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Extension of the Yang-Mills gauge theory

and

of the Poincare group

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Extension of the YM Theory

Gauge Group

Field Strength Tensors

Invariant Lagrangian

Particle Spectrum

Interactions

BCFW Recursion Relations

Amplitudes and Cross Sections

Topological Mass Generation

Supersymmetric Extension

arXiv:1006.3005
arXiv:1103.2456
arXiv:1004.4456
arXiv:1001.2808
arXiv:1007.3756
arXiv:0907.3553
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Space-time symmetry - the Poincare Group

$$[P^\mu,\ P^\nu] = 0,$$
$$[M^{\mu\nu},\ P^\lambda] = i(\eta^{\lambda\nu}\ P^\mu - \eta^{\lambda\mu}\ P^\nu),$$
$$[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),$$

10 generators = 4-translations, 3-rotations and 3-boosts

Let us add an infinite many new generators:

arXiv:1006.3005

$$L_a^{\lambda_1 \ldots \lambda_s}$$

$$[L_a^{\lambda_1 \ldots \lambda_i}, L_b^{\lambda_{i+1} \ldots \lambda_s}] = i f_{abc} L_c^{\lambda_1 \ldots \lambda_s}, \qquad s = 0, 1, 2 \ldots..$$

a,b,c =1,2,3 - are SU(2) indices, λ = 0,1,2,3 is a space-time index

Extension of the Poincare Algebra

Infinite many new generators have been added $L_a^{\lambda_1 \ldots \lambda_s}$

$$
\begin{aligned}
&[P^{\mu},\ P^{\nu}] = 0, \\
&[M^{\mu\nu},\ P^{\lambda}] = i(\eta^{\lambda\nu}\ P^{\mu} - \eta^{\lambda\mu}\ P^{\nu}), \\
&[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),
\end{aligned}
$$

$$
\begin{aligned}
&[P^{\mu},\ L_a^{\lambda_1 \ldots \lambda_s}] = 0, \\
&[M^{\mu\nu},\ L_a^{\lambda_1 \ldots \lambda_s}] = i(\eta^{\lambda_1\nu} L_a^{\mu\lambda_2 \ldots \lambda_s} - \eta^{\lambda_1\mu} L_a^{\nu\lambda_2 \ldots \lambda_s} + \ldots + \eta^{\lambda_s\nu} L_a^{\lambda_1 \ldots \lambda_{s-1}\mu} - \eta^{\lambda_s\mu} L_a^{\lambda_1 \ldots \lambda_{s-1}\nu}),
\end{aligned}
$$

$$
[L_a^{\lambda_1 \ldots \lambda_i}, L_b^{\lambda_{i+1} \ldots \lambda_s}] = i f_{abc} L_c^{\lambda_1 \ldots \lambda_s} \quad (\mu, \nu, \rho, \lambda = 0, 1, 2, 3; \qquad s = 0, 1, 2, \ldots) \tag{4}
$$

Extended Poincare algebra associated with a compact Lie group G

G.S. arXiv:1006.3005

Properties of the Algebra

The algebra is invariant with respect to the following gauge transformations:

$$L_a^{\lambda_1 \ldots \lambda_s} \to L_a^{\lambda_1 \ldots \lambda_s} + \textstyle\sum_1 P^{\lambda_1} L_a^{\lambda_2 \ldots \lambda_s} + \textstyle\sum_2 P^{\lambda_1} P^{\lambda_2} L_a^{\lambda_3 \ldots \lambda_s} + \ldots + P^{\lambda_1} \ldots P^{\lambda_s} L_a$$

$$M^{\mu\nu} \to M^{\mu\nu}, \qquad P^{\lambda} \to P^{\lambda},$$

The algebra $L_G(\mathcal{P})$ has a simple representation of the following form

$$P^{\mu} = k^{\mu},$$

$$M^{\mu\nu} = i(k^{\mu} \frac{\partial}{\partial k_{\nu}} - k^{\nu} \frac{\partial}{\partial k_{\mu}}) + i(e^{\mu} \frac{\partial}{\partial e_{\nu}} - e^{\nu} \frac{\partial}{\partial e_{\mu}}),$$

$$L_a^{\lambda_1 \ldots \lambda_s} = e^{\lambda_1} \ldots e^{\lambda_s} \otimes L_a,$$

Generalization of Yang-Mills field

$$\mathcal{A}_\mu(x, L) = \sum_{s=0}^{\infty} \frac{1}{s!} \; A^a_{\mu\lambda_1\ldots\lambda_s}(x) \; L_a^{\lambda_1\ldots\lambda_s}.$$

The gauge field $A^a_{\mu\lambda_1\ldots\lambda_s}$ carry indices $a, \lambda_1, ..., \lambda_s$ labeling the generators of *extended current algebra* $\mathcal{G}$ *associated with compact Lie group* G.

$$\boxed{A^a_{\mu\lambda_1\ldots\lambda_s}(x), \qquad s = 0, 1, 2, \ldots}$$

The non-Abelian gauge fields are defined as rank-(s+1) tensors

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A^a_μ	*four fields*	*4*
$A^a_{\mu\lambda_1} \neq A^a_{\lambda_1\mu}$	*sixteen fields*	*4x4=16*
$A^a_{\mu\lambda_1\lambda_2} = A^a_{\mu\lambda_2\lambda_1} \neq A^a_{\lambda_1\mu\lambda_2}$	*forty fields*	*4x10=40*

....

$A^a_{\mu\lambda_1\ldots\lambda_s}$	*# of fields*	$4\text{x}\dfrac{(s+3)!}{s!3!}$

$$
\begin{aligned}
\delta A^a_\mu &= (\delta^{ab}\partial_\mu + g f^{acb} A^c_\mu)\xi^b, \\
\delta A^a_{\mu\nu} &= (\delta^{ab}\partial_\mu + g f^{acb} A^c_\mu)\xi^b_\nu + g f^{acb} A^c_{\mu\nu}\xi^b, \\
\delta A^a_{\mu\nu\lambda} &= (\delta^{ab}\partial_\mu + g f^{acb} A^c_\mu)\xi^b_{\nu\lambda} + g f^{acb}(A^c_{\mu\nu}\xi^b_\lambda + A^c_{\mu\lambda}\xi^b_\nu + A^c_{\mu\nu\lambda}\xi^b), \\
\cdots\cdots\cdots &\cdot \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

The infinitesimal gauge parameters are totally symmetric rank-s tensors

$$\xi^b_{\lambda_1 \ldots \lambda_s}$$

The field strength tensors (generalized curvatures)

$$\mathcal{G}_{\mu\nu}(x,L) = \partial_\mu \mathcal{A}_\nu(x,L) - \partial_\nu \mathcal{A}_\mu(x,L) - ig[\mathcal{A}_\mu(x,L)\ \mathcal{A}_\nu(x,L)]$$

$$\boxed{\begin{array}{rcl} G^a_{\mu\nu} &=& \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}\ A^b_\mu\ A^c_\nu, \\ G^a_{\mu\nu,\lambda} &=& \partial_\mu A^a_{\nu\lambda} - \partial_\nu A^a_{\mu\lambda} + g f^{abc}(\ A^b_\mu\ A^c_{\nu\lambda} + A^b_{\mu\lambda}\ A^c_\nu\ ), \\ G^a_{\mu\nu,\lambda\rho} &=& \partial_\mu A^a_{\nu\lambda\rho} - \partial_\nu A^a_{\mu\lambda\rho} + g f^{abc}(\ A^b_\mu\ A^c_{\nu\lambda\rho} + A^b_{\mu\lambda}\ A^c_{\nu\rho} + A^b_{\mu\rho}\ A^c_{\nu\lambda} + A^b_{\mu\lambda\rho}\ A^c_\nu\ ) \\ \ldots\ldots & . & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}}$$

Homogeneous gauge transformation of field strength tensors

$$\mathcal{G}'_{\mu\nu}(x, L)) = U(\xi)\mathcal{G}_{\mu\nu}(x, L)U^{-1}(\xi)$$

$$\begin{aligned}
\delta G^a_{\mu\nu} &= g f^{abc} G^b_{\mu\nu} \xi^c \\
\delta G^a_{\mu\nu,\lambda} &= g f^{abc}(\ G^b_{\mu\nu,\lambda} \xi^c + G^b_{\mu\nu} \xi^c_\lambda \ ), \\
\delta G^a_{\mu\nu,\lambda\rho} &= g f^{abc}(\ G^b_{\mu\nu,\lambda\rho} \xi^c + G^b_{\mu\nu,\lambda} \xi^c_\rho + G^b_{\mu\nu,\rho} \xi^c_\lambda + G^b_{\mu\nu} \xi^c_{\lambda\rho} \ ) \\
\cdots\cdots \quad & . \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,
\end{aligned}$$

The Lagrangian for the rank-2 non-Abelian Tensor Gauge Field

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + ... =$$

$$\boxed{\begin{aligned} - &\quad \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \\ - &\quad \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda} \\ + &\quad \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\lambda,\nu} + \frac{1}{4} G^a_{\mu\nu,\nu} G^a_{\mu\lambda,\lambda} + \frac{1}{2} G^a_{\mu\nu} G^a_{\mu\lambda,\nu\lambda}. \end{aligned}}$$

$$\mathcal{L}_2 = \frac{1}{2} A^a_{\alpha\acute{\alpha}} H_{\alpha\acute{\alpha}\gamma\acute{\gamma}} A^a_{\gamma\acute{\gamma}} + \frac{1}{2!} \mathcal{V}^{abc}_{\alpha\acute{\alpha}\beta\gamma\acute{\gamma}} A^a_{\alpha\acute{\alpha}} A^b_{\beta} A^c_{\gamma\acute{\gamma}} + \frac{1}{2!2!} \mathcal{V}^{abcd}_{\alpha\beta\gamma\acute{\gamma}\delta\acute{\delta}} A^a_{\alpha} A^b_{\beta} A^c_{\gamma\acute{\gamma}} A^d_{\delta\acute{\delta}} + ...$$

Kinetic *Cubic* *Quartic*

Important - the coupling constants are dimensionless

The spectrum of the Yang-Mills theory

$$e_\gamma = a k_\gamma + c_1 e_\gamma^{(1)} + c_2 e_\gamma^{(2)},$$

$$e_\gamma^{(gauge)} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad e_\gamma^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad e_\gamma^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

massless gauge boson of helicity $\lambda = \pm 1$

The spectrum of the rank-2 tensor gauge field

$$e_{\gamma\acute{\gamma}} = \xi_{\acute{\gamma}} k_{\gamma} + \eta_{\gamma} k_{\acute{\gamma}} + c_1 e^{(1)}_{\gamma\acute{\gamma}} + c_2 e^{(2)}_{\gamma\acute{\gamma}} + c_3 e^{(A)}_{\gamma\acute{\gamma}},$$

$$e^{(1)}_{\gamma\acute{\gamma}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad e^{(2)}_{\gamma\acute{\gamma}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad e^{A}_{\gamma\acute{\gamma}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

massless modes of *helicity-two and helicity-zero,*

$$\lambda = \pm 2, 0, \quad \textit{charged gauge bosons}$$

$$\mathcal{L}_{s+1} + \frac{2s}{s+1}\mathcal{L}'_{s+1}\ |_{quadratic} = \frac{1}{2} A^a_{\alpha\lambda_1...\lambda_s} \mathcal{H}_{\alpha\lambda_1...\lambda_s\ \gamma\lambda_{s+1}...\lambda_{2s}} A^a_{\gamma\lambda_{s+1}...\lambda_{2s}}$$

$$\mathcal{H}_{\alpha\lambda_1...\lambda_s\ \gamma\lambda_{s+1}...\lambda_{2s}} =$$

$$\begin{aligned}
&+ \quad \frac{1}{s!}\ (\sum_p \eta_{\lambda_{i_1}\lambda_{i_2}}.......\eta_{\lambda_{i_{2s-1}}\lambda_{i_{2s}}})\ (-k^2\eta_{\alpha\gamma} + k_\alpha k_\gamma) \\
&+ \quad \frac{1}{(s+1)!}(\sum_P \eta_{\alpha\lambda_{i_1}}\ \eta_{\lambda_{i_2}\lambda_{i_3}}.......\eta_{\lambda_{i_{2s-2}}\lambda_{i_{2s-1}}}\ \eta_{\gamma\lambda_{i_{2s}}})\ k^2 \\
&- \quad \frac{1}{(s+1)!}\ (\sum_P \eta_{\rho\lambda_{i_1}}\ \eta_{\lambda_{i_2}\lambda_{i_3}}.......\eta_{\lambda_{i_{2s-2}}\lambda_{i_{2s-1}}}\ \eta_{\gamma\lambda_{i_{2s}}})\ k_\alpha k_\rho \\
&- \quad \frac{1}{(s+1)!}\ (\sum_P \eta_{\rho\lambda_{i_1}}\ \eta_{\lambda_{i_2}\lambda_{i_3}}.......\eta_{\lambda_{i_{2s-2}}\lambda_{i_{2s-1}}}\ \eta_{\alpha\lambda_{i_{2s}}})\ k_\rho k_\gamma \\
&+ \quad \frac{1}{(s+1)!}\ \eta_{\alpha\gamma}\ (\sum_P \eta_{\rho\lambda_{i_1}}\ \eta_{\lambda_{i_2}\lambda_{i_3}}.......\eta_{\lambda_{i_{2s-2}}\lambda_{i_{2s-1}}}\ \eta_{\sigma\lambda_{i_{2s}}})\ k_\rho k_\sigma
\end{aligned}$$

Summary of the Particle Spectrum

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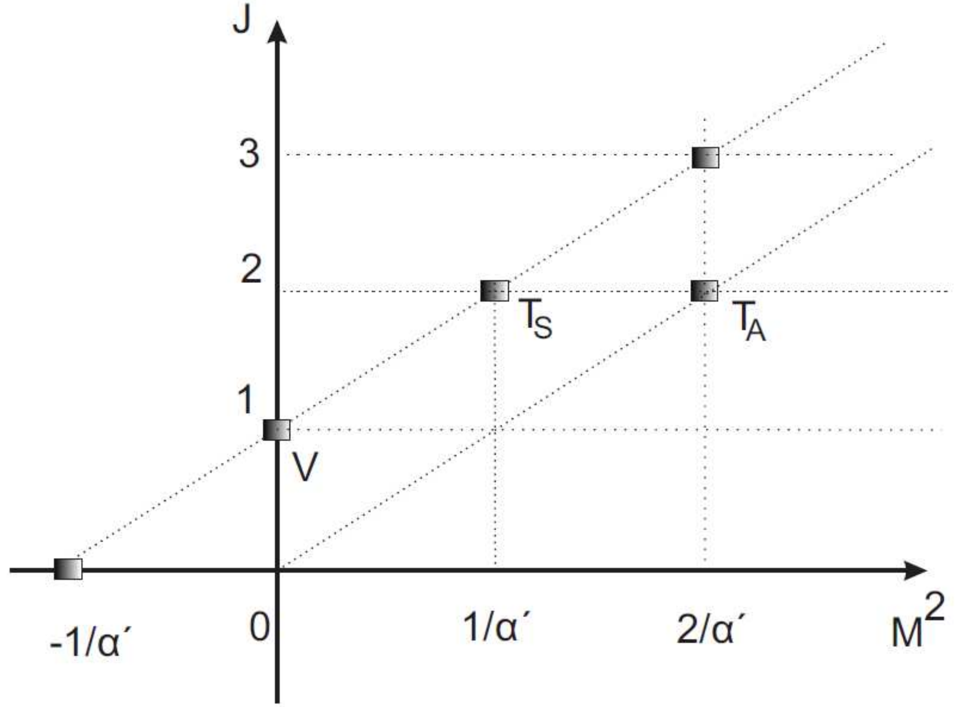
A^a_μ massless gauge boson of helicity $\lambda = \pm 1$

$A^a_{\mu\lambda_1} \neq A^a_{\lambda_1 \mu}$ $\lambda = \pm 2, 0$

$A^a_{\mu\lambda_1\lambda_2} = A^a_{\mu\lambda_2\lambda_1} \neq A^a_{\lambda_1\mu\lambda_2}$ $\lambda = \pm 3, \pm 1, \pm 1$

$$A^a_{\mu\lambda_1\ldots\lambda_s} \qquad \lambda = \pm(s+1), \quad \begin{matrix} \pm(s-1) \\ \pm(s-1) \end{matrix}, \quad \begin{matrix} \pm(s-3) \\ \pm(s-3) \end{matrix}, \quad ..$$

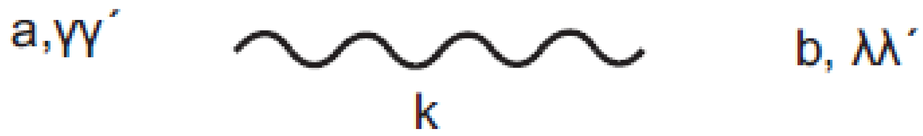


comparing with open strings spectrum

Feynman rules: Tensor gauge field propagator

$A^{a}_{\mu\lambda_1}$

$$\Delta^{ab}_{\gamma\gamma'\lambda\lambda'}(k)$$

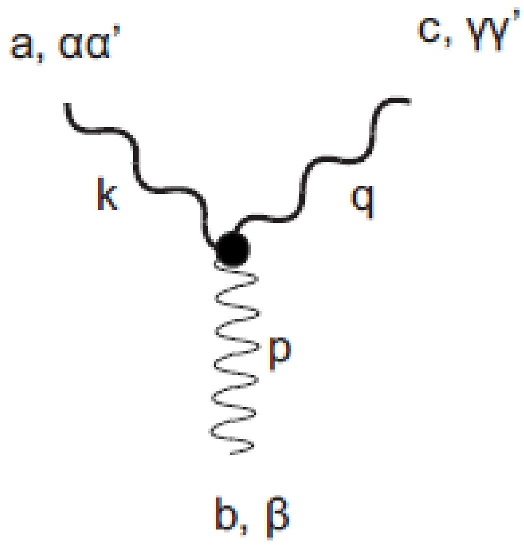


$$\boxed{\Delta^{ab}_{\gamma\acute{\gamma}\lambda\acute{\lambda}}(k) = -\frac{i}{k^2}\ (\ \frac{4}{3}\eta_{\gamma\lambda}\eta_{\acute{\gamma}\acute{\lambda}} + \frac{2}{3}\eta_{\gamma\acute{\lambda}}\eta_{\lambda\acute{\gamma}} - \eta_{\gamma\acute{\gamma}}\eta_{\lambda\acute{\lambda}}\ )\delta^{ab}.}$$

$$= -\frac{i}{k^2}\Big[(\eta^{\mu\lambda}\eta^{\nu\rho} + \eta^{\mu\rho}\eta^{\nu\lambda} - \eta^{\mu\nu}\eta^{\lambda\rho}) + \frac{1}{3}(\eta^{\mu\lambda}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\lambda})\Big]\delta^{ab}$$

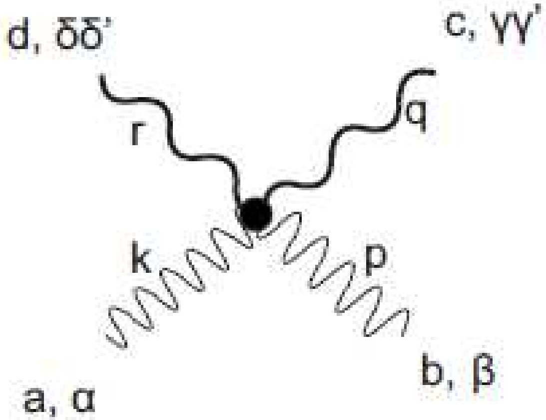
$$\lambda = \pm 2, 0.$$

The VTT vertex



$$
\begin{aligned}
\mathcal{V}^{abc}_{\alpha\acute{\alpha}\beta\gamma\acute{\gamma}}(k,p,q) \quad &= \quad (p-k)_{\gamma}(2\eta_{\alpha\beta}\eta_{\acute{\alpha}\acute{\gamma}} - \eta_{\alpha\acute{\gamma}}\eta_{\acute{\alpha}\beta} - \eta_{\alpha\acute{\alpha}}\eta_{\beta\acute{\gamma}}) \\
&+ \quad (k-q)_{\beta}(2\eta_{\alpha\gamma}\eta_{\acute{\alpha}\acute{\gamma}} - \eta_{\alpha\acute{\gamma}}\eta_{\acute{\alpha}\gamma} - \eta_{\alpha\acute{\alpha}}\eta_{\gamma\acute{\gamma}}) \\
&+ \quad (q-p)_{\alpha}(2\eta_{\beta\gamma}\eta_{\acute{\alpha}\acute{\gamma}} - \eta_{\acute{\alpha}\gamma}\eta_{\beta\acute{\gamma}} - \eta_{\acute{\alpha}\beta}\eta_{\gamma\acute{\gamma}}) \\
&- \quad (p-k)_{\acute{\alpha}}\eta_{\alpha\beta}\eta_{\gamma\acute{\gamma}} - (p-k)_{\acute{\gamma}}\eta_{\alpha\beta}\eta_{\acute{\alpha}\gamma} \\
&- \quad (k-q)_{\acute{\alpha}}\eta_{\alpha\gamma}\eta_{\beta\acute{\gamma}} - (k-q)_{\acute{\gamma}}\eta_{\alpha\gamma}\eta_{\acute{\alpha}\beta} \\
&- \quad (q-p)_{\acute{\alpha}}\eta_{\beta\gamma}\eta_{\alpha\acute{\gamma}} - (q-p)_{\acute{\gamma}}\eta_{\alpha\acute{\alpha}}\eta_{\beta\gamma}.
\end{aligned}
$$

The VVTT vertex



$$
\begin{aligned}
\mathcal{V}'^{\,abcd}_{\alpha\beta\gamma\acute{\gamma}\delta\acute{\delta}}(k,p,q,r) = 3g^2 f^{lac} f^{lbd}\big[& + \eta_{\alpha\beta}(\eta_{\gamma\acute{\delta}}\eta_{\acute{\gamma}\delta} + \eta_{\gamma\acute{\gamma}}\eta_{\delta\acute{\delta}}) \\
& - \eta_{\beta\gamma}(\eta_{\alpha\acute{\delta}}\eta_{\acute{\gamma}\delta} + \eta_{\alpha\acute{\gamma}}\eta_{\delta\acute{\delta}}) \\
& - \eta_{\alpha\delta}(\eta_{\beta\acute{\gamma}}\eta_{\gamma\acute{\delta}} + \eta_{\beta\acute{\delta}}\eta_{\gamma\acute{\gamma}}) \\
& + \eta_{\gamma\delta}(\eta_{\alpha\acute{\delta}}\eta_{\beta\acute{\gamma}} + \eta_{\alpha\acute{\gamma}}\eta_{\beta\acute{\delta}})\big] \\
3g^2 f^{lad} f^{lbc}\big[& + \eta_{\alpha\beta}(\eta_{\gamma\acute{\delta}}\eta_{\acute{\gamma}\delta} + \eta_{\gamma\acute{\gamma}}\eta_{\delta\acute{\delta}}) \\
& - \eta_{\alpha\gamma}(\eta_{\beta\acute{\delta}}\eta_{\acute{\gamma}\delta} + \eta_{\beta\acute{\gamma}}\eta_{\delta\acute{\delta}}) \\
& - \eta_{\beta\delta}(\eta_{\alpha\acute{\gamma}}\eta_{\gamma\acute{\delta}} + \eta_{\alpha\acute{\delta}}\eta_{\gamma\acute{\gamma}}) \\
& + \eta_{\gamma\delta}(\eta_{\alpha\acute{\gamma}}\eta_{\beta\acute{\delta}} + \eta_{\alpha\acute{\delta}}\eta_{\beta\acute{\gamma}})\big] \\
3g^2 f^{lab} f^{lcd}\big[& + \eta_{\alpha\gamma}(\eta_{\beta\acute{\gamma}}\eta_{\delta\acute{\delta}} + \eta_{\beta\acute{\delta}}\eta_{\delta\acute{\gamma}}) \\
& - \eta_{\beta\gamma}(\eta_{\alpha\acute{\gamma}}\eta_{\delta\acute{\delta}} + \eta_{\alpha\acute{\delta}}\eta_{\delta\acute{\gamma}}) \\
& - \eta_{\alpha\delta}(\eta_{\beta\acute{\delta}}\eta_{\gamma\acute{\gamma}} + \eta_{\beta\acute{\gamma}}\eta_{\gamma\acute{\delta}}) \\
& + \eta_{\beta\delta}(\eta_{\alpha\acute{\delta}}\eta_{\gamma\acute{\gamma}} + \eta_{\alpha\acute{\gamma}}\eta_{\gamma\acute{\delta}})\big].
\end{aligned}
$$

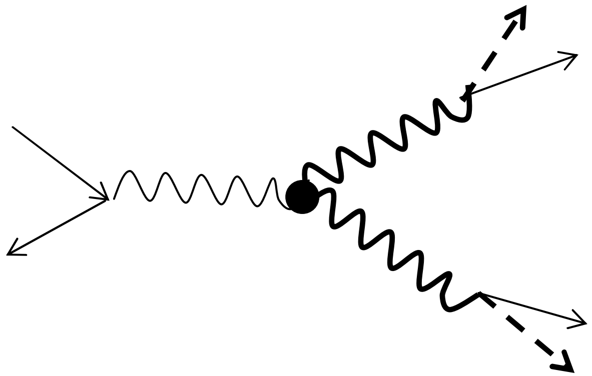
New Tensor Gauge Boson T can be creation in the channel

$$e^+ + e^- \to Z \to T^+ + T^-$$

$$T \to \nu + \tilde{e}$$

$$T \to \tilde{\nu} + e$$

S – parity conservation



standard leptons s=1/2

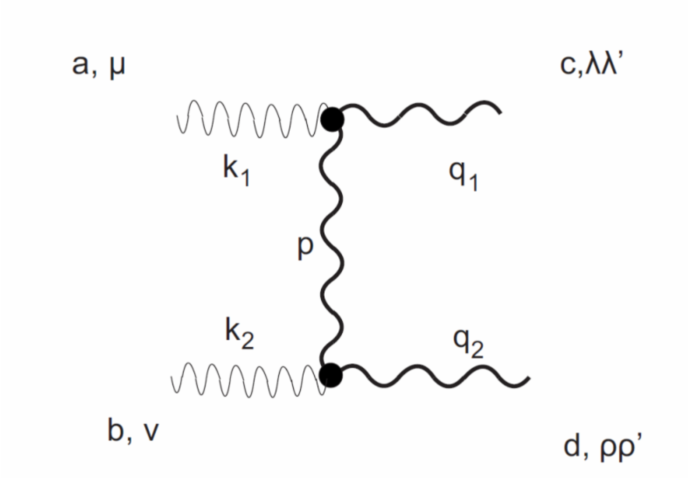
vector gauge boson

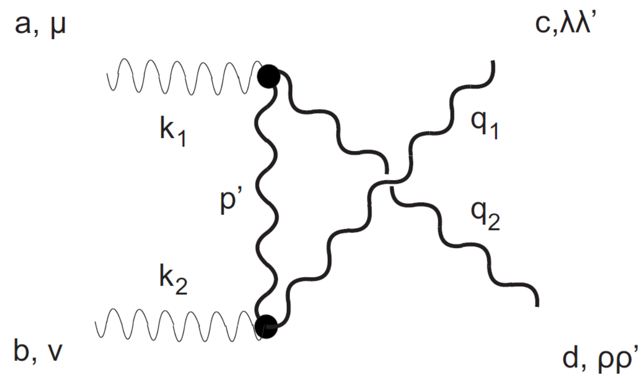
tensor leptons

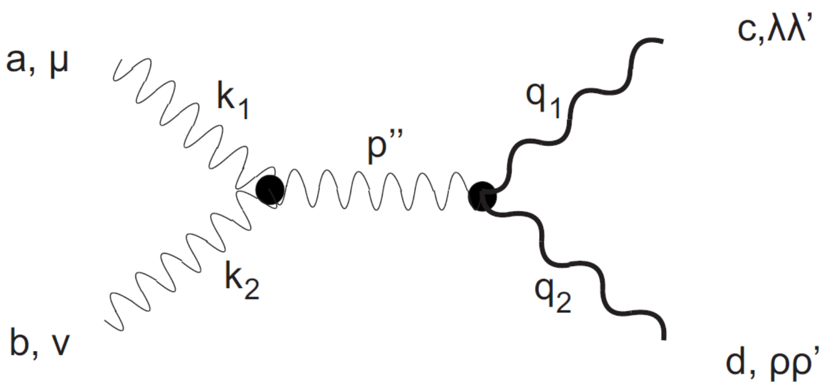
tensor boson

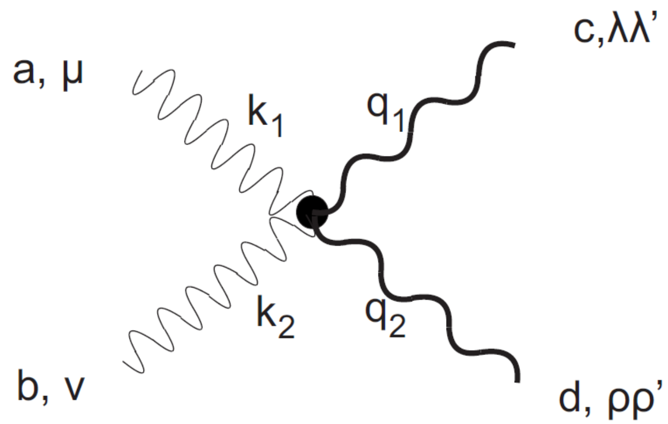
Fusion of gluons into pair of Tensor bosons

$$gg \rightarrow TT$$









$$d\sigma_{+-\rightarrow +2-2} = \frac{\alpha^2}{s}\,\frac{C_2^2(G)}{d(G)}\,\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^2\left(\frac{3+\cos^2\theta}{\sin^4\theta}\right)(1-\cos\theta)^4\,d\Omega,$$

$$d\sigma_{+-\rightarrow -2+2} = \frac{\alpha^2}{s}\,\frac{C_2^2(G)}{d(G)}\,\left(\frac{1+\cos\theta}{1-\cos\theta}\right)^2\left(\frac{3+\cos^2\theta}{\sin^4\theta}\right)(1+\cos\theta)^4\,d\Omega,$$

Comparing the above cross sections with the corresponding cross sections for the gluons one can see the factorization

$$d\sigma_{+2} = \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^2 d\sigma_{+1}, \qquad d\sigma_{-2} = \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^2 d\sigma_{-1},$$

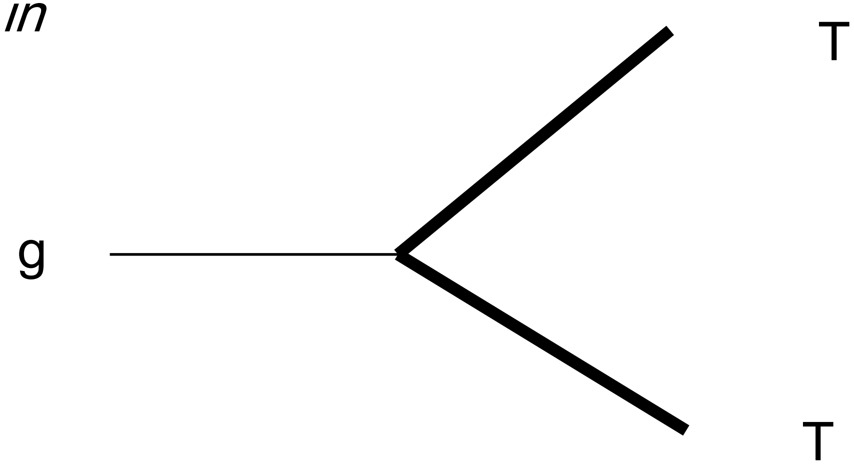
with form-factors

$$\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^2 \qquad \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^2$$

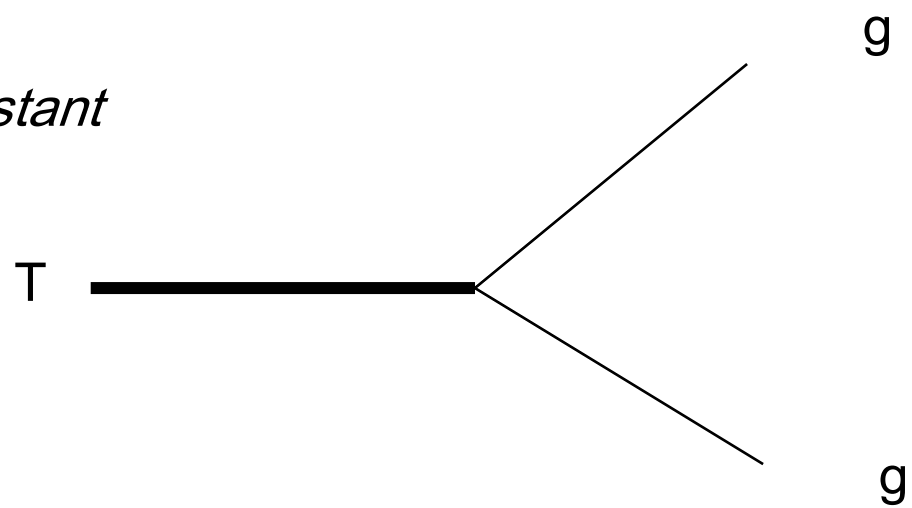
arXiv:1007.3756

Comparing Field and String Theory vertices

In field theory the vertex is gTT
and has dimensionless coupling constant ,
there are no dimensional Tgg vertices in
field theory



In string theory the vertex is Tgg
and has dimensionfull coupling constant



The ggT vector has the following momentum dependence

$$F^{\alpha\beta\gamma\gamma'} \sim \alpha' k^2 + \alpha'^2 k^4$$

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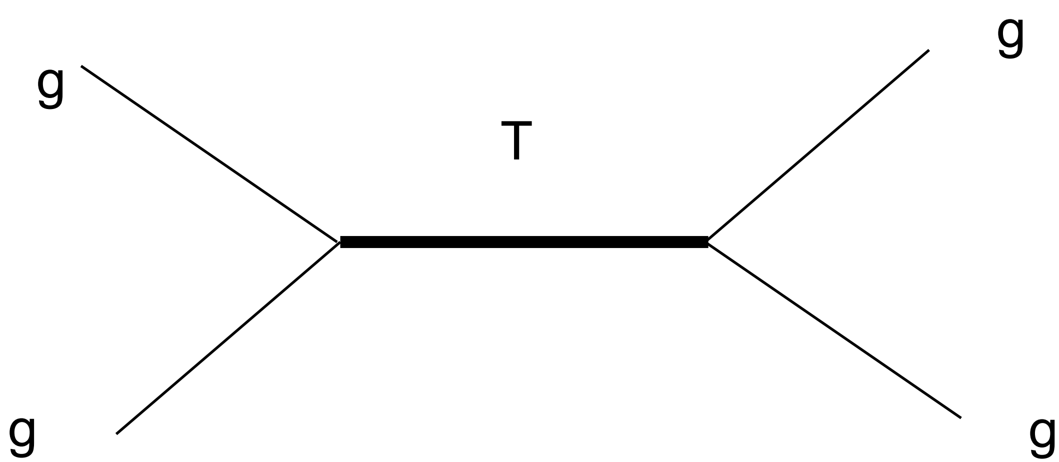
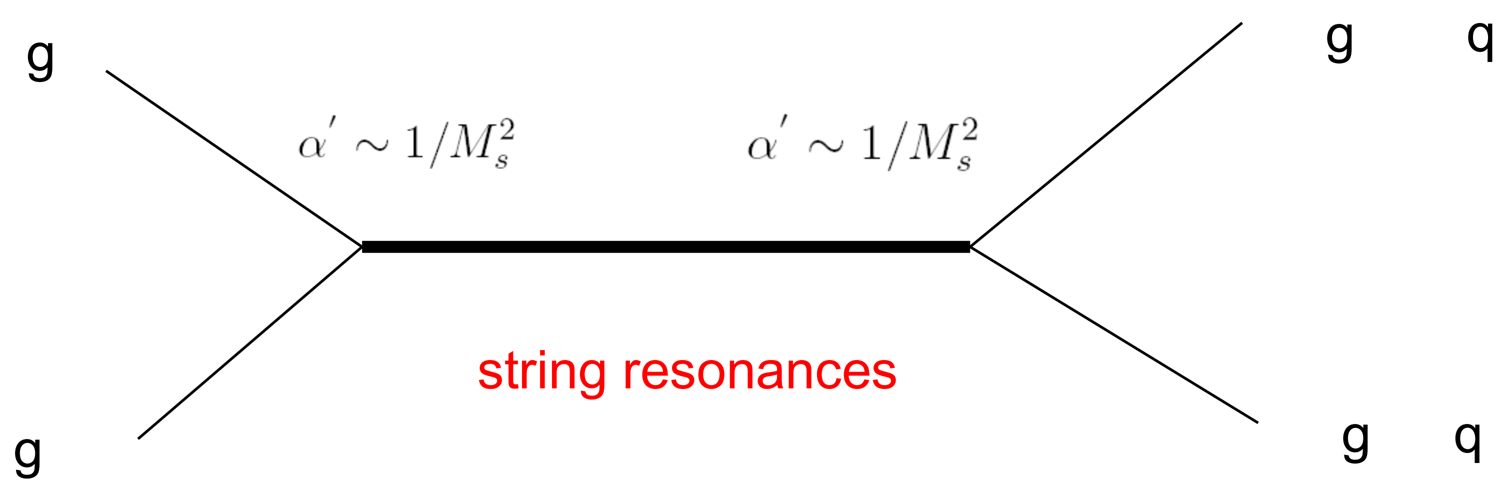
H. Goldberg,
D. Lust,
O. Schlotterer
S. Stieberger,
T. Taylor,
S. Nawata

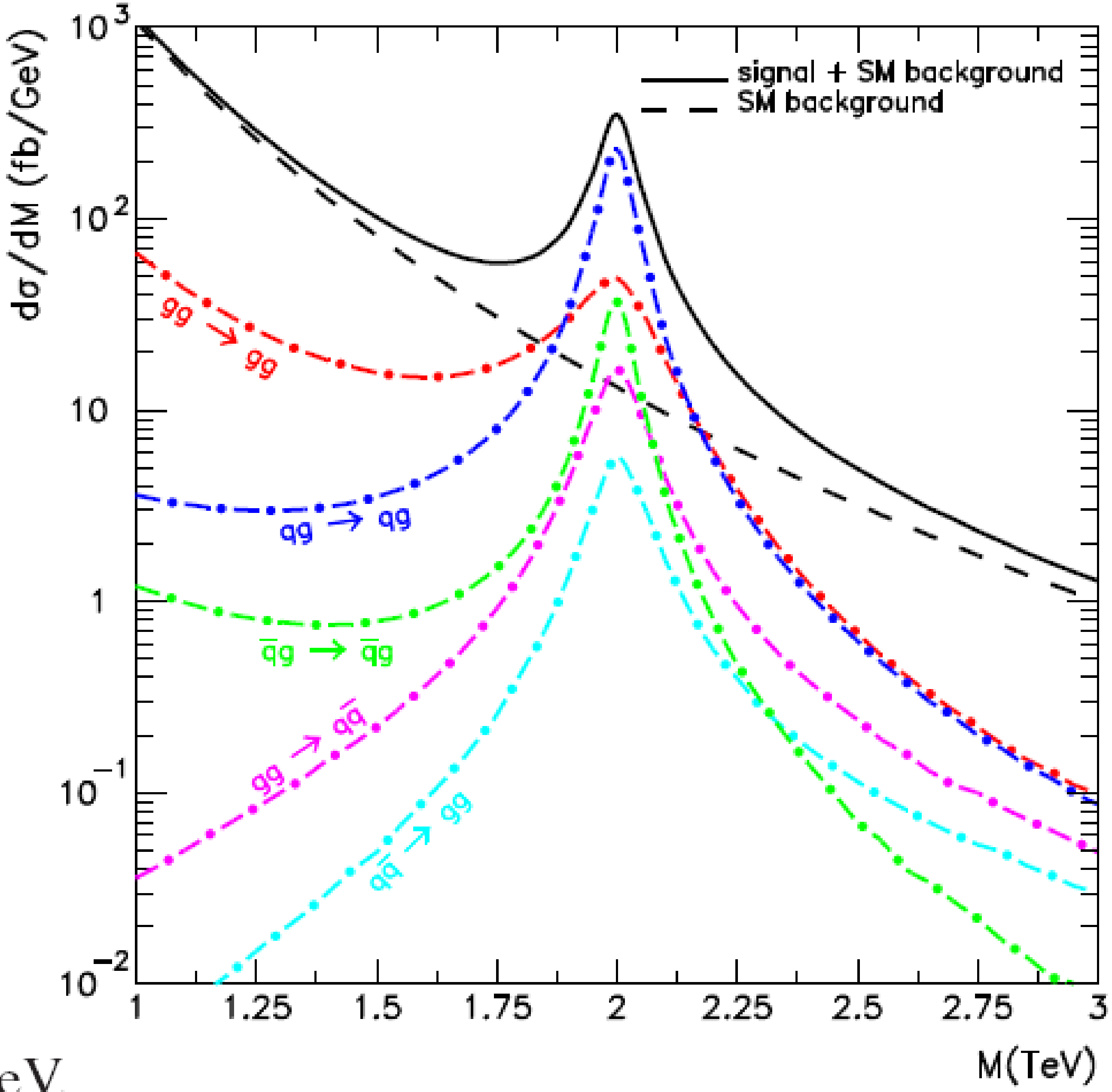
which has no linear in momenta terms and therefore has no dimensionless coupling constants



$$\alpha'^2 \mathrm{Tr}\, F^4$$

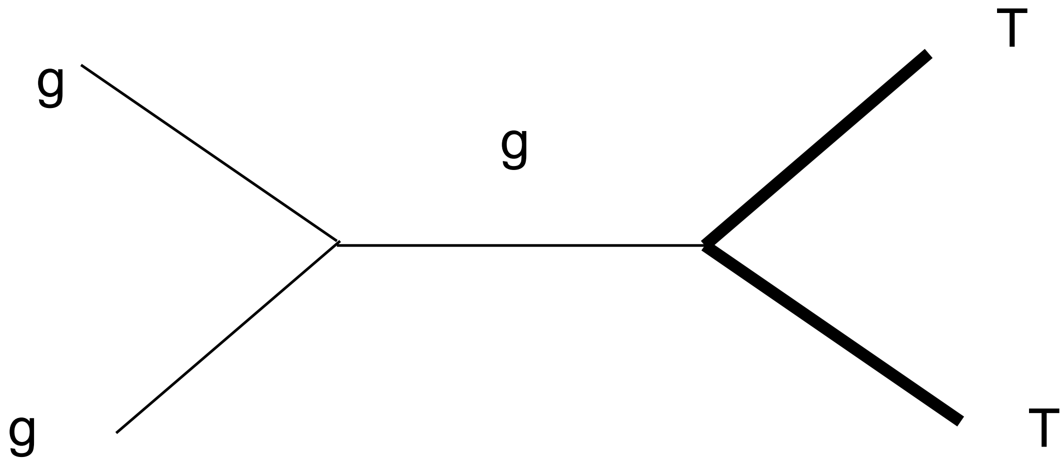


g
g
$\alpha' \sim 1/M_s^2$
$\alpha' \sim 1/M_s^2$
string resonances
g q
g q


signal + SM background
SM background
dσ/dM (fb/GeV)
M(TeV)
gg → gg
qg → qg
q̄g → q̄g
gg → qq̄
qq̄ → gg
10^3
10^2
10
1
10^-1
10^-2
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3

$M_s = 2$ TeV,

$$gg \to TT$$



$$d\sigma_{+-\to+2-2} = \frac{\alpha^2}{s}\ \frac{C_2^2(G)}{d(G)}\ \ \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^2 \left(\frac{3+\cos^2\theta}{\sin^4\theta}\right)(1-\cos\theta)^4\ d\Omega,$$

$$d\sigma_{+-\to-2+2} = \frac{\alpha^2}{s}\ \frac{C_2^2(G)}{d(G)}\ \ \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^2 \left(\frac{3+\cos^2\theta}{\sin^4\theta}\right)(1+\cos\theta)^4\ d\Omega,$$

Comparing the above cross sections with the corresponding cross sections for the gluons one can see the factorization

$$d\sigma_{+2} = \ \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^2 d\sigma_{+1}, \qquad d\sigma_{-2} = \ \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^2 d\sigma_{-1},$$

with form-factors

$$\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^2 \qquad \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^2$$

arXiv:1007.3756

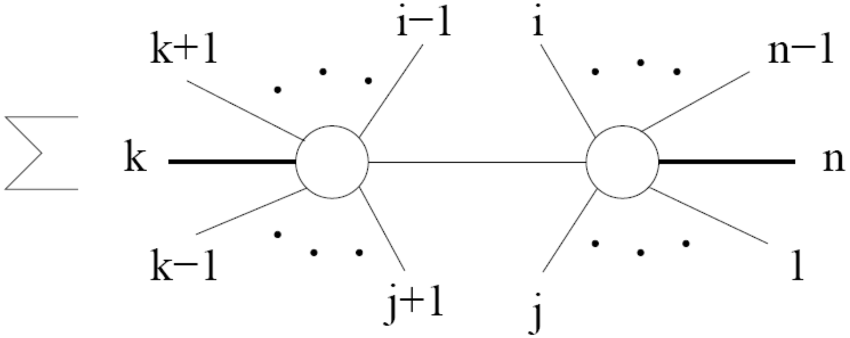
Calculating Amplitudes using spinors and BCFW recurrence relation

$$\left(\lambda_i^a \frac{\partial}{\partial \lambda_i^a} - \widetilde{\lambda}_i^{\dot a} \frac{\partial}{\partial \widetilde{\lambda}_i^{\dot a}}\right) \widehat{A}(\lambda_i, \widetilde{\lambda}_i, h_i) = -2h_i \widehat{A}(\lambda_i, \widetilde{\lambda}_i, h_i),$$

R.Britto, F.Cachazo, B.Feng and E.Witten,
L.Dixon, Z.Bern, D.Dunbar, D.Kosower and et al.
L.Dixon lectures at CERN

On-shell recurrence relation is

$$A_n = \sum_r A^h_{r+1} \frac{1}{P_r^2} A^{-h}_{n-r+1}$$



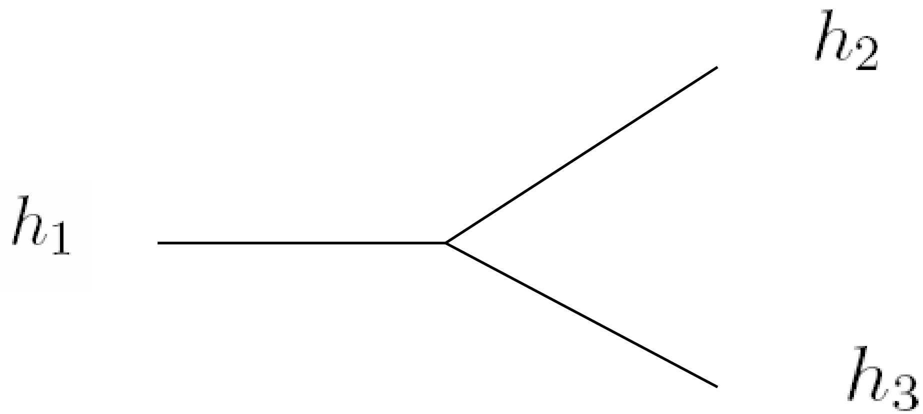
The class of dimensionless vertices appearing in Field Theory

$$M_3 = f < 1,2 >^{-2h_1-2h_2-1} < 2,3 >^{2h_1+1} < 3,1 >^{2h_2+1},$$

$$h_3 = -1 - h_1 - h_2.$$

$$M_3 = k[1,2]^{2h_1+2h_2-1}[2,3]^{-2h_1+1}[3,1]^{-2h_2+1},$$

$$h_3 = 1 - h_1 - h_2.$$

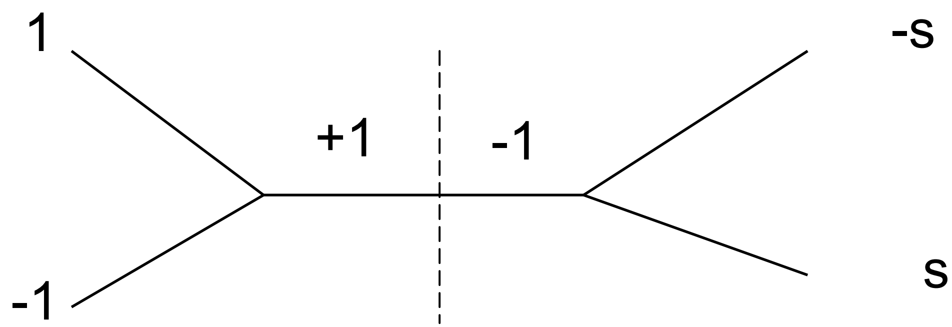


G.Georgiou, G.S.
10. arXiv:1007.3756
11. arXiv:0907.3553

$$M_4^{abcd}(+1,-1,+s,-s) =$$

$$= 2i\ g_{1ss}^2\ f^{ade} f^{bce}\ \frac{(-1)^{s+1}\ [1,3]^{s+1}\ \ < 2,4 >^{s+1}}{p_{12}^2\ p_{14}^2\ [1,4]^{s-1}\ \ < 2,3 >^{s-1}} +$$

$$+2i\ g_{1ss}^2 f^{ace} f^{bde}\ \frac{(-1)^{s+1}\ [1,3]^{s+1}\ \ < 2,4 >^{s+1}}{p_{12}^2\ p_{13}^2\ [1,4]^{s-1}\ \ < 2,3 >^{s-1}},$$



$$d\sigma_{+-\to+s-s} = \frac{\alpha^2}{s}\ \frac{C_2^2(G)}{d(G)} \ \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{2s-2}\left(\frac{3+\cos^2\theta}{\sin^4\theta}\right)(1-\cos\theta)^4\ d\Omega,$$

$$d\sigma_{+-\to-s+s} = \frac{\alpha^2}{s}\ \frac{C_2^2(G)}{d(G)} \ \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{2s-2}\left(\frac{3+\cos^2\theta}{\sin^4\theta}\right)(1+\cos\theta)^4\ d\Omega,$$

$$d\sigma_{+s} = \ \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{2s-2} d\sigma_{+1},$$

where $\sigma_{+-\to+s-s} \equiv \sigma_{+s}$

$$d\sigma_{+s} = \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{2s-2} d\sigma_{+1},$$

one can sum over all spins

$$\sum_{s=1}^{N}\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{2s-2} = \frac{(1+\cos\theta)^2}{4\cos\theta}\left(1+\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{2N-2}\right)$$

in order to get the total contribution

$$d\sigma_{tot} = \sum_{s=1}^{\infty}\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{2s-2} d\sigma_{+1} = \frac{(1+\cos\theta)^2}{4\cos\theta} d\sigma_{+1},$$

Deser, Jackiw and Templeton
Schonfeld

gauge invariance and mass term in (2+1)-dimensional gauge field theory

$$\mathcal{L}_{YMCS} = -\frac{1}{2} Tr G_{ij} G_{ij} + \frac{\mu}{2}\ \varepsilon_{ijk}\ Tr\ (A_i \partial_j A_k - ig\frac{2}{3} A_i A_j A_k),$$

$$(-k^2 \eta_{ij} + k_i k_j) e_j + i\mu\ \varepsilon_{ijl}\ k_j e_l = 0$$

the gauge field excitation becomes massive.

metric-independent density Γ in five-dimensional space-time

D=4+1

$$\Gamma = \varepsilon_{\mu\nu\lambda\rho\sigma} Tr G_{\mu\nu} G_{\lambda\rho,\sigma} = \partial_\mu \Sigma_\mu,$$

arXiv:1001.2808

$$\Sigma_\mu \quad = \quad \varepsilon_{\mu\nu\lambda\rho\sigma} Tr G_{\nu\lambda} A_{\rho\sigma}.$$

New topological current

D=3+1

$$\mathcal{P} = \frac{1}{4} \varepsilon_{\mu\nu\lambda\rho} Tr G_{\mu\nu} G_{\lambda\rho} = \partial_\mu C_\mu$$

$$C_\mu = \varepsilon_{\mu\nu\lambda\rho} Tr (A_\nu \partial_\lambda A_\rho - i \frac{2}{3} g A_\nu A_\lambda A_\rho)$$

is the Chern-Simons topological current.

Reduction to four dimensions

$$\int_{M_4} d^4x\ \Sigma \ = \ \varepsilon_{\nu\lambda\rho\sigma} \int_{M_4} d^4x\ Tr\ G_{\nu\lambda} A_{\rho\sigma}.$$

is gauge invariant up to the total divergence term

$$\delta_\xi \int_{M_4} d^4x\ \Sigma \ =$$

$$= \varepsilon_{\nu\lambda\rho\sigma} \int_{\partial M_4} Tr(G_{\nu\lambda}\xi_\sigma) d\sigma_\rho = 0.$$

Lagrangian with topological term

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_2 + \mathcal{L}_2' + \frac{m}{4}\ \Sigma$$

$$m\ \Sigma \ = \ m\ \varepsilon_{\nu\lambda\rho\sigma} Tr\ G_{\nu\lambda} A_{\rho\sigma},$$

Free field equations are

$$(-k^2\eta_{\nu\mu} + k_\nu k_\mu)e_\mu + im\ \varepsilon_{\nu\mu\lambda\rho} k_\mu b_{\lambda\rho} = 0,$$
$$(-k^2\eta_{\nu\mu}\eta_{\lambda\rho} + k_\nu k_\mu \eta_{\lambda\rho} - \eta_{\nu\mu} k_\lambda k_\mu) b_{\mu\rho} + i\frac{2m}{3}\ \varepsilon_{\nu\lambda\mu\rho} k_\mu e_\rho = 0.$$

$$k^2 = \frac{4}{3}\ m^2 = M^2.$$

$$e_\mu^{(1)} = (0,1,0,0), \quad b_{\gamma\acute{\gamma}}^{(1)} = \frac{1}{i}\frac{M}{\sqrt{\vec{k}^2+M^2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix},$$

$$e_\mu^{(2)} = (0,0,1,0), \quad b_{\gamma\acute{\gamma}}^{(2)} = -\frac{1}{i}\frac{M}{\sqrt{\vec{k}^2+M^2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$e_\mu^{(3)} = (0,0,0,\frac{M}{\sqrt{\vec{k}^2+M^2}}), \quad b_{\gamma\acute{\gamma}}^{(3)} = \frac{1}{i}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The above analysis suggests the following physical interpretation. A massive spin-1 particle appears here as a vector field of helicities $\lambda = \pm 1$ which acquires an extra polarization state absorbing antisymmetric field of helicity $\lambda = 0$, or as antisymmetric field of helicity $\lambda = 0$ which absorbs helicities $\lambda = \pm 1$ of the vector field. It is sort of "dual" description of massive spin-1 particle.

High-Rank Mass Terms

$$\Sigma_{2s+1} \quad = \quad \varepsilon_{\mu\nu\rho\sigma} \int d^4x \; Tr\{G_{\mu\nu} A_{\rho\sigma\lambda_{i_1}\lambda_{i_1}...\lambda_{i_s}\lambda_{i_s}} + ... + G_{\mu\nu,\lambda_{i_1}\lambda_{i_1}...\lambda_{i_s}\lambda_{i_s}} A_{\rho\sigma}\}.$$

$$\sum_s m_s \; \Sigma_s.$$

Super-Extension of the Poincare Group

the Coleman-Mandula theorem is the strongest no-go theorems, stating that the symmetry group of a consistent quantum field theory is the direct product of the internal symmetry group and the Poincare group.

Weakening the assumptions of the Coleman-Mandula theorem by allowing both commuting and anticommuting symmetry generators, allows a nontrivial extension of the Poincare algebra, namely the super-Poincare algebra.

$$[P^\mu,\ P^\nu] = 0,$$
$$[M^{\mu\nu},\ P^\lambda] = i(\eta^{\lambda\nu}\ P^\mu - \eta^{\lambda\mu}\ P^\nu),$$
$$[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),$$

$$[P^\mu,\ \Theta_\alpha] = 0,$$
$$[M^{\mu\nu},\ \Theta_\alpha] = \frac{i}{2}(\gamma^{\mu\nu}\Theta)_\alpha, \qquad \gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$$

$$[\Theta_\alpha, \Theta_\beta] = -2\ (\gamma^\mu C)_{\alpha\beta} P_\mu$$

$$[P^\mu,\ P^\nu] = 0, \tag{9}$$
$$[M^{\mu\nu},\ P^\lambda] = i(\eta^{\lambda\nu}\ P^\mu - \eta^{\lambda\mu}\ P^\nu),$$
$$[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),$$

$$[P^\mu,\ L_a^{\lambda_1\ldots\lambda_s}] = 0, \tag{10}$$
$$[P^\mu,\ Q^i_\alpha] = 0,$$
$$[M^{\mu\nu},\ L_a^{\lambda_1\ldots\lambda_s}] = i(\eta^{\lambda_1\nu}L_a^{\mu\lambda_2\ldots\lambda_s} - \eta^{\lambda_1\mu}L_a^{\nu\lambda_2\ldots\lambda_s} + \ldots + \eta^{\lambda_s\nu}L_a^{\lambda_1\ldots\lambda_{s-1}\mu} - \eta^{\lambda_s\mu}L_a^{\lambda_1\ldots\lambda_{s-1}\nu}),$$
$$[M^{\mu\nu},\ Q^i_\alpha] = \frac{i}{2}(\gamma^{\mu\nu}Q^i)_\alpha, \qquad \gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$$

$$[L_a^{\lambda_1\ldots\lambda_n}, L_b^{\lambda_{n+1}\ldots\lambda_s}] = if_{abc}L_c^{\lambda_1\ldots\lambda_s}, \quad s = 0, 1, 2, \ldots$$
$$\{Q^i_\alpha,\ Q^j_\beta\} = -2\ \delta^{ij}(\gamma^\mu C)_{\alpha\beta}P_\mu, \qquad i = 1, \ldots, N$$
$$[L_a^{\lambda_1\ldots\lambda_s}, Q^i_\alpha] = 0\ . \tag{11}$$

arXiv:1103.2456 Antoniadis, Brink, G.S.