On Stability and Transport of Cold Holographic Matter

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arXiv:1108.1798 [hep-th] M. Ammon, J. Erdmenger, SL, S. Muller, A. O'Bannon and J. Shock

Outline

- System of interest(finite density, zero temperature)
- Motivations to study the stability of the system
- Methods of stability study
- Some transport properties of the system
- Conclusion

System of interest

D3: dual to N=4 SYM(adjoint matter)

D7: dual to N=2 hypermultiplet(quenched fundamental matter)

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	$\times$	$\times$	$\times$	$\times$						
D7	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		

$AdS_5 \times S^5$ $$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}(dr^2 + r^2 d\Omega_5^2),$$

R-R five form $$F^{(5)} = \frac{4}{R^4}(r^3 dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dr) - 4R^4 d\Omega_5,$$

D7 wraps the S^3 of S^5

$SO(6)_R \rightarrow SO(4)\times U(1)_R \sim SU(2)_L \times SU(2)_R \times U(1)_R$

Flavor field content: two complex scalars ϕ^m and two Weyl fermions $\psi_\pm$

System of interest

Fundamental (flavor) fields have a $U(1)_B$ symmetry.

$$S_{D7} = -T_{D7}\int d^8\xi\sqrt{-\det(g_{ab}+(2\pi\alpha')F_{ab})} + \frac{(2\pi\alpha')^2}{2}T_{D7}\int P[C^{(4)}]\wedge F\wedge F,$$

$$y(\rho) = \frac{1}{6}c\mathcal{N}^{-1/3}\left(\frac{d^2}{(2\pi\alpha')^2}-c^2\right)^{-1/3} B\left(\frac{\mathcal{N}^2\rho^6}{\mathcal{N}^2\rho^6+\frac{d^2}{(2\pi\alpha')^2}-c^2};\frac{1}{6},\frac{1}{3}\right)$$

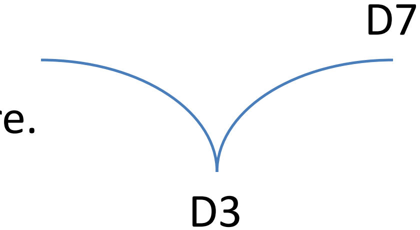
$$A_t(\rho) = \frac{1}{(2\pi\alpha')}\frac{1}{\varepsilon}y(\rho),$$

$$c = \gamma\mathcal{N}(2\pi\alpha')^3\left(\mu^2-M^2\right)M,$$

$$d = \gamma\mathcal{N}(2\pi\alpha')^4\left(\mu^2-M^2\right)\mu,$$

$$\mathcal{N} = T_{D7}2\pi^2 = \frac{\lambda N_c}{(2\pi)^4}$$

M: quark mass
μ: chemical potential
μ>M black hole embedding



Quenched flavor fields at finite baryonic density, zero temperature.
0709.0570 A. Karch and A. O'Bannon

Why the system is interesting?

- Finite entropy density at zero temperature. $\lim_{T\to 0} s = \frac{1}{2}\sqrt{\lambda}\langle J^t \rangle$
- Large Nc QCD, chiral density wave develops at high density. $\langle \overline{\psi}(x)\psi(y)\rangle = e^{i\mathbf{P}\cdot(\mathbf{x}+\mathbf{y})}\int d^4q\, e^{-iq(x-y)} f(q)$
- CS term induced spatially modulated phase in other holographic system. (Einstein) Maxwell-CS, BF bound
- Charged scalar at finite chemical potential susceptible to BEC.

Whether an instability from any one of the above is realized in our system?

Fluctuations of the D7 brane fields

$$S_{D7} = -T_{D7}\int d^8\xi\sqrt{-\det(g_{ab}+(2\pi\alpha')F_{ab})} + \frac{(2\pi\alpha')^2}{2}T_{D7}\int P[C^{(4)}]\wedge F\wedge F,$$

Embedding: $y(\rho)+(2\pi\alpha')\chi, \qquad \phi=(2\pi\alpha')\varphi$

Gauge fields: $A_b=\delta_b^t A_t+\delta A_b$

decoupled fluctuations: $\varphi=\Phi(\rho)e^{i\omega t-ikx_3}\mathcal{Y}^m$ similarly for δA_1 and δA_2.

coupled fluctuations:

$$\chi=\Phi(\rho)e^{i\omega t-ikx_3}\mathcal{Y}^m,\ \delta A_t=\varepsilon\chi,\ \delta A_3=-\frac{\omega}{k}\varepsilon\chi,\ \delta A_i=0, \qquad \varepsilon=\frac{M}{\mu}$$

$$\chi=\delta A_t=\delta A_3=0,\ \delta A_i=\Phi^{\pm}(\rho)e^{i\omega t-ikx_3}\mathcal{Y}^{m,\pm},$$

$$\chi=\delta A_t=0,\ \delta A_3=\Phi(\rho)e^{i\omega t-ikx_3}\mathcal{Y}^m,\ \delta A_i=\frac{-ik\eta^{xx}}{m(m+2)\eta^{S3}}\Phi(\rho)e^{i\omega t-ikx_3}\nabla_i\mathcal{Y}^m,$$

m: the eigenvalue of the S^3 spherical harmonics

i: index of coordinates on S^3

EOMs of the fluctuations

$$\partial_\rho(\sqrt{-\eta}\eta^{\rho\rho}\eta^{xx}\partial_\rho\Phi) - \sqrt{-\eta}\left(\omega^2\eta^{tt}\eta^{xx} + k^2(\eta^{xx})^2 + m(m+2)\eta^{S3}\eta^{xx}\right)\Phi = 0,$$

$$\partial_\rho(\sqrt{-\eta}\eta^{S3}\eta^{\rho\rho}\partial_\rho\Phi^\pm) - \left(\sqrt{-\eta}\left(\eta^{S3}(\omega^2\eta^{tt} + k^2\eta^{xx}) + (\eta^{S3})^2(m+1)^2\right)\right.$$
$$\left.\pm 4(\rho^2 + y^2)(\rho + yy')(m+1)\sqrt{\tilde{g}}\right)\Phi^\pm = 0,$$

IR limits:

$$\Phi'' + \frac{\bar{\omega}^2}{\bar{\rho}^4}(1-\varepsilon^2)\Phi = 0$$
$$\Phi^{\pm\prime\prime} + \frac{2}{\bar{\rho}}\Phi^{\pm\prime} + \frac{\bar{\omega}^2}{\bar{\rho}^4}(1-\varepsilon^2)\Phi^\pm = 0,$$

massless scalars in AdS_2
BF bound not violated

Low frequency expansion

$$\Phi(\bar\rho) = \bar\rho\, e^{-\frac{i\Omega}{\bar\rho}}(1+\mathcal{O}(\bar\rho^5)) \approx \bar\rho - i\Omega$$

$$\Phi(\bar\rho) = (a_1 - i\Omega a_0)\Phi_S(\bar\rho) + (b_1 - i\Omega b_0)\Phi_V(\bar\rho)$$

0907.2694 Faulkner, Liu, McGreevy and Vegh

S: source
V: VEV

$$\Omega \equiv \bar\omega\sqrt{1-\varepsilon^2},$$

$$G(\Omega) = \frac{b_1 - i\Omega b_0}{a_1 - i\Omega a_0}$$

a's and b's depend on k, m, ε. Can be computed numerically without difficulty

Dispersion at Ω<<1: $\Omega = -ia_1 / a_0$

An instability would mean $\frac{a_1}{a_0} \to 0^-$ at certain value of k and m. NOT FOUND!

$$G^{\pm}(\Omega) = \frac{b_0^{\pm} - i\Omega b_{-1}^{\pm}}{a_0^{\pm} - i\Omega a_{-1}^{\pm}}.$$

For $\Phi^-(\bar\rho)$ we found that $\frac{a_0^-}{a_{-1}^-} \to D(\varepsilon)\bar k^2 > 0$ as $\bar k \to 0,$ Diffusion mode

Zig-Zag method for QNM search

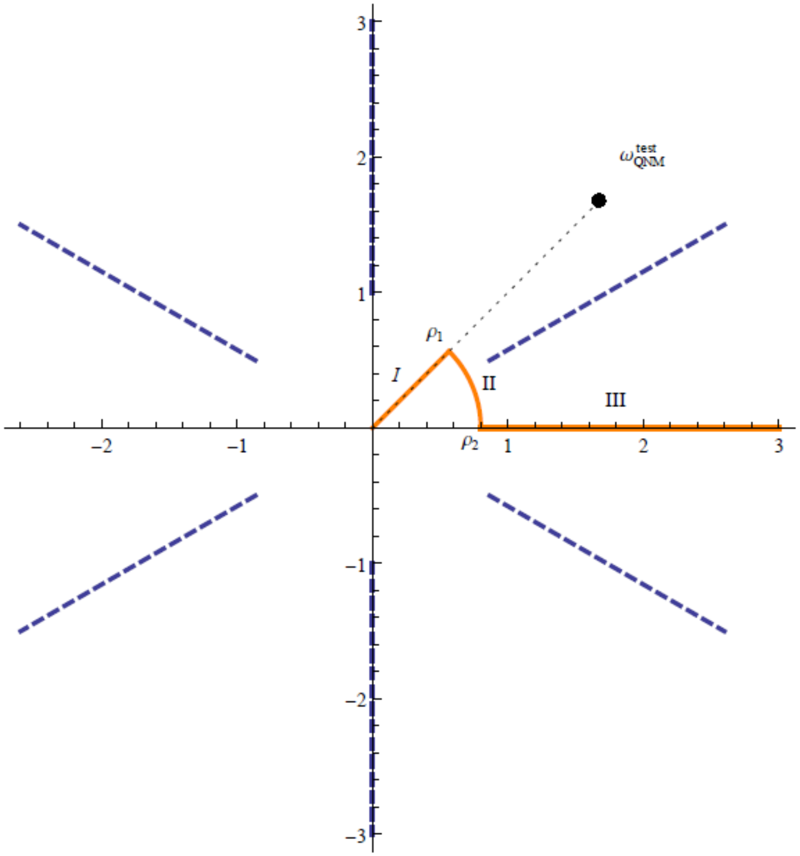
near the boundary $\Phi(\bar{\rho},\bar{\omega},\bar{k}) \approx A(\bar{\omega},\bar{k})\Phi_S(\bar{\rho}) + B(\bar{\omega},\bar{k})\Phi_V(\bar{\rho})$.

QNM: zeros of $|A(\bar{\omega},\bar{k})/B(\bar{\omega},\bar{k})|$, minimization in the complex ω plane.
Difficulty: Near the horizon, the outgoing wave is exponentially small while the ingoing wave is exponentially large at QNM. Therefore numerically hard to separate.

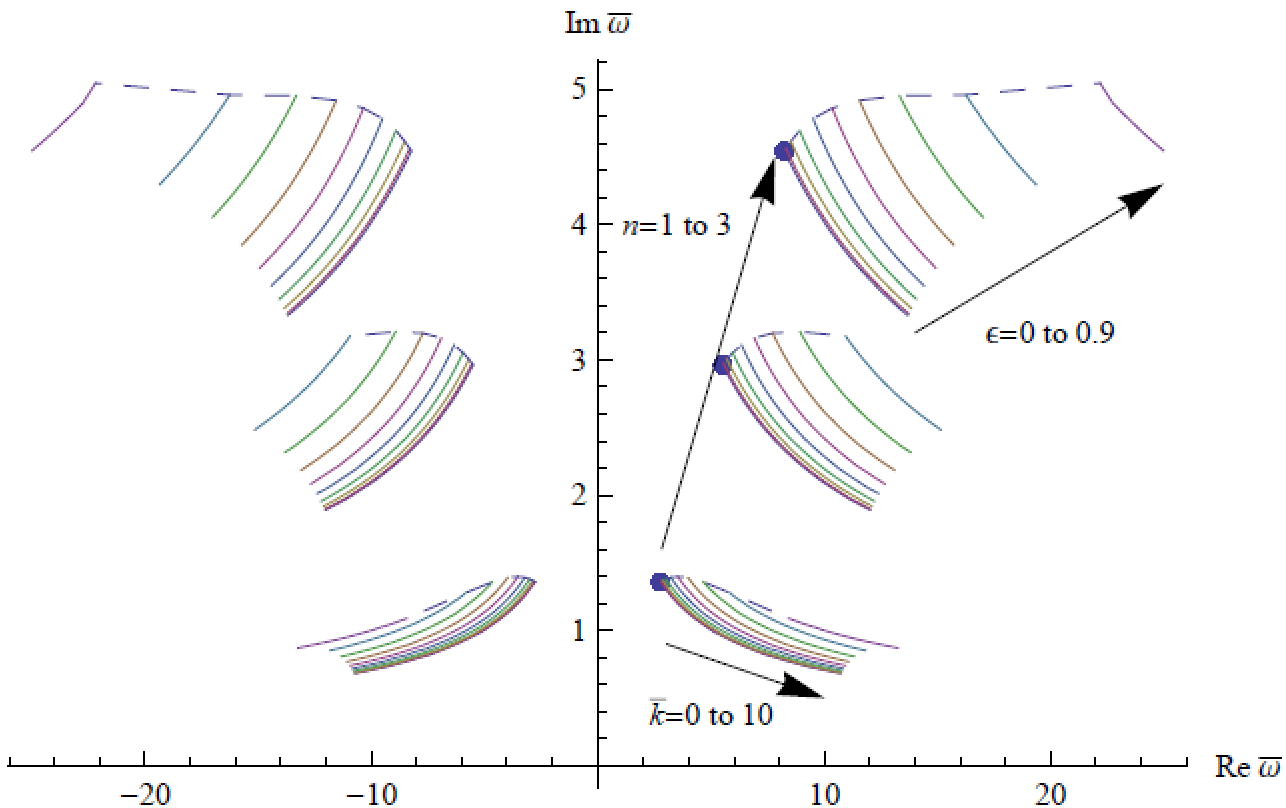
$$\Phi(\bar{\rho}) = \bar{\rho}\, e^{-\frac{i\Omega}{\bar{\rho}}}(1 + \mathcal{O}(\bar{\rho}^5)).$$

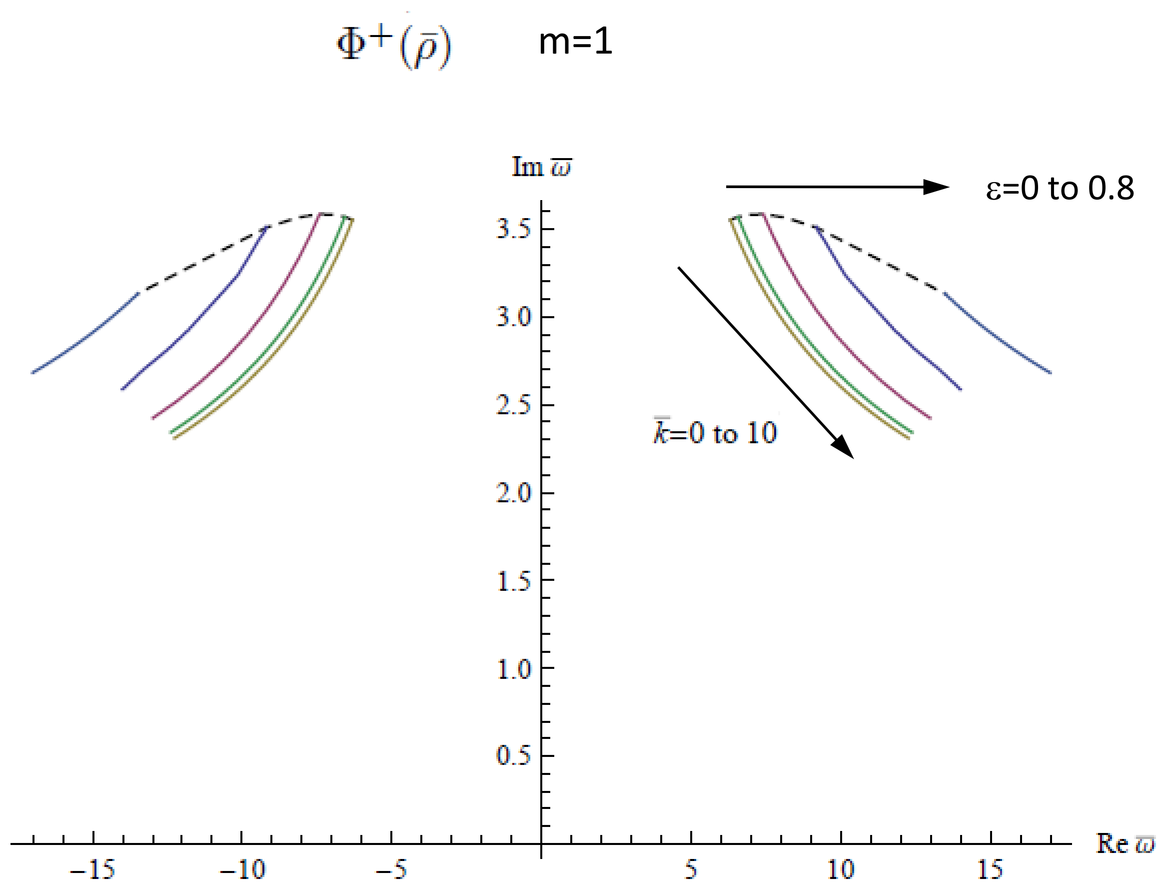
anti-Stokes line
$\arg\bar{\rho} = \arg\bar{\omega}^{\text{test}}_{\text{QNM}}$

$\Phi(\bar{\rho})$ analytic away from the singularities and branch cuts

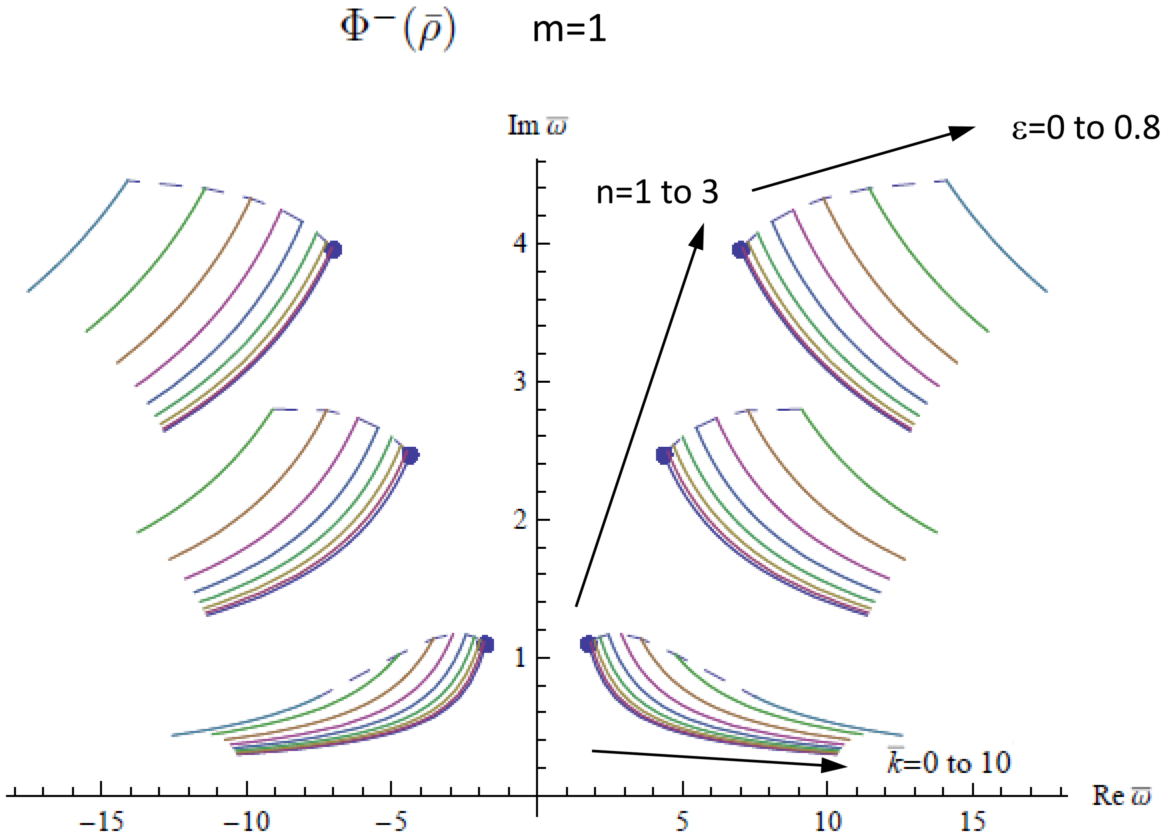


$\Phi(\bar{\rho})$ m=0



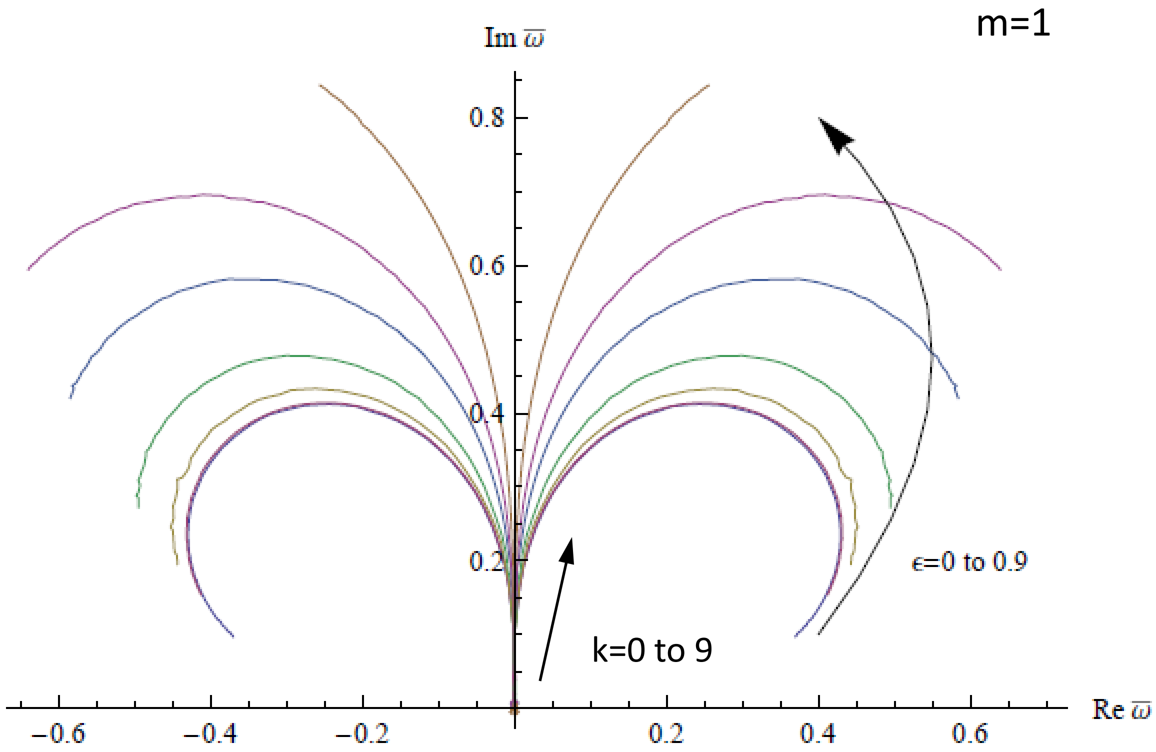


Φ+(ρ̄) m=1
Im ϖ
ε=0 to 0.8
k̄=0 to 10
Re ϖ
3.5
3.0
2.5
2.0
1.5
1.0
0.5
−15
−10
−5
5
10
15

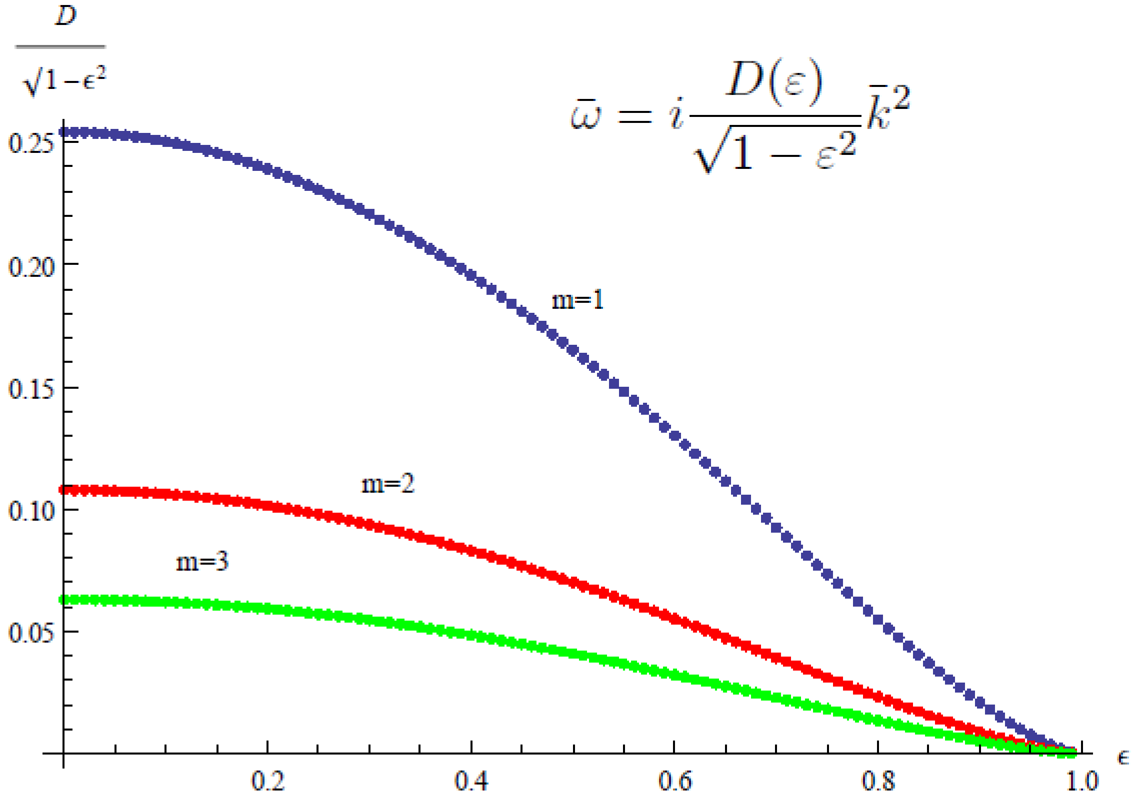


$\Phi^-(\bar{\rho})$ m=1
Im $\bar{\omega}$
ε=0 to 0.8
n=1 to 3
$\bar{k}$=0 to 10
Re $\bar{\omega}$
−15
−10
−5
5
10
15
1
2
3
4

Diffusion mode for Φ^-



Diffusion constants



$$\bar{\omega} = i\frac{D(\varepsilon)}{\sqrt{1-\varepsilon^2}}\bar{k}^2$$

$$\varepsilon = \frac{M}{\mu}$$

density effect

Interpretation of the diffusion

fluctuations $\chi = \delta A_t = \delta A_3 = 0,\ \delta A_i = \Phi^{\pm}(\rho) e^{i\omega t - ikx_3} \mathcal{Y}^{m,\pm},$

Dual operators transform as scalars under Lorentz and a vector under $SU(2)_R$.

At m=1, $\mathcal{O}^I = \bar{\phi}^m \sigma^I_{mn} \phi^n \quad I = 1, 2, 3$

hep-th/0304032 Kruczenski, Mateos, Myers and Winters

Diffusion of Noether charge associated with $SU(2)_R$ symmetry?

--R-Spin diffusion

Note standard baryonic charge diffusion, if present, should show up as fluctuations of Φ. NOT FOUND.

The baryonic charge diffusion does not survive the zero temperature limit while the R-spin diffusion does!

Summary&Outlook

- We have not found any instability in finite baryonic density, zero temperature system.
- We have developed a numerical technique, zig-zag method for QNM search.
- We have found an R-spin diffusion mode associated with the $SU(2)_R$ symmetry. The R-spin diffusion persists at zero temperature, in contrast to the baryonic charge diffusion.
- Fermions? Nonlinear instability? Backreaction?

Thank you!

Scaling of the variables

$$\bar{N} \equiv \mathcal{N}/c.$$

$$\bar{\rho}^6 \equiv \rho^6 \bar{N}^2 \left(\frac{1}{\varepsilon^2} - 1\right)^{-1}, \qquad \bar{y}^6 \equiv y^6 \bar{N}^2 \left(\frac{1}{\varepsilon^2} - 1\right)^{-1},$$

$$\bar{\omega}^2 \equiv \omega^2 \left(\bar{N}^2 / \left(\frac{1}{\varepsilon^2} - 1\right)\right)^{1/3}, \qquad \bar{k}^2 \equiv k^2 \left(\bar{N}^2 / \left(\frac{1}{\varepsilon^2} - 1\right)\right)^{1/3}$$

Master equations

$$\Phi'' + p_1(\bar{\rho})\Phi' + q_1(\bar{\rho}, \bar{\omega}, \bar{k}, m)\Phi = 0,$$
$$\Phi^{\pm\prime\prime} + p_2(\bar{\rho})\Phi^{\pm\prime} + q_2^{\pm}(\bar{\rho}, \bar{\omega}, \bar{k}, m)\Phi^{\pm} = 0,$$

$$p_1(\bar{\rho}) = \frac{3\bar{\rho}^5}{1+\bar{\rho}^6},$$
$$q_1(\bar{\rho}, \bar{\omega}, \bar{k}, m) = \frac{-m(m+2)\bar{\rho}^4}{1+\bar{\rho}^6} + \frac{\bar{\omega}^2(1+\bar{y}'^2)}{(\bar{\rho}^2+\bar{y}^2)^2} - \frac{\bar{k}^2\bar{\rho}^6}{(1+\bar{\rho}^6)(\bar{\rho}^2+\bar{y}^2)^2},$$
$$p_2(\bar{\rho}) = \frac{\bar{\rho}^2(2+5\bar{\rho}^6) + (-2+\bar{\rho}^6)\bar{y}^2 + 4\bar{\rho}(1+\bar{\rho}^6)\bar{y}\bar{y}'}{\bar{\rho}(1+\bar{\rho}^6)(\bar{\rho}^2+\bar{y}^2)},$$
$$q_2^{\pm}(\bar{\rho}, \bar{\omega}, \bar{k}, m) = -\frac{(m+1)^2\bar{\rho}^4}{1+\bar{\rho}^6} \mp 4(m+1)\sqrt{\frac{\bar{\rho}^6}{1+\bar{\rho}^6}}\frac{\bar{\rho}+\bar{y}\bar{y}'}{\bar{\rho}(\bar{\rho}^2+\bar{y}^2)}$$
$$+ \frac{\bar{\omega}^2(1+\bar{y}'^2)}{(\bar{\rho}^2+\bar{y}^2)^2} - \frac{\bar{k}^2\bar{\rho}^6}{(1+\bar{\rho}^6)(\bar{\rho}^2+\bar{y}^2)^2}.$$

Zero sound

Two coupled fluctuations:

$$f_1 = \delta A_t - \varepsilon\chi, \qquad f_2 = \omega\eta^{\rho t} y' \chi + \omega\eta^{tt}/\eta^{xx}\delta A_t - k\delta A_3.$$

$$\partial_\rho f_2 = \omega\partial_\rho(\eta^{tt}/\eta^{xx})(f_1).$$

$$(\varepsilon^2 - 1)\partial_\rho(\eta^{tt}/\eta^{xx})\sqrt{-\eta}\partial_\rho f_1 + \partial_a(\sqrt{-\eta}\eta_S^{ab}\eta^{xx}\partial_b f_1) + \omega\sqrt{-\eta}\eta^{xx2} f_2 = 0.$$

f_1 and f_2 not gauge invariant. Pure gauge solution should be considered.

$$\frac{\omega^2}{k^2} = \frac{1-\varepsilon^2}{3-\varepsilon^2} = \frac{\mu^2 - M^2}{3\mu^2 - M^2},$$

Karch-O'Bannon background

$$\langle \mathcal{O}_M \rangle = (2\pi\alpha')c$$

$$\langle J^t \rangle = d$$

$$\mathcal{O}_M = i\bar{\psi}\psi + q^\dagger(M+\Psi)q + \tilde{q}^\dagger(M+\Psi)\tilde{q}.$$