓		✓	✓
W_3				✓			✓		✓		✓	✓		✓	✓	✓
W_4					✓			✓		✓	✓		✓	✓	✓	✓



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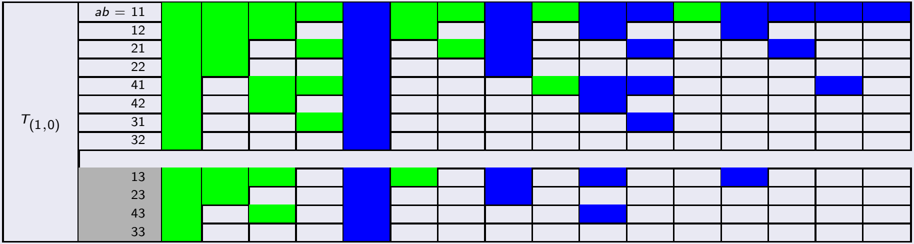
W_3				✓			✓		✓		✓	✓		✓	✓	✓
W_4					✓			✓		✓	✓		✓	✓	✓	✓



Flavor Symmetries

The Space Group Selection Rule SGSR

A coupling $\Phi_1 \cdot \Phi_2 \cdot \ldots \cdot \Phi_L$ is allowed in the superpotential only if the product of conjugacy classes for the fields contains the identity element.

[Dixon *et. al.*'86, Erler *et. al.*'92]

- The point group becomes a discrete symmetry of the $4D$ QFT. $\Rightarrow$ Assume that the field Φ_i belongs to $T_{(n_i,m_i)}$, then the L-point couplings does not vanish, provided

$$\sum_{i=1}^{L} n_i = 0 \mod 2\,, \quad \sum_{i=1}^{L} m_i = 0 \mod 4\,.$$

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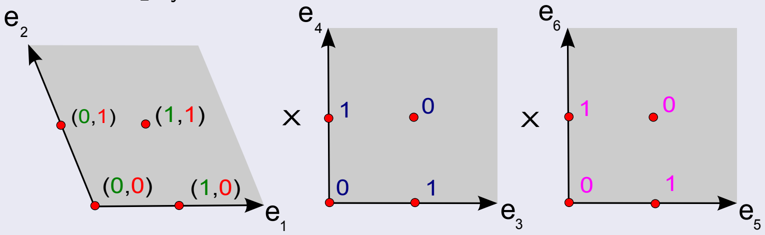
- The location of the singularities in the extra dimensions introduces four additional $\mathbb{Z}_2$ symmetries.



Flavor Symmetries

Permutation symmetries

There are some fixed points/tori which have exactly the same matter and can not be distinguished by means of the CFT.

[Kobayashi *et. al.*'07]

For $\mathbb{Z}_2 \times \mathbb{Z}_4$ we obtain

$$G_{\text{Flavor}} = \frac{\left(\frac{D_4 \times D_4}{\mathbb{Z}_2}\right) \times \left(\frac{D_4 \times \mathbb{Z}_4}{\mathbb{Z}_2}\right) \times \left(\frac{D_4 \times \mathbb{Z}_4}{\mathbb{Z}_2}\right)}{\mathbb{Z}_2 \times \mathbb{Z}_4} = \frac{D_4^4 \times \mathbb{Z}_4}{\mathbb{Z}_2^4}.$$

Wilson lines break the flavor group blockwise, they affect only the non-Abelian parts.

$\rightarrow$ Topography + Flavor symmetry structure allow for the two light families to complete GUT multiplets transforming as a flavor doublet.

R-Symmetries

There is a discrete subgroup of $SO(6) \subset SO(9,1)$ which survives compactification and orbifolding.

→ Potential candidates for R-symmetries in the low energy

- Lattice isometries acting only in each plane fulfill these conditions.

[Font *et. al.*'88]

- For the specific case of $\mathbb{Z}_2 \times \mathbb{Z}_4$, non vanishing couplings in the superpotential must satisfy

$$\sum_{\alpha=1}^{L} R^1_\alpha = 1 \bmod 2, \qquad \sum_{\alpha=1}^{L} R^2_\alpha = 1 \bmod 4, \qquad \sum_{\alpha=1}^{L} R^3_\alpha = 1 \bmod 4.$$

VERY IMPORTANT: If Higgses have R-charges $(1,0,0)$, the Higgs bilinear is neutral under all selection rules!

Trilinear Top-Yuakawa Coupling

- Selection rules leave a few possibilities for a trilinear coupling
- All except a purely untwisted coupling turn out to be disfavored because
  1. Models contain more than one heavy family
  2. There is no shift embedding with the desired features
  3. Rule out the possibility for the light families to be complete GUT multiplets
- Untwisted trilinear coupling is verified to exist in 75% of all $SO(10)$ models

$\rightarrow$ Left- and right-chiral top and up type Higgs live in the bulk

$\rightarrow$ Down-type Higgs likely to live also in the untwisted sector.

Conclusions

- The $\mathbb{Z}_2 \times \mathbb{Z}_4$ seems to confirm the $\mathbb{Z}_{6-\mathrm{II}}$ location picture.
- The presence of a $\mathbb{Z}_2$ plane favors the Higgses to arise as a vector-like pair from the bulk.
- The amount of twisted sectors gives more flexibility for the light families to be accommodated in the extra dimensions.

Prospects

- Explicit construction of $\mathbb{Z}_2 \times \mathbb{Z}_4$ models is work in progress
- Algorithms developed for classifying embeddings and obtaining matter spectra can be extended to other $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds

$\varepsilon\upsilon\chi\alpha\rho\iota\sigma\tau\acute{\omega}$