The relevance of different D-brane wrappings in holographic dimerization

Konstadinos Sfetsos

Department of Engineering Sciences, University of Patras, GREECE

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Based on:

- arXiv:1106.1200 [hep-th] and in progress
  (with N. Karaiskos and E. Tsatis).

Motivation

Holographic lattices and dimers

- Impurities with localized fermions.
- Lattices by breaking translation and rotational invariance.
- How dimers arise in a phase transition.
- What are the different D-brane wrappings one can use?
- What are their distinct characteristics?

Some notable references

- Flux-brane stablilization: [Bachas-Douglas-Schweigert 00], [Bachas-Petropoulos 00], [Camino-Paredes-Ramallo 01].
- AdS/CFT defects: [Karch-Randall 01],[DeWolfe-Freedman-Ooguri 01]
- Holographic Lattices/Dimers: [Kachru-Karch-Yaida 09], [Mueck 10].

Outline

Wrapping a D5-brane in D3-brane geometries

- Different D5-brane wrappings.
- Essential features and differences between them.

Holographic lattices

- Lattices with D5 and anti-D5-branes.
- Holographic dimers.

Concluding remarks

Wrapping D5-brane in D3-brane geometries

The theory is type-IIB with black D3-brane metric (also an F_5)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{R^2}d\vec{x}_{||}^2 + R^2 d\Omega_5^2 \ , \quad f(r) = \frac{r^4 - \mu^4}{R^2 r^2} \ .$$

The Hawking temperature $T = \dfrac{\mu}{\pi R^2}$.

- A D5-brane probe wraps t, r and a 4-dim submanifold of S^5.
- The non-trivial world-volume gauge field component F_{tr} stabilizes the azimuth θ to a discrete set of values.

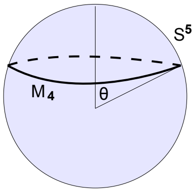


Figure: Azimuthal angle θ at its minimum.

Different D5-brane embeddings

- The embedding angle θ enters in the S^5-metric

$$d\Omega_5^2 = d\theta^2 + \cos^2\theta d\Omega_q^2 + \sin^2\theta d\Omega_{4-q}^2 \ , \quad q = 0, 1, 2$$

  and similarly by writing S^5 as an S^1 fibration over CP_2.
- The submanifolds M_4 are:

$$S^4 \ , \qquad S^2 \times S^2 \ , \qquad S^1 \times S^3 \ , \qquad CP^2 \ .$$

- The D5-brane can be interpreted as a bound state of n fundamental strings. The energy E_q depends on the embedding and obeys

$$E_q \leqslant n T_f \ , \quad \forall q \ .$$

- A single D5 inserts a half-BPS operator in antisymmetric rep. of $SU(N)$ with n boxes in the Y-T [Gomis-Passerini 06].

- Integrating out the corresponding degrees of freedom introduces fermions in the fundamental rep. of $SU(N)$ at a point impurity with occupation number n coupled to $\mathcal{N}=4$ SYM.
- The classical configurations give for the submanifolds of S^5:

q	M_4	Minima	E_q in units of NT_f
0	S^4	$\pi\nu = \theta - \sin\theta\cos\theta$	$\frac{2}{3\pi}\sin^3\theta$
2	$S^2 \times S^2$	$\theta = \pi\nu/2$	$\frac{1}{\pi}\sin\pi\nu$
1	$S^1 \times S^3$, CP_2	$\sin\theta = (4\nu/3)^{1/2}$	$\nu\sqrt{1-32\nu/27}$

where $\nu = \frac{n}{N}$.

- We have $0 \leqslant \nu \leqslant 1$ (for $q = 0, 2$) and $0 \leqslant \nu \leqslant \frac{27}{32}$(for $q = 1$).
- Little puzzle: in the last case there is an upper bound in $n \leqslant \frac{27}{32}N < N$, the number allowed by the rep.

- The free energy of a single D5-brane is

$$F_{\text{IIB}} = -E_q N \sqrt{\lambda}\, T \ , \quad \lambda = {}'\text{t Hooft coupling} \ .$$

- Different wrappings result to different free energies.

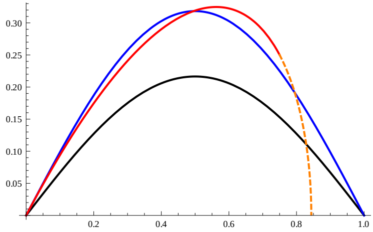


Figure: E_0 (black), E_1 (red) and E_2 (blue) as a function of ν. E_1 and E_2 intersect for $\nu \simeq 0.50$.

- Min. of $F(T)$: The less symmetric are favorable.
- For $\nu \gtrsim 0.50$ a change from $S^1 \times S^3$ or CP_2 to $S^2 \times S^2$.

Small digression: Generalizations

In a similar fashion one may consider:

- Wrapping of a D(8-p) brane on submanifolds of S^{8-p} in the background of a Dp-brane.
- Also in beta-deformed backgrounds.

Holographic lattices

Lattice with D5-branes

Taking V_3/a^3 D5-branes one may construct a cubic lattice of size a.

- The free energy for given D5-brane wrapping is

$$F_q = \underbrace{-\frac{\pi^2}{8} V_3 N^2 T^4}_{\text{black}-\text{D3}} \underbrace{-\frac{V_3}{a^3} E_q \sqrt{\lambda} N T}_{\text{probe}-\text{D5}} + \dots \ , \quad q = 0, 1, 2 \ .$$

- Probe approximation valid if

$$Ta \gg \left(\frac{\lambda}{N^2} \right)^{1/6} \ .$$

- The true free energy is that for F_1 which changes to F_2 when $\nu \simeq 0.50$. Discontinuity in derivative w.r.t. ν.

Lattice with pairs of D5- a anti-D5-branes

Let a D5 and an anti-D5 separated by Δx. Correspond to adding fermions in the fundamental and anti-fundamental of $SU(N)$.

- Could remain disconnected going down to the horizon with

$$F_{\rm disc} = 2F_{D5} \ .$$

- Could connect at some point $r_{\rm turn}$ in the bulk.

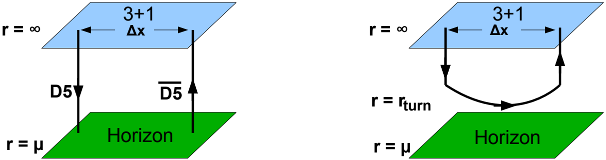


Figure: D5-anti-D5 pair: Disconnected (left) and connected (right).

- The disconnected (connected) configuration prevails for high (low) temperature.

The free energy of the connected configuration

Free energy: From the (renormalized) Euclidean classical action.

- The turning point r_{turn} relates to Δx and T as

$$\pi T \Delta x = \frac{1}{2} B(3/4, 1/2) \frac{\sqrt{z_0^4 - 1}}{z_0^3} \, {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{1}{z_0^4}\right) , \quad z_0 = \frac{r_{\text{turn}}}{\pi R^2 T} .$$

- The free energy is

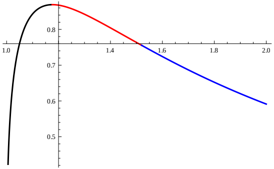
$$F_{\text{conn}} = -F_{\text{disc}} \tilde{F}_{\text{conn}} ,$$

where

$$\tilde{F}_{\text{conn}} = \frac{z_0}{4} B\left(-1/4, 1/2\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}; \frac{1}{z_0^4}\right) ,$$

measures the deviation from F_{disc}.

- For fixed T, two values of z_0; two branches for free energy.
- Similar to $q\bar{q}$ potential in AdS/CFT [Rey-Theisen-Yee 98], [Brandhuber-Itzhaki-Sonnenschein-Yankielowicz 98].
- Upper branch is expected to be unstable as in the $q\bar{q}$ potential [Avramis-KS-Siampos 06].
- For large z_0 (small T), the connected configuration prevails.
- For $z_0 < 1.52$ (for larger T) the disconnected configuration prevails.



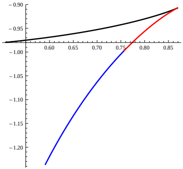


Figure: $\pi T \Delta x$ (left) vs. z_0 and $\tilde{F}_{\rm conn}$ (right) vs. $\pi T \Delta x$.

Holographic dimers

- Dimers are systems used in condensed matter to describe: superconductivity; transitions between conducting and insulating phases (Mott insulators).
- Easy to construct 2-dim lattices (stacks of them, to evade the Mermin-Wagner-Hohenberg-Coleman theorem).

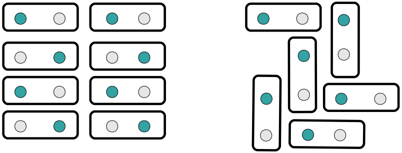


Figure: Dimerized configurations (extremes cases).

  - Left: Broken translation symmetry as in Valence Bond Solid (VBS) order.
  - Right: More random/generic configurations.
- Same free energy in this order in N, λ. Tunneling mechanisms through brane recombination (beyond next neighbor).

Concluding remarks

- Used D5-branes wrapped on 4-dim submanifolds of S^5 to construct holographic lattices that can dimerize below a critical T.
- Different D5-brane wrappings are classified. Lead to a discontinuity of the chemical potential

$$\frac{\partial F}{\partial \nu} \,.$$

  Characteristic of systems undergoing growth, i.e. in a crystallization process each time an additional layer on the crystal face is completed.
- The condition determining the amimuthal angle θ in the gravity computation should be extractable from the Green functions of the impurity fermions. How these encode the different wrappings?
- Probe D5-brane backreaction. Effect on the holographic lattice and dimerization.