Andrea Borghese

A geometric bound on F-term inflation

Based on: A.B., D.Roest, I.Zavala, [1203.2909]
A geometric bound on F-term inflation

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right] \]
predictions on cosmological observables are (so far) perfectly consistent with observations
[Guth, 81; Linde, 82]

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IS IT POSSIBLE TO EMBED INFLATION IN A UV-COMPLETE THEORY?
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4 dimensional lagrangians
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Inflationary lagrangians
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Supergravity lagrangians

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String Theory

Low energy limit

10 dimensional Supergravity

4 dimensional lagrangians

Inflationary lagrangians

Compactification

CORFÙ 22/09/12
IS IT POSSIBLE TO EMBED INFLATION IN A UV-COMPLETE THEORY?

STRING THEORY

low energy limit

10 dimensional Supergravity

compactification

Supergravity lagrangians

Supergravity is like a bridge between the EFT of inflation and the UV complete theory
SUPERGRAVITY SPECTRA

dozen of scalar fields

during inflation we have a deSitter (dS) space-time in which SUSY is broken completely

non-supersymmetric configurations high probability of tachyonic directions (no stable dS vacua in $\mathcal{N} = 4, 8$)
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$H^2$ Hubble scale
(during inflation is given by the value of the scalar potential)

$m^2$ inf inflaton mass

$m^2$ other scalars

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SCALARS ARRANGE THEMSELVES IN MANIFOLDS

for $\mathcal{N} > 2$ they are coset manifolds

for $\mathcal{N} = 1,2$ they are complex manifolds such as Hodge-Kähler, special Kähler or quaternionic-Kähler
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SGOLDSTINI

Super-Higgs mechanism $= \text{Higgs mechanism} \ " + 1/2 "$

For every broken SUSY we have a spin-1/2 field called Goldstino

the Goldstini are “eaten up” by the gravitini
the gravitini eventually become massive

$\eta^i \propto N^i_a \chi^a$

$i = 1, \ldots, \mathcal{N}$

$a$ labels spin-1/2 fields
sGoldstini are the supersymmetric partners of Goldstini

\[ \eta^i \propto N^i_a \chi^a \]

\[ \varepsilon^j \]

\[ \tilde{\eta}^{ij} \propto N^{ij}_\alpha \phi^\alpha \]

\( \mathcal{N}^2 \) directions in the scalar manifold

\( \alpha \) labels the scalar fields
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- \( N = 1 \):
  - One complex direction corresponding to two real scalar d.o.f.

- \( N = 2 \):
  - 4 complex directions
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correspond to a
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and 3 symmetric and 3 symmetric

- \( N = 8 \):
  - 64 complex directions
  - 28 anti-symmetric
  - 36 symmetric
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\( \mathcal{N} = 8 \)

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Used to check perturbative stability of critical points in 4D supergravity

[Gomez-Reino, Scrucca, 06-07; Gomez-Reino, Louis, Scrucca, 08; A.B., Roest, 10; A.B., Linares, Roest, 11]
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\[ \mathcal{N} = 1 \quad \text{only chiral-multiplets} \]

theory completely specified by

\[ V(\phi) = e^K \left( -3 W \bar{W} + K^{\alpha \bar{\beta}} D_{\alpha} W D_{\bar{\beta}} \bar{W} \right) \]

Kähler potential \( \mathcal{K} = \mathcal{K}(\phi^\alpha, \bar{\phi}^{\bar{\alpha}}) \)

super potential \( \mathcal{W} = \mathcal{W}(\phi^\alpha) \)

\[ N_\alpha = e^{K/2} D_\alpha \mathcal{W} \]

two real directions

[Covi, Gomez-Reino, Gross Louis, Palma, Scrucca, 08]
A geometric bound on F-term inflation

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$m_{sG}^2 = \frac{1}{2} (m_1^2 + m_2^2)$

$\eta_{sG} \equiv \frac{m_{sG}^2}{V} \leq \frac{2}{3\gamma} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \sqrt{\epsilon} + \frac{\gamma}{1+\gamma} \epsilon - \frac{1+\gamma}{\gamma} \tilde{R}$

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| \[\gamma \equiv \frac{V}{3|m_{3/2}|^2} \] |

| \[\tilde{R} = \mathcal{R}_{\alpha \bar{\beta} \gamma \delta} \hat{N}^\alpha \hat{N}^{\bar{\beta}} \hat{N}^\gamma \hat{N}^\delta \] |

| \[\epsilon \equiv \frac{\mathcal{K}^{\alpha \bar{\beta}} \mathcal{D}_\alpha V \mathcal{D}_{\bar{\beta}} V}{2V^2} \] |

Ratio between Hubble scale and gravitino mass

sectional curvature related to the plane spanned by sGoldstino directions

first slow-roll parameter

[Covi, Gomez-Reino, Gross Louis, Palma, Scrucca, 08]
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\[ \gamma \equiv \frac{V}{3|m_{3/2}|^2} \quad \text{take the limit} \quad \gamma \to \infty \]

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single field inflation implies \( \eta_{sG} \geq \frac{1}{2} \)

slow-roll inflation implies \( \epsilon \ll 1 \)

canonical kinetic terms for all scalars imply \( \tilde{R} = 0 \)
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**GENERALISATION TO EXTENDED SUPERGRAVITY**

\[ \mathcal{N} = 1 \quad \eta_{sG} \equiv \frac{m_{sG}^2}{V} \leq \frac{2}{3\gamma} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \sqrt{\epsilon} + \frac{\gamma}{1+\gamma} \epsilon - \frac{1+\gamma}{\gamma} \tilde{R} \]

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a similar bound can be obtained in the case of extended supergravities

\[ \mathcal{N} = 2 \]

\[ \eta_{sG} \leq c_0 (f(\gamma) + g(\gamma) \bar{R}) + c_{1/2} \frac{1}{\sqrt{1+\gamma}} \sqrt{\varepsilon} + c_1 \frac{\gamma}{1+\gamma} \varepsilon \]

\[ \mathcal{N} = 8 \]
VIABILITY OF INFLATION IN F-TERM SUPERGRAVITY

- Geometry is tightly entangled with dynamics of scalar fields

Constraints on inflationary dynamics:
- average sGoldstino mass is bounded from above by first slow roll parameter and geometric data

- Similarities in the analysis for minimal and extended supergravity
THANK YOU!

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