Supersymmetric backgrounds of M-theory and AdS(4)/CFT(3)

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Based on

1207.3082, 1107.5035 [Gabella, Martelli, A.P., Sparks]
1110.6400 [Martelli, A.P., Sparks]

Introduction

Prototype

M2 branes on $\mathbb{C}_4/\mathbb{Z}_k \to$ ABJM theory

Dual supergravity background Freund-Rubin compactification

$$\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k \ , \qquad G \ = \ N \, \mathrm{vol}_{\mathrm{AdS}_4} \ .$$

In general: $\mathrm{AdS}_4 \times Y_7$ where Y_7 Sasaki-Einstein .

Motivation

Beyond Freund-Rubin

I. Turn on internal flux

$$G = m\,\mathrm{vol}_{\mathsf{AdS}_4} + F\,,$$
$$g = e^{2\Delta}(g_{\mathsf{AdS}_4} + g_{Y_7}) \quad \text{warped product}\,.$$

II. Deformation of Y_7 (non-product geometries)

e.g. gauging Killing vectors of $Y_7 \rightarrow$ gauge fields propagating in AdS_4

back-reaction $\rightarrow$ asymptotically locally anti-deSitter solutions .

$\mathcal{N} = 2$ SUSY $\mathrm{AdS}_4 \times Y_7$ solutions

Ansatz

$$g = e^{2\Delta}(g_{\mathrm{AdS}_4} + g_{Y_7}) \; ,$$
$$G = m \, \mathrm{vol}_{\mathrm{AdS}_4} + F \; .$$

Supersymmetry: Killing spinor ϵ

$$\delta_\epsilon \psi = \nabla_m \epsilon + \frac{1}{288} \left(\Gamma_m{}^{npqr} - 8 \delta_m{}^n \Gamma^{pqr} \right) G_{npqr} \epsilon = 0 \; , \quad \psi \text{ gravitino} \; .$$

$\mathcal{N} = 2$ SUSY $\text{AdS}_4 \times Y_7$ solutions

Killing spinor: $Spin(3,1) \times Spin(7) \subset Spin(10,1)$

$$\mathcal{N} \ = \ 2 \ \ \rightarrow \ \ \epsilon \ = \ \sum_{i=1}^{2} \psi_i^+ \otimes \chi_i + (\psi_i^+)^c \otimes \chi_i^c$$

ψ_i^+ : positive chirality spinors on AdS_4

Majorana spinors : $\chi_i \ = \ \chi_i^c \ \rightarrow$ Sasaki-Einstein

Dirac spinors : $\chi_i \ \neq \ \chi_i^c \ \rightarrow$ $SU(2)$ structure

$\mathcal{N} = 2$ SUSY $AdS_4 \times Y_7$ solutions

$SU(2)$ structure in 7d

$7 \;\rightarrow\; 3 + 4$ decomposition

$$g_7 \;=\; e^7 \otimes e^7 + e^6 \otimes e^6 + e^5 \otimes e^5 + g_4 \; .$$

$$J_1,\; J_2,\; J_3 \quad \text{almost-complex structures on } g_4 \; .$$

Killing spinor equation $\rightarrow$

- differential conditions on the $SU(2)$ structure
- F determined in terms of the geometry

$\mathcal{N} = 2$ SUSY $\text{AdS}_4 \times Y_7$ solutions

u(1) R-symmetry

Killing vector field ξ rotating χ_i .

1-form σ defines a contact structure

$$d\sigma \;\propto\; J_3 + e^5 \wedge e^6 \;\;\Rightarrow\;\; \sigma \wedge (d\sigma)^3 \;\propto\; \mathrm{vol}_7 \; .$$

$$\xi \cdot d\sigma \;=\; 0 \; , \quad \xi \cdot \sigma \;=\; 1 \; .$$

$\mathcal{N} = 2$ SUSY $\mathrm{AdS}_4 \times Y_7$ solutions

Exact solution

Additional assumption: extra Killing vector

- g_4 is conformal to a Kähler-Einstein metric .
- susy conditions reduce to a single 2nd order non-linear ODE for the warp factor .

$\mathcal{N} = 2$ SUSY $\mathrm{AdS}_4 \times Y_7$ solutions

Two solutions:

1. [Corrado, Pilch, Warner]
2. new solution

same topology as S^7 but different metric .

$\mathcal{N} = 2$ SUSY $AdS_4 \times Y_7$ solutions

AdS(4)/CFT(3) application

Free Energy

$$\mathcal{F} = -\log Z \ , \qquad Z \ \text{ partition function} \ .$$

gravity side

$$Z = e^{-I_{EH}} \ .$$

$$\mathcal{F}_{\rm AdS} = \frac{\pi}{2G_4} = N^{3/2}\sqrt{\frac{32\pi^6}{9\int_{Y_7}\sigma\wedge(d\sigma)^3}} \ .$$

Exact computation by localization techniques [Martelli, Sparks, Yau] .

Squashed 3-sphere

Supersymmetric gauge theories on compact curved manifolds amenable to localization techniques .

SUSY parameter ϵ

$$\nabla_\mu \epsilon \; = \; \frac{i}{2} \gamma_\mu \epsilon \quad \text{admits the round } S^3 \text{ as a solution}$$

[Hama, Hosomichi, Lee]

$$D_\mu \epsilon \; = \; \frac{i}{2f(\theta)} \gamma_\mu \epsilon \; ,$$

$$D_\mu \; = \; \nabla_\mu - i A_\mu \; , \quad f(\theta) \; = \; \frac{1}{b^4 \cos^2 \theta + \sin^2 \theta} \; .$$

Squashed 3-sphere

Solution

$$ds_3^2 = \frac{d\theta^2}{b^4\cos^2\theta + \sin^2\theta} + \cos^2\theta d\varphi_1^2 + b^{-4}\sin^2\theta d\varphi_2^2 \ ,$$

$$A = \frac{1}{2(b^4\cos^2\theta + \sin^2\theta)}\left(d\varphi_1 - b^{-2}d\varphi_2\right) \ .$$

Gravity dual ? First addressed in [1110.6400] .

Squashed 3-sphere

(Euclidean) $\mathcal{N} = 2$ gauged supergravity

Bosonic sector: g metric $+$ $U(1)$ gauge field (graviphoton)

susy variation:

$$\left(\nabla_\mu + \tfrac{1}{2}\Gamma_\mu - iA_\mu - \tfrac{i}{4}F_{\nu\rho}\Gamma^{\nu\rho}\Gamma_\mu\right)\epsilon \;=\; 0\ .$$

Squashed 3-sphere

Solution

Euclidean AdS

$$ds_4^2 = \frac{y^2 - f^{-2}(\theta)}{(y^2-1)(y^2-b^4)}dy^2 + \frac{y^2 - f^{-2}(\theta)}{f^{-2}(\theta)}d\theta^2 + (y^2-1)\cos^2\theta d\varphi_1^2 + \frac{y^2-b^4}{b^4}\sin^2\theta d\varphi_2^2 \; .$$

gauge field: instanton $\rightarrow$ zero stress-energy tensor

$$A = \frac{1}{2(y+f(\theta))}\left[(y\,f(\theta)-1)d\varphi_1 + (b^2 - b^{-2}y\,f(\theta))d\varphi_2\right] \; .$$

Squashed 3-sphere

consistency check

In the limit $y \to \infty$

- bulk metric + gauge field $\rightarrow$ boundary metric + gauge field
- bulk KSE + Killing spinors $\rightarrow$ boundary KSE + Killing spinors

Squashed 3-sphere

gravitational Free Energy

$\rightarrow I_{grav}$: gravity contribution infinite

- finite via holographic renormalisation

$\rightarrow I_F$: instanton contribution finite

Total:

$$\mathcal{F} = \left(b + \frac{1}{b}\right)^2 \frac{\pi}{8G_4} .$$

Squashed 3-sphere

field theory Free Energy

supersymmetric localization techniques $\rightarrow$ Z reduces to a matix integral

$U(N)^G$ Chern-Simons quiver gauge theory, CS levels k_I, $I = 1, \ldots, G$

$$Z_b \;=\; \frac{1}{N!^G} \int \left(\prod_{I=1}^{G} \prod_{i=1}^{N} \frac{d\lambda_i^I}{2\pi} \right) \exp\left[-F_b\left(\lambda_i^I\right)\right] \;.$$

Squashed 3-sphere

large N

For $\mathcal{N} \geq 2$ it is possible to extract large N value

[Martelli, Sparks], [Cheon, Kim, Kim], [Jafferis, Klebanov, Pufu, Safdi]

It matches with the gravity side !