# Non-commutative branes and model building in the type IIB matrix model

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*Based on:*
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# Introduction and Motivation

Grand scope in modern physics: Describe Nature at Planck scale.

Unification of all fundamental interactions.

Exciting proposal: Existence of extra dimensions
(supported by string / M theory).

If extra dimensions have anything to do with Physics:
determine their 4D, low-energy consequences
(compactification, dimensional reduction, 4D models).

$\rightsquigarrow$ try to make contact with low energy phenomenology.

Profound conceptual problems as well as low-energy physics questions may be addressed in the framework of matrix models.

Relations to string theory and non-commutative geometry

- Stringy matrix models: conjectured to be non-perturbative definitions of string / M theory...

  [Banks, Fischler, Shenker, Susskind '96]

  [Ishibashi, Kawai, Kitazawa, Tsuchiya '97]
- Non-commutative structures
  $\rightsquigarrow$ short-distance modification to space-time.
- Account for gauge theory - gravity.

Relevance for low-energy physics:

- Laboratories to study systems of branes, interactions, stability issues (analytically and numerically).

Are there backgrounds in matrix models which can account for low-energy physics?

or

Is realistic model building within the matrix models possible?

# Overview

The type IIB / IKKT Matrix Model

Intersecting non-commutative branes and chiral fermions

(Matrix) Model Building

Conclusions - Open questions

# IKKT Matrix Model

Proposed as a non-perturbative definition of type IIB superstring theory - Candidate for quantum theory of fundamental interactions.
[Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

Action: $S = -\frac{\Lambda^4}{g^2} Tr(\frac{1}{4}[X_a, X_b][X^a, X^b] + \frac{1}{2}\bar{\psi}\Gamma^a[X_a, \psi])$.

- $X_a, a = 0, \ldots, 9$ : ten Hermitian matrices $\in Mat(N; \mathbb{C})$.
- $\psi$: 16-component Majorana-Weyl spinor.

Symmetries:

- $U(N)$ gauge symmetry ($N \to \infty$).
- Global rotational and translational symmetry.
- $\mathcal{N} = 2$ supersymmetry.

# Some important properties...

- Obtained from dimensional reduction of $\mathcal{N} = 1$, $D = 10$ SYM to a point. Therefore there are no space-time dimensions to start with.
- No geometrical prerequisites.
  Not defined on any predetermined spacetime background.
- Spacetime and metric (should) *emerge* as solution of the model.
- Then non-abelian gauge fields - gravitons correspond to fluctuations around given solution.

# Solving the model

Varying the action w.r.t. $X_a$ and setting $\psi = 0$...

$\rightsquigarrow$ Equations of Motion: $[X_a, [X^a, X^b]] = 0.$

Split $X_a$ in two sets: $X^a = \begin{pmatrix} X^\mu \\ Y^i \end{pmatrix}$, $\mu = 0, 1, 2, 3, \quad \alpha = 1, ..., 6.$

Basic solution: $X^a = \begin{pmatrix} \bar{X}^\mu \\ 0 \end{pmatrix}$, with $[\bar{X}^\mu, \bar{X}^\nu] = i\theta^{\mu\nu}$

$\theta^{\mu\nu}$: constant antisymmetric tensor.

$\rightsquigarrow$ Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$.

$\hookrightarrow$ 4D Spacetime emerging as a solution of the model corresponding to a single, non-commutative, flat brane.

# Gauge theory and gravity

Fluctuations around the solution,

$$\begin{pmatrix} X^\mu \\ \phi^i \end{pmatrix} = \begin{pmatrix} \bar{X}^\mu \\ 0 \end{pmatrix} + \begin{pmatrix} A^\mu \\ \phi^i \end{pmatrix}$$

give rise to an abelian gauge theory coupled to scalar fields.

The above solution may be easily generalized to include non-abelian gauge fields: $\begin{pmatrix} \bar{X}^\mu \\ 0 \end{pmatrix} \otimes \mathbb{1}_n$.

$\rightsquigarrow$ $n$ coincident branes $\rightsquigarrow$ $U(n)$ gauge theory.

Deformations of the basic brane solution
$\rightsquigarrow$ more general curved submanifolds $\mathcal{M}^4_\theta \subset \mathbb{R}^{10}$ $\rightsquigarrow$ gravity.
[Steinacker '07]

# Other solutions

- Compact solutions (such as fuzzy tori or spheres) may be obtained upon appropriate deformations of the model.
  [Myers '99][Iso, Kimura, Tanaka, Wakatsuki '01][Kimura '01][Kitazawa '02]

  e.g. the addition of a Chern-Simons term $\propto \epsilon_{ijk} Tr(X^i X^j X^k)$ allows for fuzzy sphere solution.

  (For such solutions without additional terms see talk by H. Steinacker)

- Also, there exist non-compact solutions associated to Lie-type non-commutativity
  (based on nilpotent and solvable Lie algebras).
  [A.C. 1108.1107 [hep-th]]

# Multiple brane backgrounds

Consider higher-dimensional non-commutative branes.

$2n$-dimensional quantum plane solutions: $[X^a, X^b] = i\Theta^{ab}$.

$\rightsquigarrow$ "$D(2n-1)$-brane".

Such solutions can be combined via block matrices:

$$X^a = \begin{pmatrix} X^a_{(1)} & 0 & \dots & 0 \\ 0 & X^a_{(2)} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & X^a_{(k)} \end{pmatrix}.$$

$\rightsquigarrow$ each block may describe a stack of branes ($X^a_{(i)} \to X^a_{(i)} \otimes \mathbb{1}_{n_i}$)
$\rightsquigarrow$ $k$ stacks of (intersecting) non-commutative branes.

Assume a common $\mathbb{R}^4_{0123}$ (space-time-filling branes), i.e.

$$X^\mu = \begin{pmatrix} X^\mu_{(1)} & 0 & \dots & 0 \\ 0 & X^\mu_{(2)} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & X^\mu_{(k)} \end{pmatrix} = \begin{pmatrix} \bar{X}^\mu & 0 & \dots & 0 \\ 0 & \bar{X}^\mu & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \bar{X}^\mu \end{pmatrix},$$

$$Y^i = \begin{pmatrix} Y^i_{(1)} & 0 & \dots & 0 \\ 0 & Y^i_{(2)} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & Y^i_{(k)} \end{pmatrix},$$

where $X^a = (X^\mu, Y^i), i = 1, \dots, 6$.

Q: Model building based on such backgrounds?

# Fermions and Chirality

Most important issue: Accommodate chiral fermions.
$\rightsquigarrow$ Study fermions in a background of two intersecting branes, in particular off-diagonal modes:

$$\Psi = \begin{pmatrix} 0 & \Psi_{(12)} \\ \Psi_{(21)} & 0 \end{pmatrix}.$$

Dirac operator:

$$\not{D}_6 \Psi_{(12)} = \Gamma_i [Y^i, \Psi_{(12)}] = \Gamma_i (Y^i_{(1)} \Psi_{(12)} - \Psi_{(12)} Y^i_{(2)}).$$

Mass operator:

$$(\not{D}_6)^2 \Psi_{(12)} = Y^i_{(1)} Y^i_{(1)} \Psi_{(12)} + \Psi_{(12)} Y^i_{(2)} Y^i_{(2)} + \Sigma^{(1)}_{ij} \Theta^{ij}_{(1)} \Psi_{(12)} - \Sigma^{(2)}_{ij} \Psi_{(12)} \Theta^{ij}_{(2)}.$$

Zero (massless) modes?

Explicit analysis shows:

- There is always a would-be chiral mode localized at the intersection of two branes.
  - But one has to be careful about the actual 4D chirality.

Case by case analysis:

- $\mathbb{R}^2 \cap \mathbb{R}^2$ (two $D5$ branes): $\subset \mathbb{R}^6$, two remaining directions $\rightsquigarrow$ chirality on the full model is spoiled.
  - Message: Need to saturate full "internal" $\mathbb{R}^6$.
    [A.C., Steinacker, Zoupanos '09][Aoki '10]
- $\mathbb{R}^2 \cap \mathbb{R}^4$ (one $D5$ and one $D7$ brane): works out fine.
- $\mathbb{R}^4 \cap \mathbb{R}^4$ (two $D7$ branes): the intersection is (generically) 6D $\rightsquigarrow$ need to compactify, e.g. $\mathbb{R}^4 \times T^2$ or $\mathbb{R}^4 \times S^2$.
  - Add flux on the compact space $\rightsquigarrow$ chirality in 4D.
  - $\#$ of zero-modes determined by the flux.

With these results at hand, try to realize realistic models.

# Towards realistic scenarios

Is model building within the IKKT matrix model possible?

Minimal requirements:

- Standard model gauge group (plus additional $U(1)$s).
- Chiral fermion spectrum.
- Correct hypercharge assignment.

Guarantee the above and then impose more...

Similar to model building in the context of type II orientifold vacua.

# Minimal model

Consider 4 branes $D_a, D_b, D_c, D_d$ with gauge group

$$\begin{aligned} G &= U(3)_C \times U(2)_L \times U(1)_c \times U(1)_d, \\ &= SU(3)_C \times SU(2)_L \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d. \end{aligned}$$

There is essentially one way to embed the branes in $\mathbb{R}^{10}$ such that the SM fermions are accommodated at their intersections:

| Table 1 | | |
|---|---|---|
| brane | gauge group | brane embedding |
| $D_a$ | $U(3)_C$ | D7 along $\mathbb{R}^4_{4567}$ |
| $D_b$ | $U(2)_L$ | D7 along $\mathbb{R}^4_{6789}$ |
| $D_c$ | $U(1)_c$ | D7 along $\mathbb{R}^4_{6789}$ |
| $D_d$ | $U(1)_d$ | D7 along $\mathbb{R}^4_{4589}$ |

All branes *must* be *D7* $\rightsquigarrow$ 6D intersections

$\rightsquigarrow$ Compactification and fluxes required.

The particle assignment is:

| Table 2 | | | |
|---|---|---|---|
| Intersection | Representation | Particle | flux |
| $D_a \cap D_b$ | $(\bar{3}, 2)(-1, 1, 0, 0)$ | $Q_L$ | $N'_\beta - N_\beta$ |
| $D_a \cap D_c$ | $(\bar{3}, 1)(-1, 0, 1, 0)$ | $d_R$ | $N''_\beta - N_\beta$ |
| $D_a \cap D_d$ | $(\bar{3}, 1)(-1, 0, 0, 1)$ | $u_R$ | $N'_\alpha - N_\alpha$ |
| $D_d \cap D_b$ | $(1, 2)(0, 1, 0, -1)$ | $l_L$ | $N_\gamma - N''_\gamma$ |
| $D_d \cap D_c$ | $(1, 1)(0, 0, 1, -1)$ | $e_R$ | $N'_\gamma - N''_\gamma$ |

Note:

- $D_b \cap D_c$ is not chiral $\rightsquigarrow$ *no* exotic leptons $(1, 2)(0, 1, -1, 0)$.

Matrix realization:

$$\Psi = \begin{pmatrix} 0_2 & 0 & l_L & Q_L \\ & 0 & e_R & d_R \\ & & 0 & u_R \\ & & & 0_3 \end{pmatrix}.$$

Note:

- The lower-diagonal blocks are related to the upper by the 10D Majorana condition $\rightsquigarrow$ antiparticles in the conjugate rep.

Hypercharge assignment:

$$Y = \begin{pmatrix} 0_2 & & \\ & -\sigma_3 & \\ & & -\frac{1}{3}\mathbb{1}_3 \end{pmatrix} = -\frac{1}{3}Q_a - Q_c + Q_d.$$

## Chirality and fluxes

All branes are $D7 \rightsquigarrow$ compactification is required.

Consider compactified intersections $D_i \cap D_j$ carrying flux $m_{ij}$.
$\rightsquigarrow |m_{ij}|$ zero-modes $\psi_{(ij)}$ with chirality $\mathrm{sign}(m_{ij})$.

Note: $\psi_{(ji)}$ see flux $m_{ji} = -m_{ij}$, and have opposite chirality.

For the standard model realization:
If, compactifying on $K$ (e.g. $S^2_N$ with quantization parameter $N$, i.e. $N \times N$ matrices),...

$$D_a = K_{45} \times K_{67}, \quad D_b = K'_{67} \times K_{89},$$
$$D_c = K''_{67} \times K'_{89}, \quad D_d = K'_{45} \times K''_{89}.$$

then...

$$D_a \cap D_b = K_{67} = D_a \cap D_c,$$
$$D_a \cap D_d = K_{45},$$
$$D_b \cap D_d = K_{89} = D_c \cap D_d.$$

$K$ and $K'$ may have different quantization parameters $N$ and $N'$.

$\rightsquigarrow$ e.g. $\psi_{(ab)}$ feels a flux $N_\beta - N'_\beta$, $\psi_{(ad)}$ feels a flux $N_\alpha - N'_\alpha$, etc.
$(\alpha \sim 45, \quad \beta \sim 67, \quad \gamma \sim 89)$

Then, choosing

$$N_\gamma - N''_\gamma = 3, \quad N'_\gamma - N''_\gamma = -3,$$
$$N''_\beta - N_\beta = -3, \quad N'_\beta - N_\beta = 3, \quad N'_\alpha - N_\alpha = -3,$$

gives the correct chiralities and generations of the standard model fermions.

This provides a realization of the SM spectrum in 4-dimensional non-commutative spacetime.

# Stability

Interactions between the branes $\rightsquigarrow$ 1-loop effective action.

Q: Are configurations like the one for the standard model stable?

Difficult to answer. But there are indications:

- The sign of the effective potential for the interaction of $D_a, D_b$ depends on the relative flux $\Theta_{(ab)} = \Theta_{(a)} - \Theta_{(b)}$.
- For $\text{rank}(\Theta_{ab}) \leq 4 \rightsquigarrow$ Attractive interaction
  Low-dimensional branes tend to form bound states.
- For $D7$ branes the interaction may be attractive or repulsive depending on the eigenvalues of the flux matrix.
  - There indeed exist flux configurations forming bound state!
  - Q: Why would this be a preferred vacuum?...

# Conclusions

Main messages:

- Within the IKKT matrix model, semi-realistic model building is indeed possible.
- The standard model gauge group and chiral spectrum with three generations can be realized.
- Stability issues can be addressed.

# Further prospects

- Impose more theoretical and phenomenological requirements.
- Study phenomenological consequences.
- Implement generalized Green-Schwarz mechanism for anomaly cancellation.
- Further / more detailed analysis of the 1-loop effective action.

- Explore more sophisticated compactifications.
  e.g. fuzzy manifolds of special holonomy, fuzzy twisted tori.
- Relation to string backgrounds with non-geometric fluxes (work in progress...).