A Holographic Approach to Non-relativistic Logarithmic CFT's

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Introduction

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- The duality has become an important tool in studying strongly correlated conformal field theories (CFT)
- Here we will consider gravity duals to Logarithmic CFT's

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- In conformal field theories correlation functions behave as

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \delta_{ij}\frac{c}{|x-y|^{2\Delta}}$$

and there exist an Hamiltonian that is diagonizable

$$[H, \mathcal{O}_i] = E_0\mathcal{O}_i$$

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- Correlators involve logarithmic terms

$$\begin{aligned} \langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle &= 0 \\ \langle \mathcal{O}_i^{\log}(x) \mathcal{O}_j(y) \rangle &= \delta_{ij} \frac{b}{|x-y|^{2\Delta}} \,, \\ \langle \mathcal{O}_i^{\log}(x) \mathcal{O}_j^{\log}(y) \rangle &= \delta_{ij} \frac{b}{|x-y|^{2\Delta}} (2 \log |x-y| + \lambda) \end{aligned}$$

Massive Gravity

Supplement Einstein-Hilbert gravity with cosmological constant and interaction with up to four derivatives of the metric.

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- Topologically Massive Gravity (TMG) [Deser, Jackiw, Templeton; 1988]
  Lorentz Chern-Simons term:

$$\mathcal{L}_{\mathrm{LCS}} = \epsilon^{\mu\nu\rho}\Gamma^{\gamma}_{\mu\alpha}\left(\partial_\nu \Gamma^{\alpha}_{\rho\gamma} + \frac{2}{3}\Gamma^{\alpha}_{\nu\beta}\Gamma^{\beta}_{\rho\gamma}\right)$$

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- Einstein term with cosmological constant
- New Massive Gravity (NMG) [Bergshoeff, Hohm, Townsend, 2009]
  R^2 term in 3 dimensions:

$$\mathcal{L}_{\mathrm{R}^2} = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2$$

New Massive Gravity in d dimensions

$$S = \frac{1}{\kappa^2} \int d^d x \sqrt{-g} \left\{ \sigma R - 2\lambda m^2 + \frac{1}{m^2} G^{\mu\nu} S_{\mu\nu} \right\}$$

with

$$S_{\mu\nu} = \frac{1}{(d-2)} \left(R_{\mu\nu} - \frac{1}{2(d-1)} R g_{\mu\nu} \right)$$

After linearization around an AdS background one can distinguish two cases, depending on the value of

$$\bar{\sigma} = \sigma - \frac{\lambda}{m^2} \frac{1}{d-1}$$

- $\bar{\sigma} = 0 \Rightarrow$ critical gravity
- $\bar{\sigma} \neq 0 \Rightarrow$ non-critical gravity

Non-critical Gravity

The linearized Lagrangian has the general structure of [Bergshoeff, Hohm, Rosseel, Townsend, 2010]

$$\mathcal{L}_2 = \bar{\sigma}\mathcal{L}_{EH}(h) - \frac{1}{m^4\bar{\sigma}(d-1)^2(d-2)^2}\mathcal{L}_{R^2}(k)$$

- 2 massless spin-2 modes, 2 massive spin-2 modes

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  - Unitary for $\bar{\sigma} < 0$
- $d > 3$
  - $\mathcal{L}_{EH}$ and $\mathcal{L}_{R^2}$ have opposite sign $\Rightarrow$ non-unitary

Critical Gravity

The linearized equations of motion can be reduced to:

$$\left(\Box - \frac{4\Lambda}{(d-1)(d-2)} - M^2\right)\left(\Box - \frac{4\Lambda}{(d-1)(d-2)}\right) h_{\mu\nu} = 0$$

with

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- At $\bar{\sigma} = 0$ the massive modes degenerate with the massless (or boundary) modes.
- A new solution can be found, which solves $\left(\Box - \frac{4\Lambda}{(d-1)(d-2)}\right)^2 h^{\log}_{\mu\nu} = 0$, but does not solve $\left(\Box - \frac{4\Lambda}{(d-1)(d-2)}\right) h^{\log}_{\mu\nu} = 0$. This is the logarithmic mode.

AdS/Log CFT

- In $d = 4$, the log mode is $h_{\mu\nu}^{\log} = (i\tau + \log(\cosh^2 \rho))h_{\mu\nu}^{M=0}$

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- Following the AdS/CFT dictionary for computing correlation functions on the boundary, at the critical point the correlators form a Logarithmic Conformal Field Theory.

[Skenderis, Taylor, van Rees, 2008; Grumiller, Hohm, 2010]

$$\langle \mathcal{O}^{M=0}(x)\mathcal{O}^{M=0}(y)\rangle = 0\,,$$

$$\langle \mathcal{O}^{\log}(x)\mathcal{O}^{\mathrm{M}=0}(y)\rangle = \frac{b_L}{|x-y|^{2\Delta}}\,,$$

$$\langle \mathcal{O}^{\log}(x)\mathcal{O}^{\log}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}\left(-2b_L\log|x-y| + \lambda\right)\,.$$

where b_L is the 'new anomaly'

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- Symmetries are realized geometrically on the gravitational side by the Lifshitz metric [Kachru, Liu, Mulligan; 2008]

$$ds^2_{\mathrm{Lif_{d+1}}} = L^2 \left(\frac{1}{r^{2z}} dt^2 + \frac{1}{r^2} dr^2 + \frac{1}{r^2} dx^a dx_a \right) .$$

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$$(\Box - m_1^2)(\Box - m_2^2)\phi = 0$$

- Taken in a Lifshitz background, we find logarithmic modes at the critical point $m_1^2 = m_2^2$

$$\phi^{\log}(t, \mathbf{x}) = \frac{1}{2\Delta - (d + z - 1)} \log(r)\, \phi^{\mathrm{s}}(t, \mathbf{x})$$

Non-relativistic Logarithmic CFT

By calculating correlation functions on the boundary of the Lifshitz spacetime we argue that there is an non-relativistic scaling version of an Logarithmic Conformal Field Theory, defined by

[Bergshoeff, de Haan, WM, Rosseel '11]

$$\langle \mathcal{O}^{\rm s}(t_1, \mathbf{x}_1)\mathcal{O}^{\rm s}(t_2, \mathbf{x}_2)\rangle = 0\,,$$
$$\langle \mathcal{O}^{\rm log}(t_1, \mathbf{x}_1)\mathcal{O}^{\rm s}(t_2, \mathbf{x}_2)\rangle = \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^{2\Delta}} f(\chi)\,,$$
$$\langle \mathcal{O}^{\rm log}(t_1, \mathbf{x}_1)\mathcal{O}^{\rm log}(t_2, \mathbf{x}_2)\rangle = \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^{2\Delta}} \left(-g(\chi)\log|\mathbf{x}_1 - \mathbf{x}_2| + \lambda\right)\,,$$

with $f(\chi), g(\chi)$ functions of the scale invariant variable $\chi = \frac{\mathbf{x}^z}{t}$

Conclusions and Outlook

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- We proposed a non-relativistic LCFT, purely through holographic reasoning.
- Non-relativistic LCFT correlation functions agree with what one would expect from scaling arguments.
- We worked with a scalar model, which shares a lot of features of massive (critical) gravity.
- Further steps would be to consider the full gravitational theory, since both TMG and NMG exhibit Lifshitz vacua.

The End

Thank you for your attention!
Questions?