Toward a superpotential for the Papadopoulos–Tseytlin ansatz.

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Introduction and motivation.

- Several illustrious solutions based on the conifold. Great insight into physics of strongly coupled gauge theories.
- Share some properties:
  - Poincaré symmetric in 4d $\Rightarrow$ Domain Wall.
  - Preserve $\mathcal{N} = 1$ susy.
  - Attainable from the PT ansatz. Known (empirical) superpotentials.
- Supersymmetrization of the PT ansatz. It's an $\mathcal{N} = 4$ gauged sugra.
- Systematic, unifying picture for these solutions? Sugra origin for the superpotentials?

- We are led consider 1/4 BPS Domain Walls of 5d $\mathcal{N} = 4$ gauged sugra.
- Similar to how the FGPW flow was understood in $\mathcal{N} = 2$ [Ceresole, Dall'Agata, Kallosh, Van Proeyen '01].
- A general, methodical procedure can give new solutions.
- If the sugra is a consistent truncation: solution of string theory!

Domain Walls

- Start from

$$S = \frac{1}{2\kappa_d^2} \int \Big[R - G_{xy}(\phi)\, d\phi^x \lrcorner\, d\phi^y - 2\, V(\phi) \Big] *_d 1$$

- Gravity solutions with (d-1)-dimensional Poincaré invariance

$$ds^2 = dr^2 + a^2(r)\, \eta_{\mu\nu} dx^\mu dx^\nu$$

supported by radial-dependent scalars.

- Special class: BPS

$$V = \frac{d-2}{2} \left[(d-2)\, G^{xy}\, \partial_x W\, \partial_y W - (d-1)\, W^2 \right]$$

- The quantity W is called (fake) Superpotential. Gives $1^{\rm st}$ order equations!

$$\frac{a'}{a} = \pm W\,, \qquad\qquad \phi^{x\,\prime} = \mp (d-2)\, G^{xy}\, \partial_y W$$

- The e.o.m.'s follow. Stability is ensured.

- In the sugra context, these are BPS equations coming from vanishing of fermionic variations.

- Conifold solutions are susy and admit superpotential. BPS Domain Walls of a suitable sugra?

$\mathcal{N} = 4$ d=5 supergravity.

- The theory is characterized by the number of vectors and the gauging. Gravity multiplet $\{g,\ 6\,A,\ \sigma\}$ and vector-tensor multiplet $\{A,\ 5\,\Phi\}$.

- The scalar manifold is

$$SO(1,1) \times \frac{SO(5,n)}{SO(5) \times SO(n)}$$

- Gauging described by embedding tensor [Schön, Weidner '06];

$$\Theta \quad \supset \quad f^{MNR}, \quad \xi^{MN}, \quad \cancel{\xi^{M}}.$$

- Fermionic variations read schematically (in DW ansatz)

$$\begin{aligned} \delta\psi &= \frac{a'}{a}\,\epsilon + P\,\epsilon \\ \delta\chi &= \sigma'\,\epsilon + \partial_\sigma P\,\epsilon \\ \delta\lambda &= \phi'_x\,\epsilon + D_x P\,\epsilon + K_x\,\epsilon \end{aligned}$$

- Gauging information contained in the gravitino shift matrix

$$P = P(\sigma, \phi^x) \qquad \supset \qquad f^{MNR}\,, \quad \xi^{MN}$$

- Vanishing of variations will give:
  - Superpotential and BPS equations.
  - Projectors describing the embedding of $\mathcal{N} = 1$ into $\mathcal{N} = 4$.
  - Algebraic constraints involving the scalars.

- R-symmetry is $USp(4) \simeq SO(5)$. Choose preferred direction with a vector $\mathcal{A}$ that breaks

$$SO(5) \quad \rightarrow \quad SO(4) \simeq SU(2)_{+} \times SU(2)_{-}$$

- Simultaneously $\mathcal{A}$ defines a chirality. Project to singlets under one of the SU(2)'s.

- We are left with one SU(2), situation similar to the $\mathcal{N} = 2$ case.

- Distinguished $USp(4)$ matrix P_{ab}. We can form a vector

$$\mathcal{A}^{a} \;\sim\; \epsilon^{abcde} \, P_{bc} \, P_{de}$$

- Gravitino variation solved by the projectors

$$\mathcal{A}\,\epsilon = \epsilon$$

that selects the $SU(2)_+$ subspace, and

$$P\,\epsilon = W\,\epsilon = \frac{a'}{a}\,\epsilon$$

that singles out one of the residual spinors as required for 1/4 BPS.

- The superpotential reads

$$W = \pm\sqrt{2\,Tr\,P^2 \pm X}$$

with

$$X = \sqrt{8\left[Tr\,P^2\right]^2 - 16\,Tr\,P^4}$$

- Dilatino and gaugino variations yield BPS equations plus algebraic constraints ($\partial_\sigma \mathcal{A} = 0 = \partial_\sigma(P/W)\ \ldots$).

Type IIB on squashed SE.

- Is an $\mathcal{N}=4$ sugra with 2 vectors [Cassani, Dall'Agata, AF '10], [Gauntlett, Varela '10].

- Explicit embedding tensor: we can construct P and the rest of the quantities.

- Reduces to 3 non-trivial modes:

$$Tr\,(\mathcal{W}^2)\,, \quad (\textit{first}) \qquad Tr(\mathcal{W}^2\overline{\mathcal{W}}^2) \qquad (\textit{first and last})$$

- One has to kill the (irrelevant, $\Delta=8$) D-term source for AdS asymptotics.

- Most general DW based on SE takes the D3-brane form

$$ds^2 = h^{-1/2}(\rho)\ ds^2(M_4) + h^{1/2}(\rho)\ ds^2(M_6)$$

with transverse space

$$ds^2(M_6) = \frac{e^{2\rho}}{(1 - V_0\, e^{-6\rho})^{2/3}} \left[d\rho^2 + \eta^2 + (1 - V_0\, e^{-6\rho})\, ds^2(B_{KE}) \right]$$

- CY with blown-up 4-cycle at V_0 [Benvenuti, Mahato, Pando-Zayas, Tachikawa '05].

- Reduces in appropriate limits to known solutions (GPPZ, [Benini, Canoura, Cremonesi, Nuñez, Ramallo '06])

$T^{1,1}$ within the PT ansatz.

- Type IIB on $T^{1,1}$ is $\mathcal{N} = 4$ with 3 vectors [Cassani, AF '10], [Bena, Giecold, Graña, Halmagyi, Orsi '10].
- PT ansatz is a subsector of it (consistent truncation).
- Contains 2 fluxes $\{P, Q\}$ and 9 scalars $\{p, x, g, a, b, h_1, h_2, K, \chi\}$. Some have nice geometric interpretation.
- Algebraic constraints can be used to put h_1 and h_2 in terms of the rest. Satisfied in the known susy solutions.
- Taking this into account

$$W = e^{-2p-2x-g} a\, S + e^{4p}\, S^{-1} \left[C + e^{-2x+\phi} P^2 \,(b - C)\,(b\, C - 1) \right]$$

with

$$C \equiv \frac{1 + a^2 + e^{2g}}{2a} \qquad S \equiv \frac{\sqrt{a^4 + 2a^2(-1 + e^{2g}) + (1 + e^{2g})^2}}{2a}$$

- Can be thought of as a superpotential for the baryonic branch (supplemented with the algebraic constraints).

- Reduces to the ones given in PT in the pertinent limits except for [Pando-Zayas, Tseytlin '00]. It is non-susy!

- The susy superpotential is

$$W = e^{-2p-2x}\cosh y + e^{4p} + e^{4p-2x}\sqrt{e^{2x+\phi}P^2\sinh^2 y + \frac{1}{4}[Q+P(f_1-f_2)]^2}$$

Solution related to fractional D3-branes on the resolved conifold.

- Numeric solutions (IR singular). The uplifted projectors show Myers effect

$$\begin{aligned} \epsilon - \frac{1}{8}\,\Gamma^{0123}\,\Gamma^{AB}\,J_{AB}\,\epsilon &= 0 \\ \epsilon - i\Gamma^{0123}\left(\cos\beta\,\epsilon + \sin\beta\,\frac{1}{8}\,\Gamma^{AB}\,J_{AB}\,\epsilon^c\right) &= 0 \end{aligned}$$

Summary and conclusions.

- General construction of 1/4 BPS Domain Walls in $\mathcal{N} = 4$ sugra. Unifying picture for susy solutions on the conifold. Superpotential for the baryonic branch.

- New solutions: general SE and susy resolved conifold. Include modes outside PT.

- Inspiration for fake superpotentials and non-susy solutions.

- Application to other consistent truncations (Romans $SU(2) \times U(1)$).

- Similar constructions in d=4.