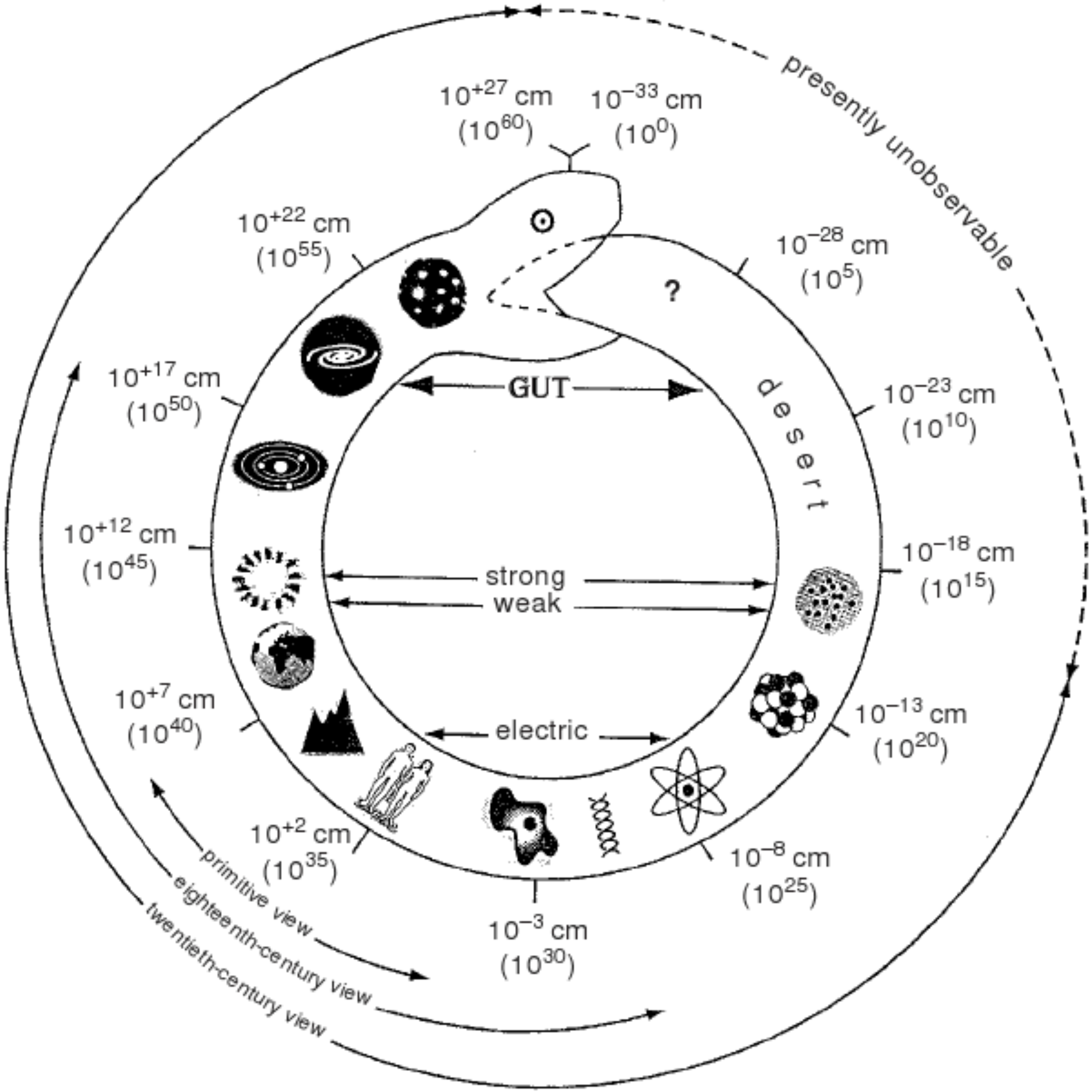
Subir Sarkar

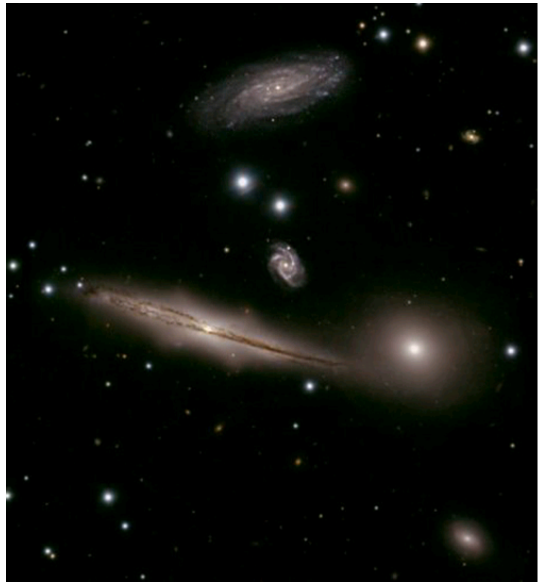
Oxford Physics

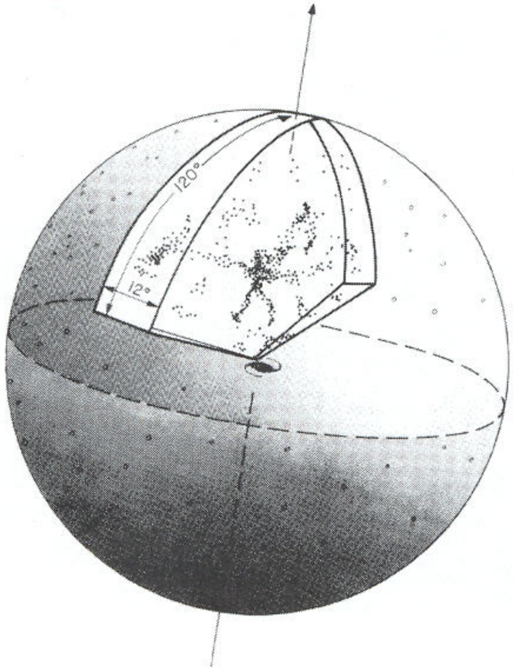
- The universe observed
- Reconstructing our thermal history
- Dark matter
- The early universe

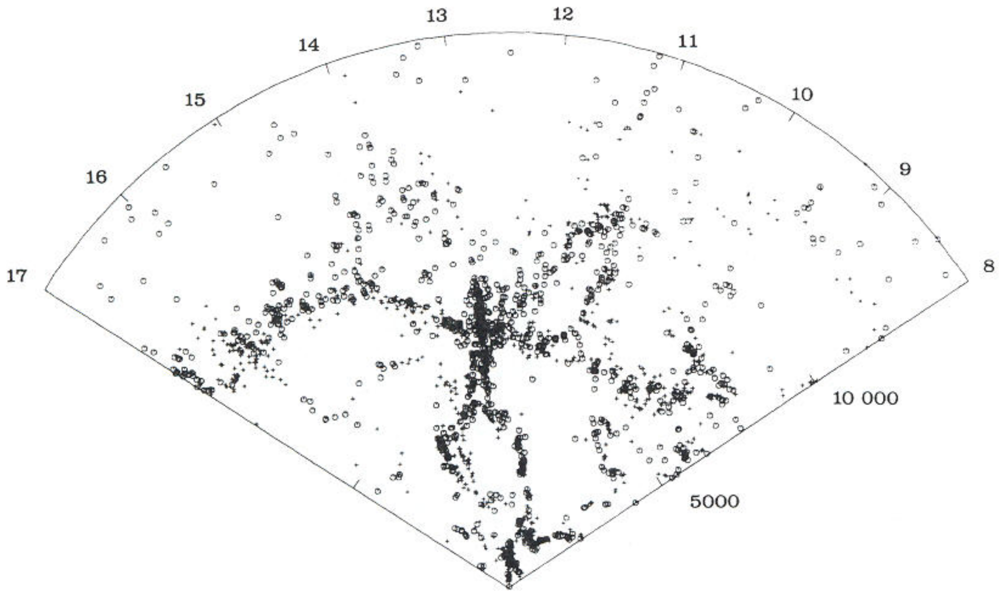


10+27 cm (10^60)
10−33 cm (10^0)
10+22 cm (10^55)
10−28 cm (10^5)
10+17 cm (10^50)
10−23 cm (10^10)
10+12 cm (10^45)
10−18 cm (10^15)
10+7 cm (10^40)
10−13 cm (10^20)
10+2 cm (10^35)
10−8 cm (10^25)
10−3 cm (10^30)
GUT
strong
weak
electric
desert
?
presently unobservable
primitive view
eighteenth-century view
twentieth-century view

# The universe appears complex and structured on many scales ....

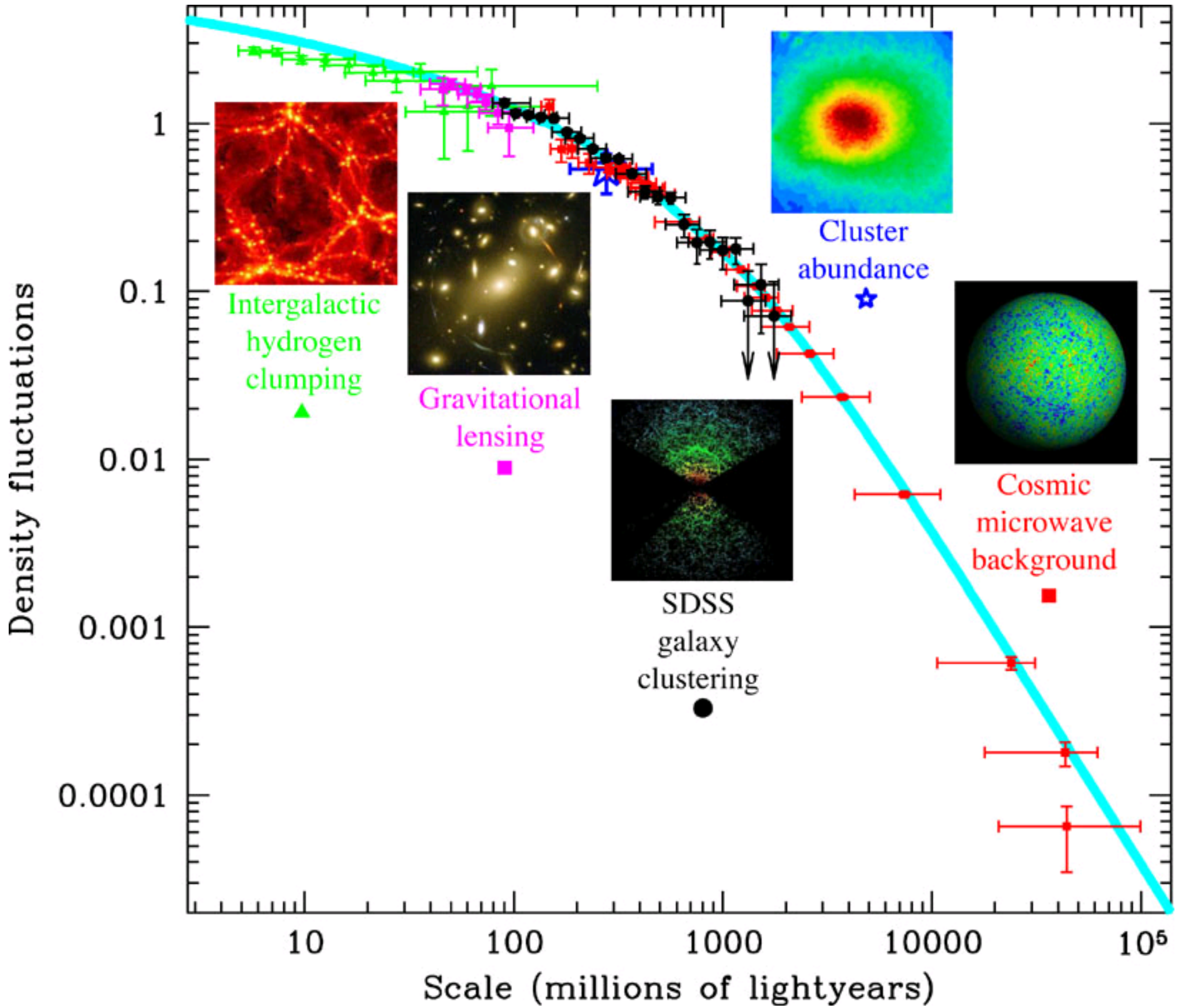





**How can we possibly describe it by a simple mathematical model?**

Although the universe is lumpy, it seems to become smoother and smoother when averaged over larger and larger scales …



# The universe certainly looks *isotropic* around us ...

e.g. this is the distribution of the 31000 brightest radio sources at ~1-6 cm

**But is the universe *homogeneous*?**

The largest anisotropy at large-scales on the sky is the CMB dipole
(1 part in ~$10^3$) ... believed to be due to our 'peculiar (non-Hubble) motion'

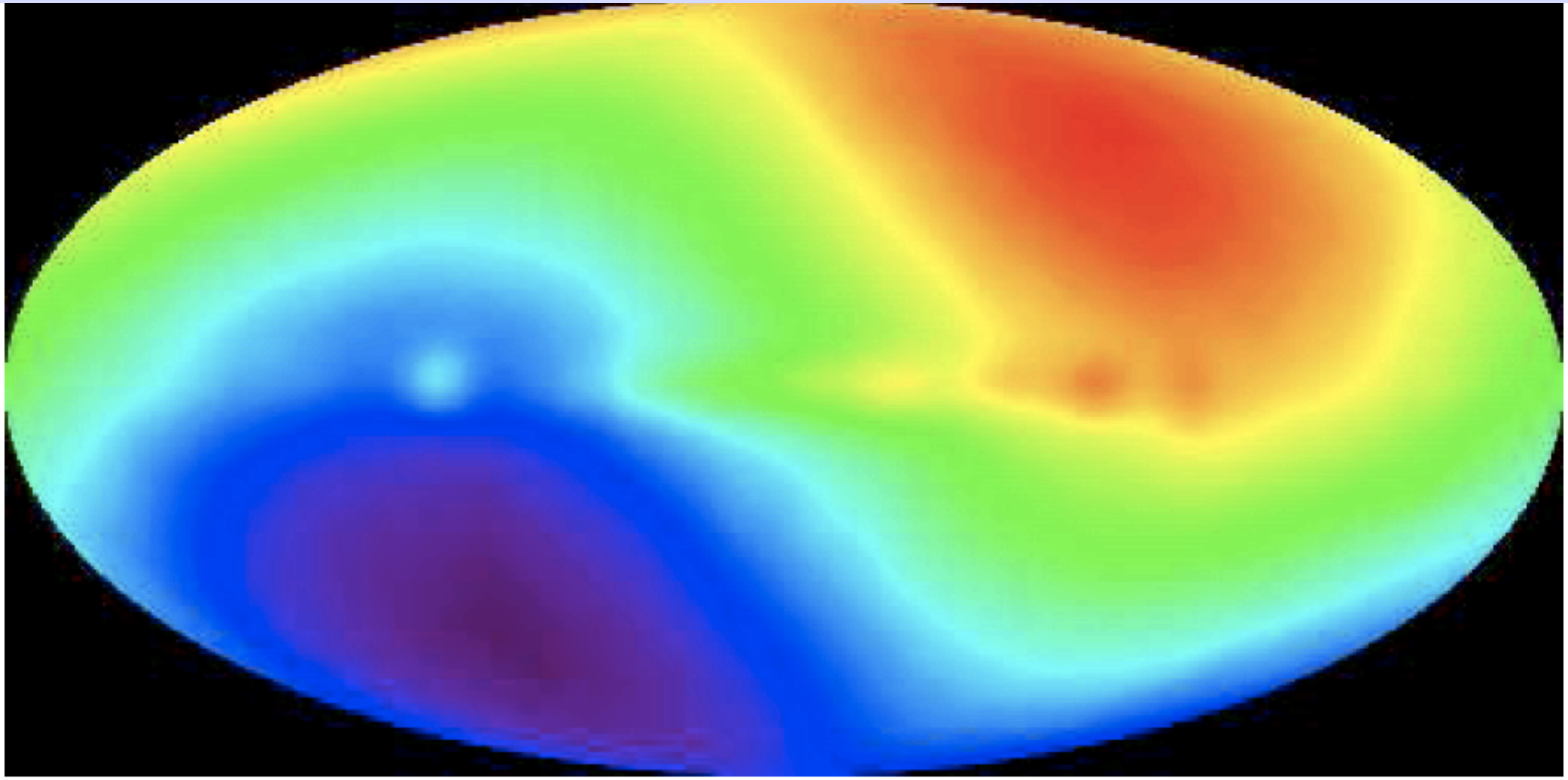
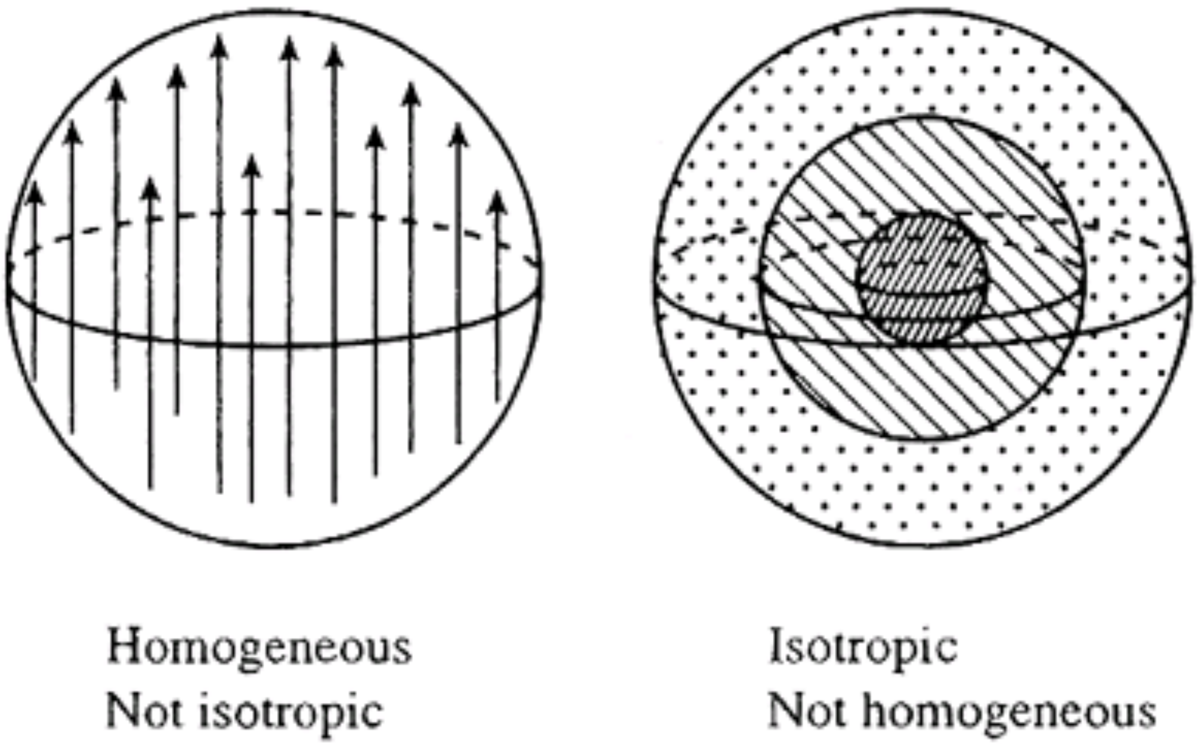


FIG. 18: The dipole in CMB as measured by the COBE satellite. The temperature range is T=2.721K (violet) to 2.729K (red). The inferred dipole velocity of the Solar System is $v = 368 \pm 2$ km/s and of the Local Group,

... after transforming to this frame, the CMB is isotropic to 1 part in ~$10^5$

However the *local* inhomogeneity (beyond which we converge to the CMB frame) extends out to at least ~260 Mpc [Colin *et al*, MNRAS **414** (2011) 264]

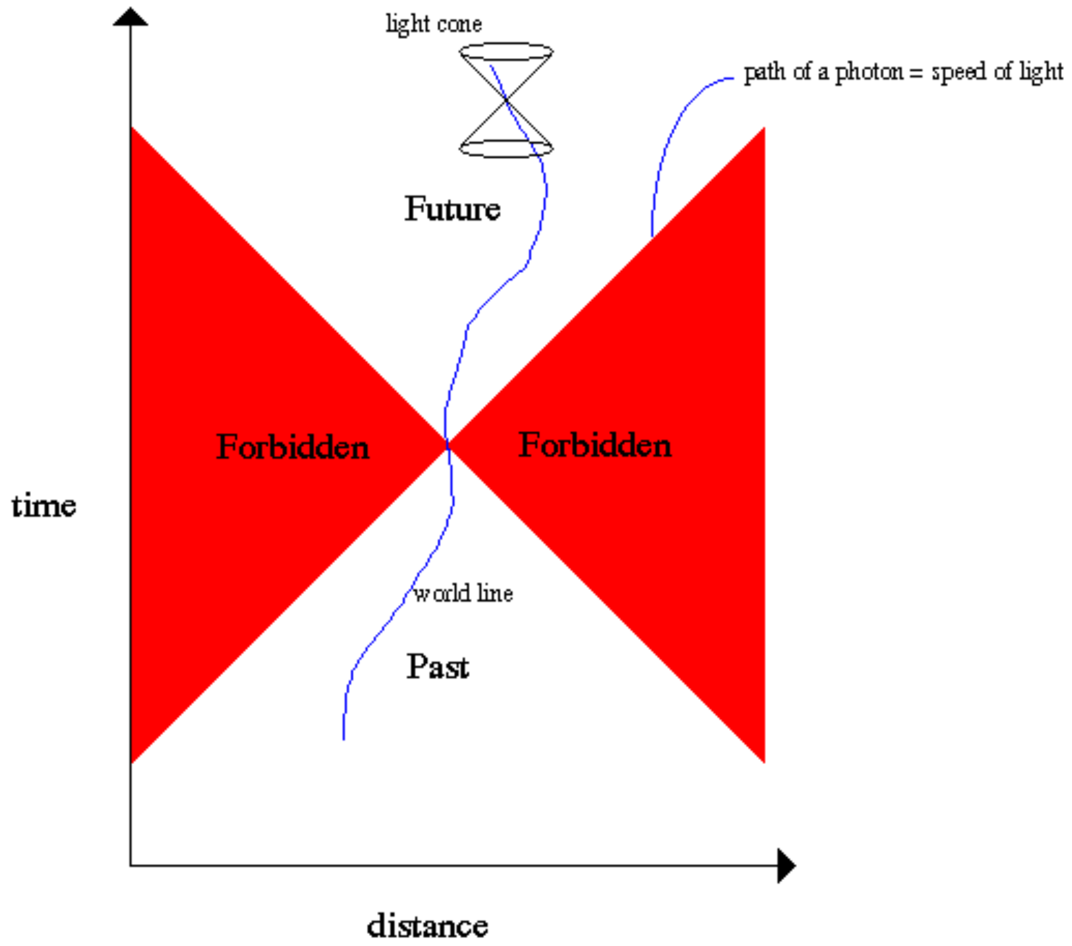
# Also isotropy does not necessarily imply homogeneity




… unless it is so about *every* point in space

But we cannot move (very far) in space so must assume that our position is typical – "The Cosmological Principle" (Milne 1935)

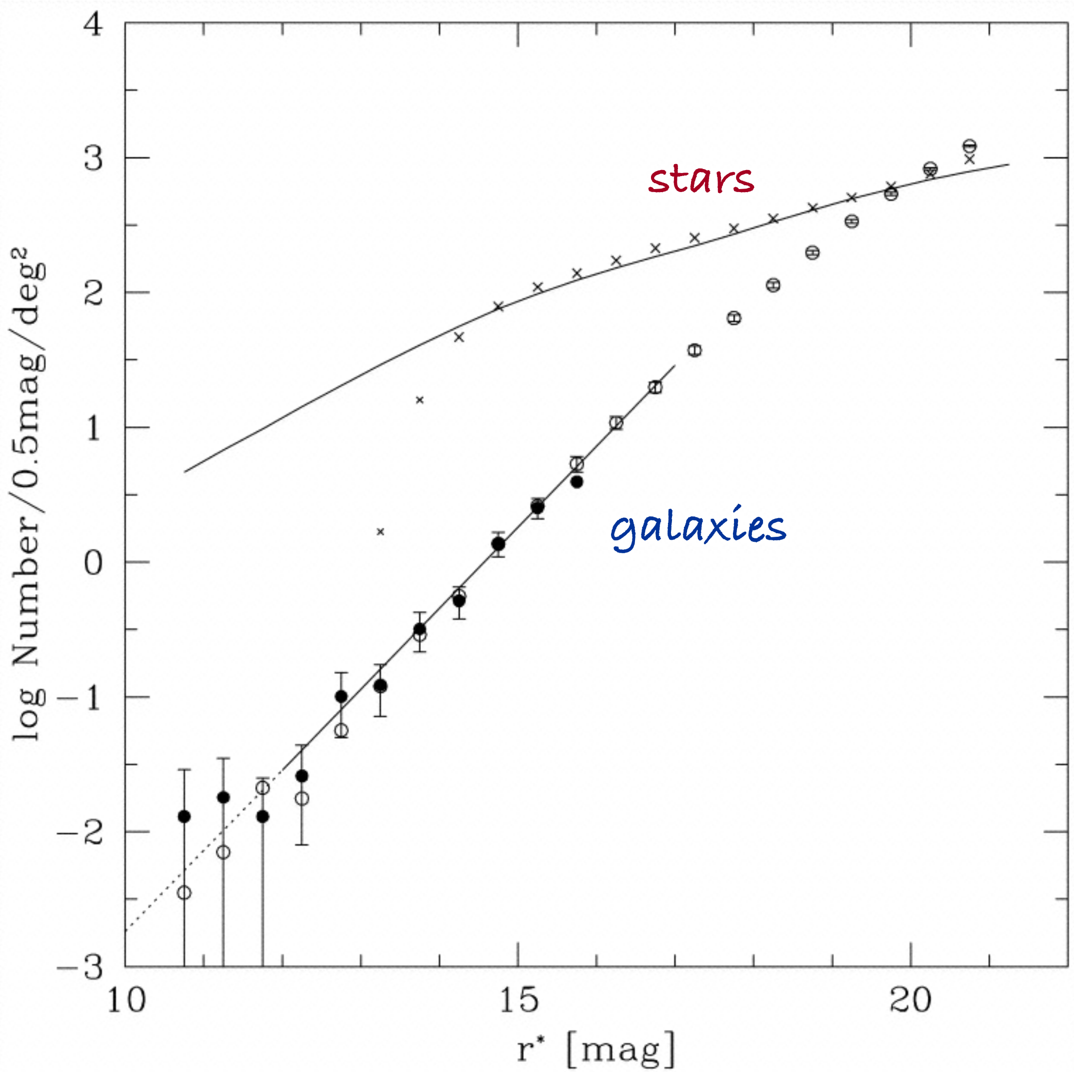
# All we can *ever* learn about the universe is contained within our past light cone




We *cannot* move over cosmological distances and check that the universe looks the same from 'over there' as it does from here … so there are *fundamental limits* to what we can know about the universe

Hubble showed that the distribution of galaxies is **homogeneous**,

i.e. $N(>S) \propto S^{-3/2} \Rightarrow N(<m) \propto 10^{0.6m}$, where $m \equiv -2.5 \log (S/S_0)$

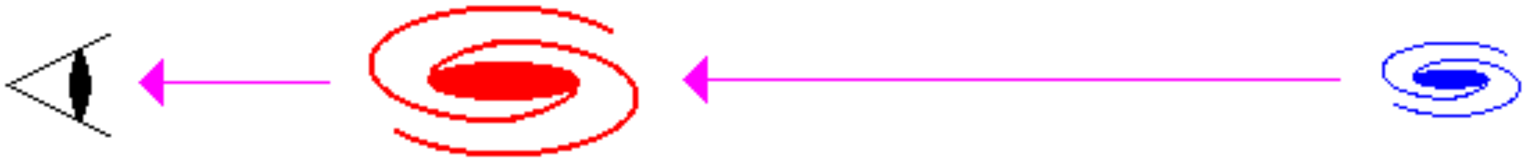



Here is the test done on galaxies in the **Sloan Digital Sky Survey**

Note that for stars, $N(<m) \propto 10^{0.4m}$, reflecting their 2D distribution

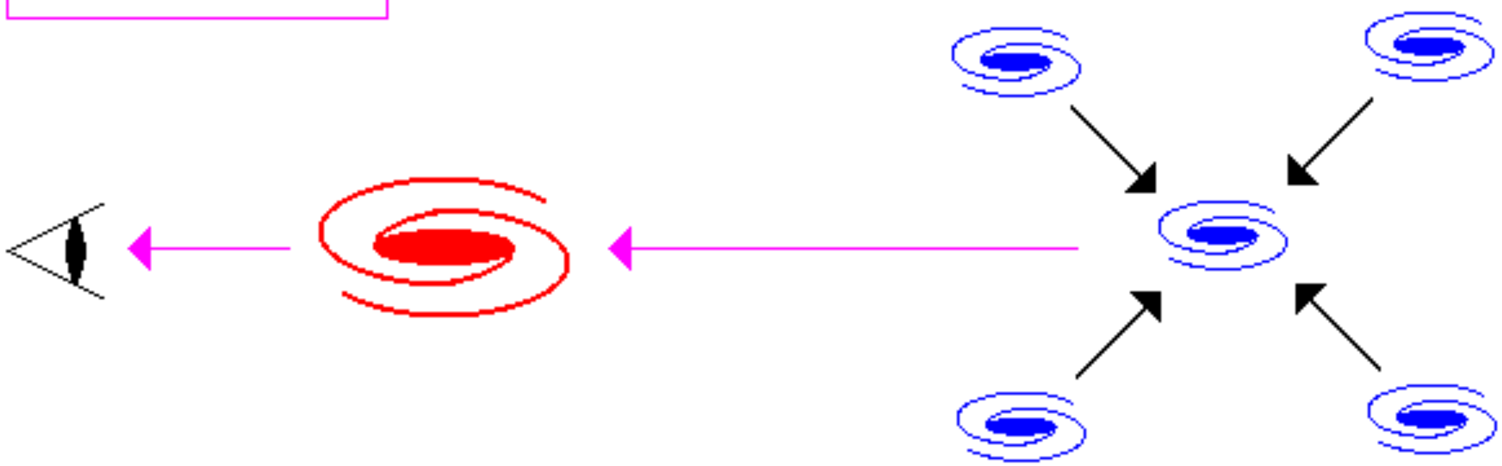
# Such tests are complicated however by *evolution effects*

## Color Evolution



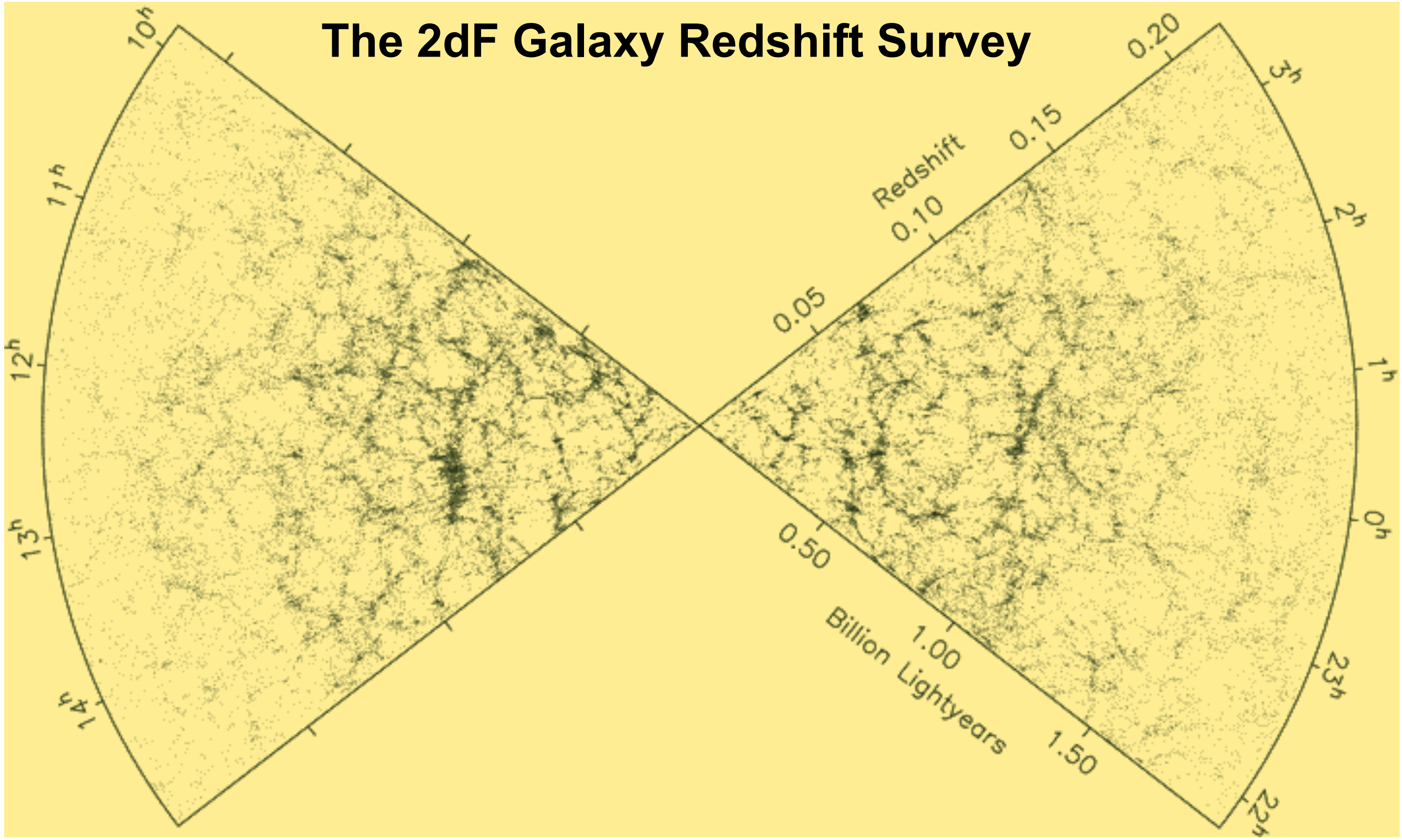
distant galaxies are bluer since we are looking back in time, and are seeing them at a younger age, younger stars = hotter stars = bluer stars

## Number Evolution



small galaxies merger at early epochs to form present–day galaxies. More galaxies are seen as we look back into the past.

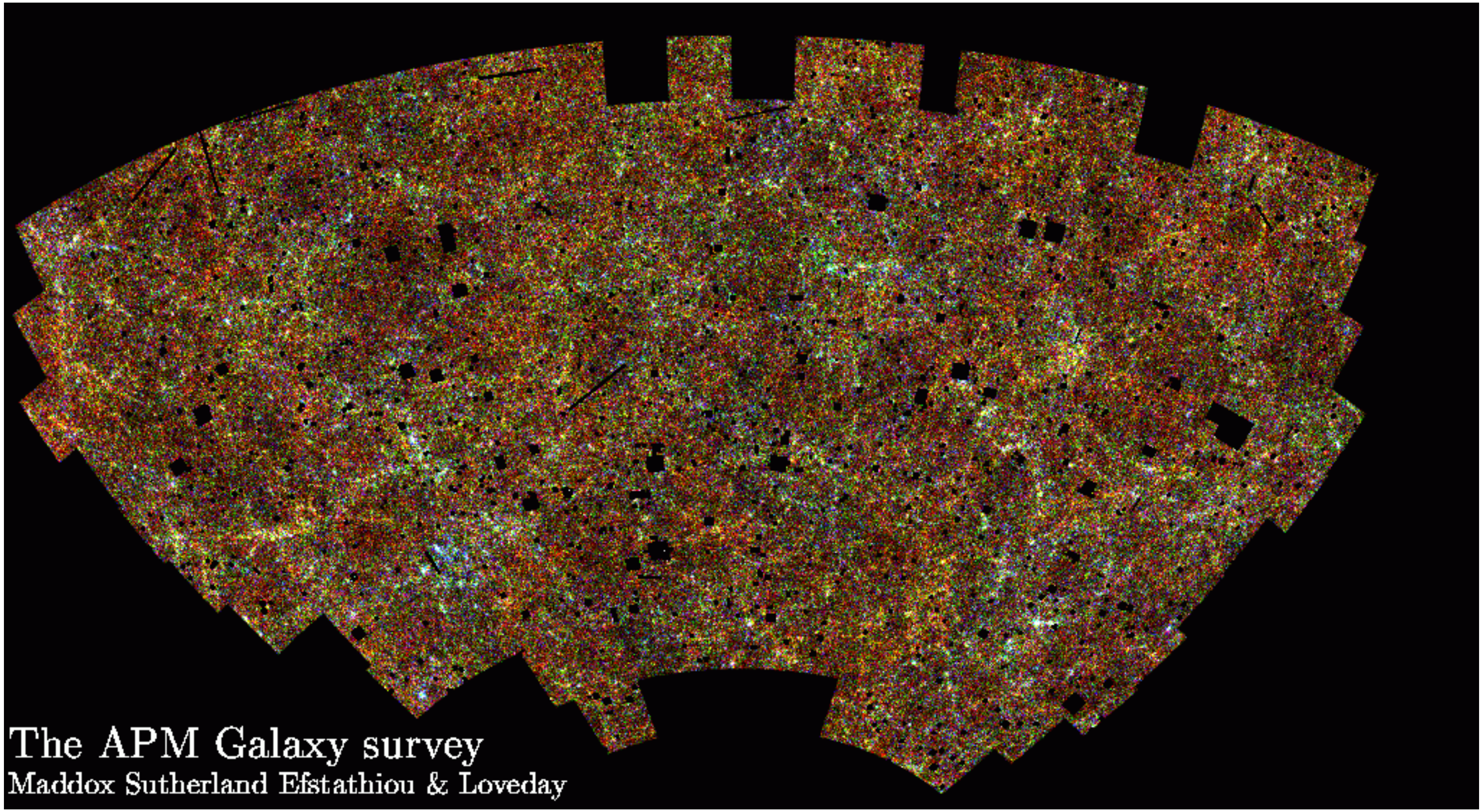
Einstein "anticipated" (without any data!) that the universe is **homogeneous and isotropic** when averaged over large scales




The distribution of galaxies is *fractal* on small scales … But averaged over very large scales (> 100 Mpc) the galaxy distribution supposedly is homogeneous

**However there is *still* structure ('walls', 'voids') on the largest scales probed … so the scale of homogeneity is still not well established**

# A consistency test of homogeneity is the *scaling* of the galaxy angular correlation function with the survey depth




If the distribution is indeed homogeneous on large scales (with small overdensities on small scales), then the characteristic **angular scale of clustering** should be *smaller* for fainter galaxies (which are on average further away) than for the nearby brighter ones …

# This is indeed true for the APM survey which measured the positions of 2 million galaxies extending upto ~600 Mpc

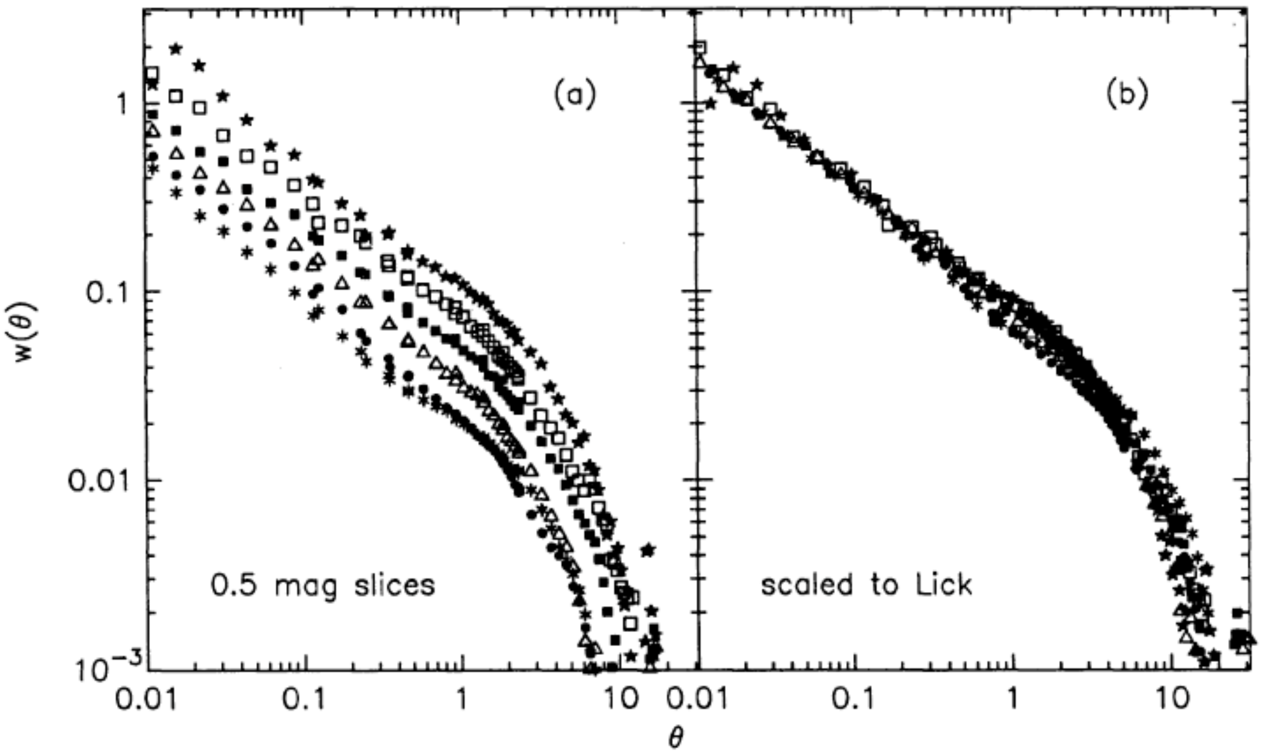



**Figure 2.** (a) Shows angular correlation functions for six 0.5 mag slices in the range $17.5 \leq b_J \leq 20.5$. (b) Shows the results from (a) scaled to the depth of the Lick survey as described in the text. Maddox *et al*, MNRAS 242 (1990) 43p

The angular correlation function $w(\theta)$ – defined as the *excess* probability over average of finding two galaxies within an angle $\theta$ of each other – was found to scale with the depth of the survey $D_*$ as: $w(\theta) = (r_0/D_*)\ W(\theta D_*/r_0)$

... **as is indeed expected for a homogeneous distribution** (with clustering scale $r_0$)

For a **fractal distribution** (with *no* intrinsic scale), $w(\theta)$ should *not* change with $D_*$

Equivalently the probability of finding two galaxies at a distance $r$ from each other is: $dP_{1,2} = n^2[1+\xi(r)]dV_1 dV_2$

The two-point correlation function (2PCF) is found to behave as a power-law: $\xi(r) = \left(\frac{r_0}{r}\right)^{\gamma}$ ... so becomes harder to measure as the distribution tends towards homogeneity

The conditional density (Coleman & Pietronero 1992) is defined as:

$$\Gamma(r) = \frac{\langle n(\boldsymbol{r}) n(\boldsymbol{r}+\boldsymbol{x})\rangle}{\langle n \rangle}$$

... and is simply related to the correlation function as:

$$\xi(r) = \frac{\Gamma(r)}{\langle n \rangle} - 1$$

But use of the 2-point correlation function implicitly *assumes* there is homogeneity on large scales - in order that an 'average density' and `small fluctuations' can be defined!

… so ought to analyse data without any *prior* assumptions about the nature of the distribution (Gabrielli *et al*, *Statistical Physics of Cosmic Structures*, 2005)

**Count number of galaxies in a spheres of different radius, centred on each galaxy in survey.**

Scaling exponent

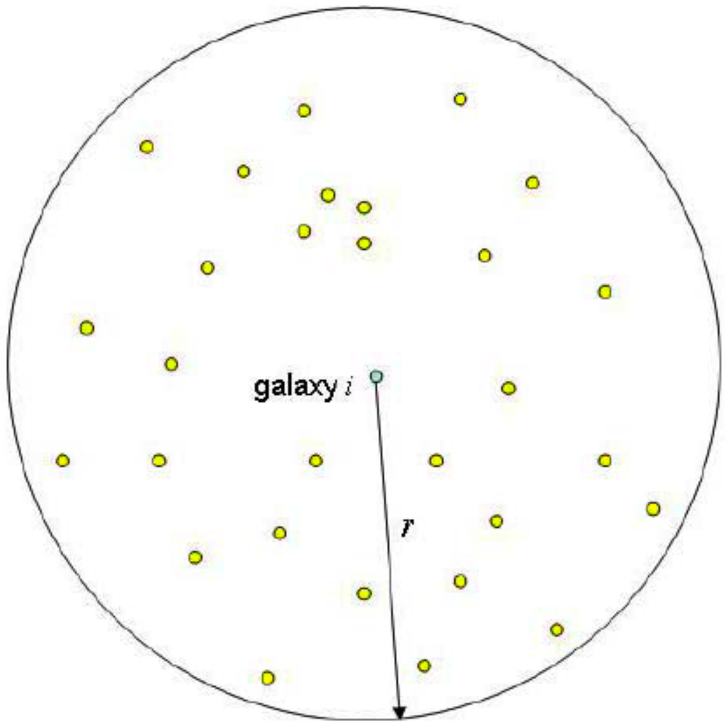
$$D_q = \frac{\tau(q)}{q-1}$$

Set of generalised dimensions

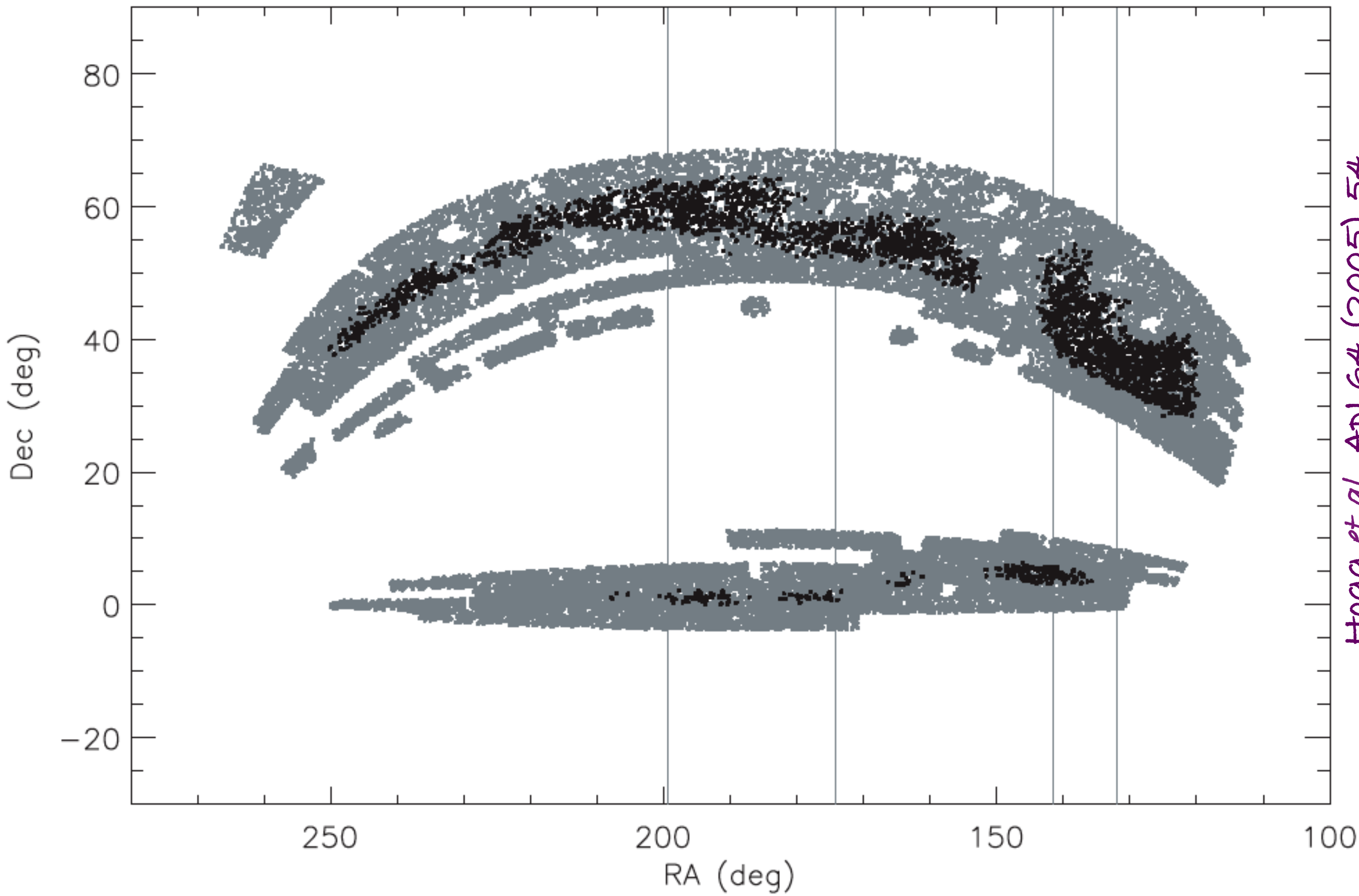
2PCF

$$D_2 = 3 - \frac{d[\log \xi(r)]}{d[\log(r)]}$$

Correlation dimension



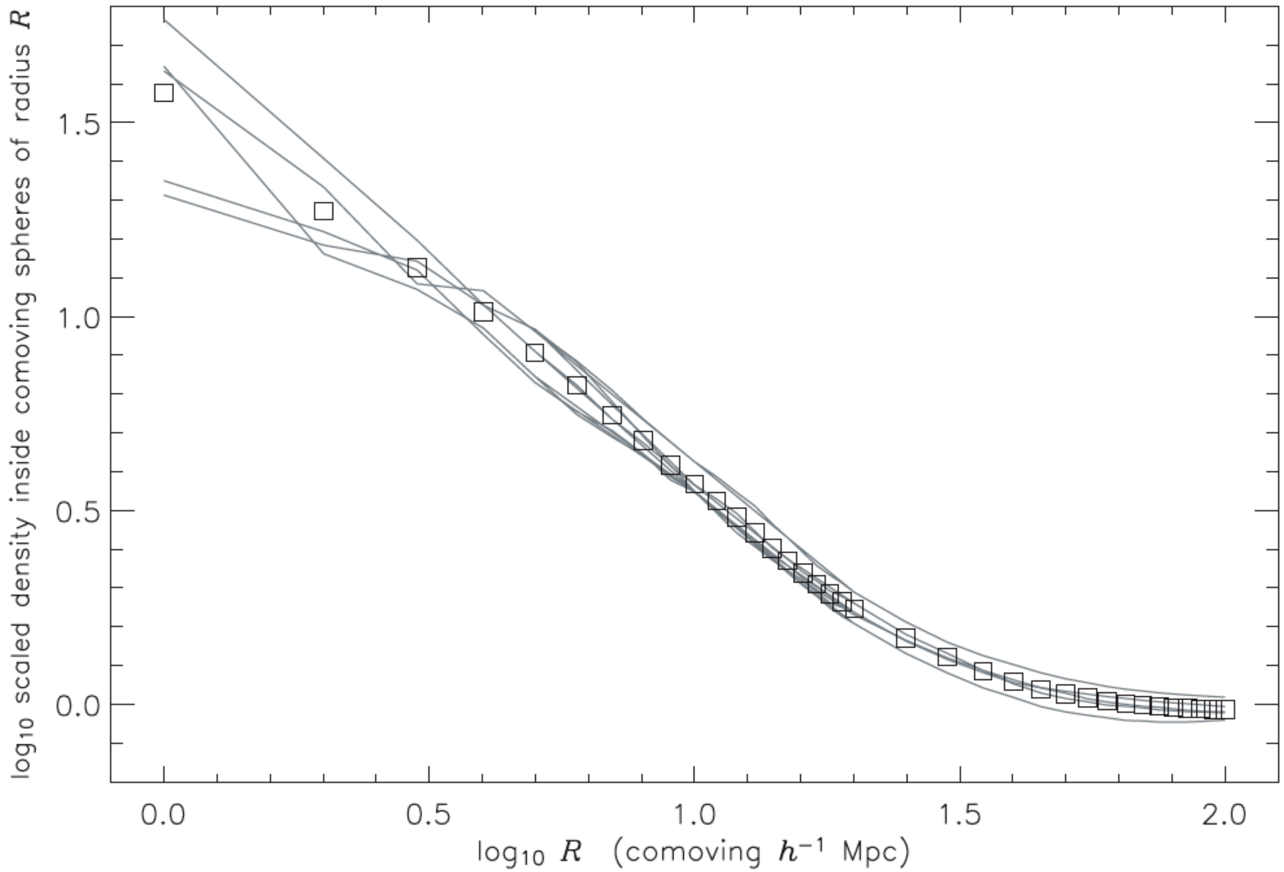
This ought to grow as $r^3$ *beyond* the **homogeneity scale … taking care that the test sphere is contained within the survey volume (and that luminosity selection dos not introduce bias)**




Hogg et al, ApJ 64 (2005) 54

This test has been performed on a sample of 3658 Luminous Red Galaxies with $0.2 < z < 0.4$ (occupying a volume 2 Gpc³) in the **Sloan Digital Sky Survey**

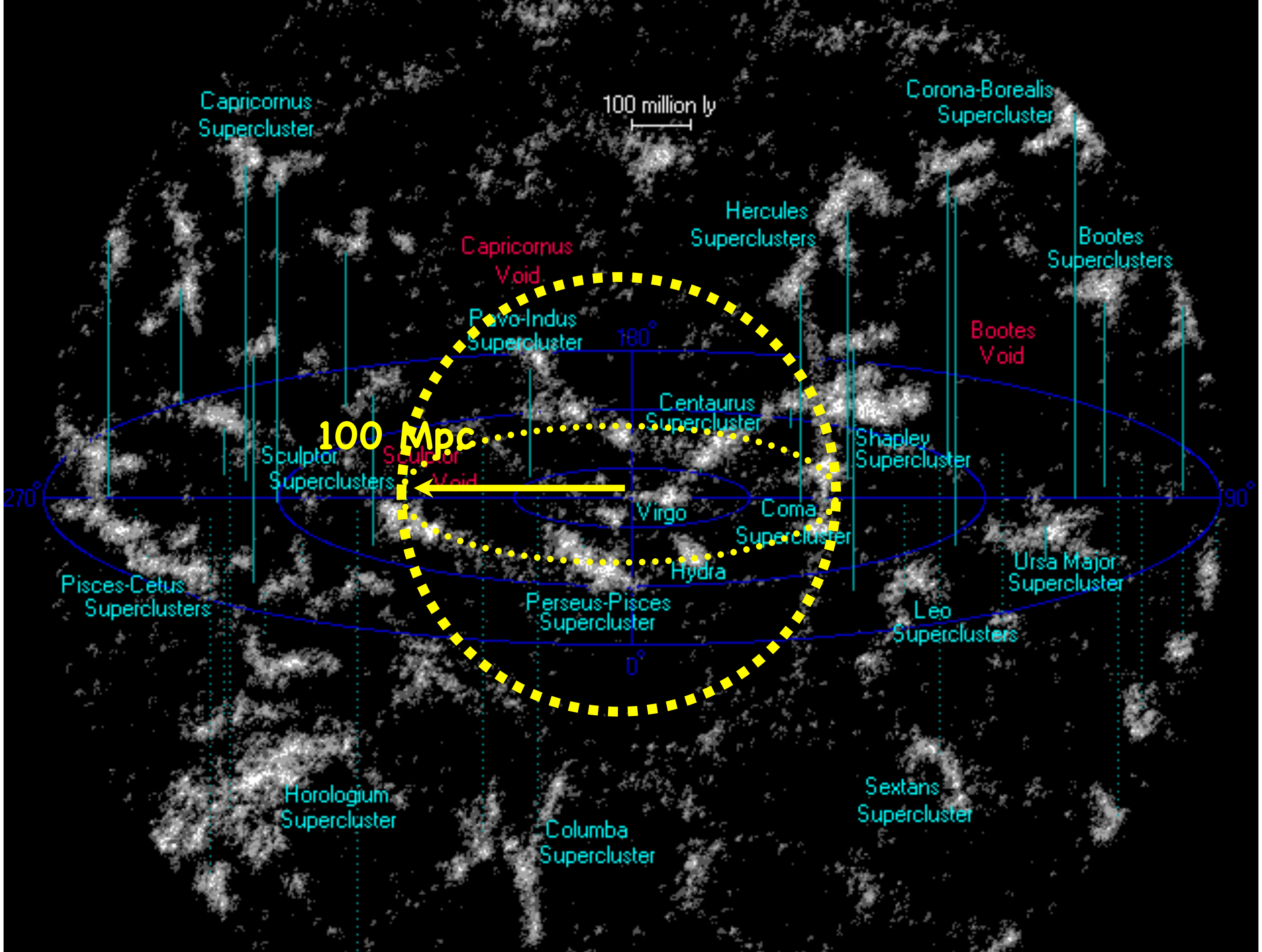
Actual counts in the SDSS grow as $\sim r^2$ on small scales where the distribution is fractal, but tend to homogeneity beyond ~100 Mpc …



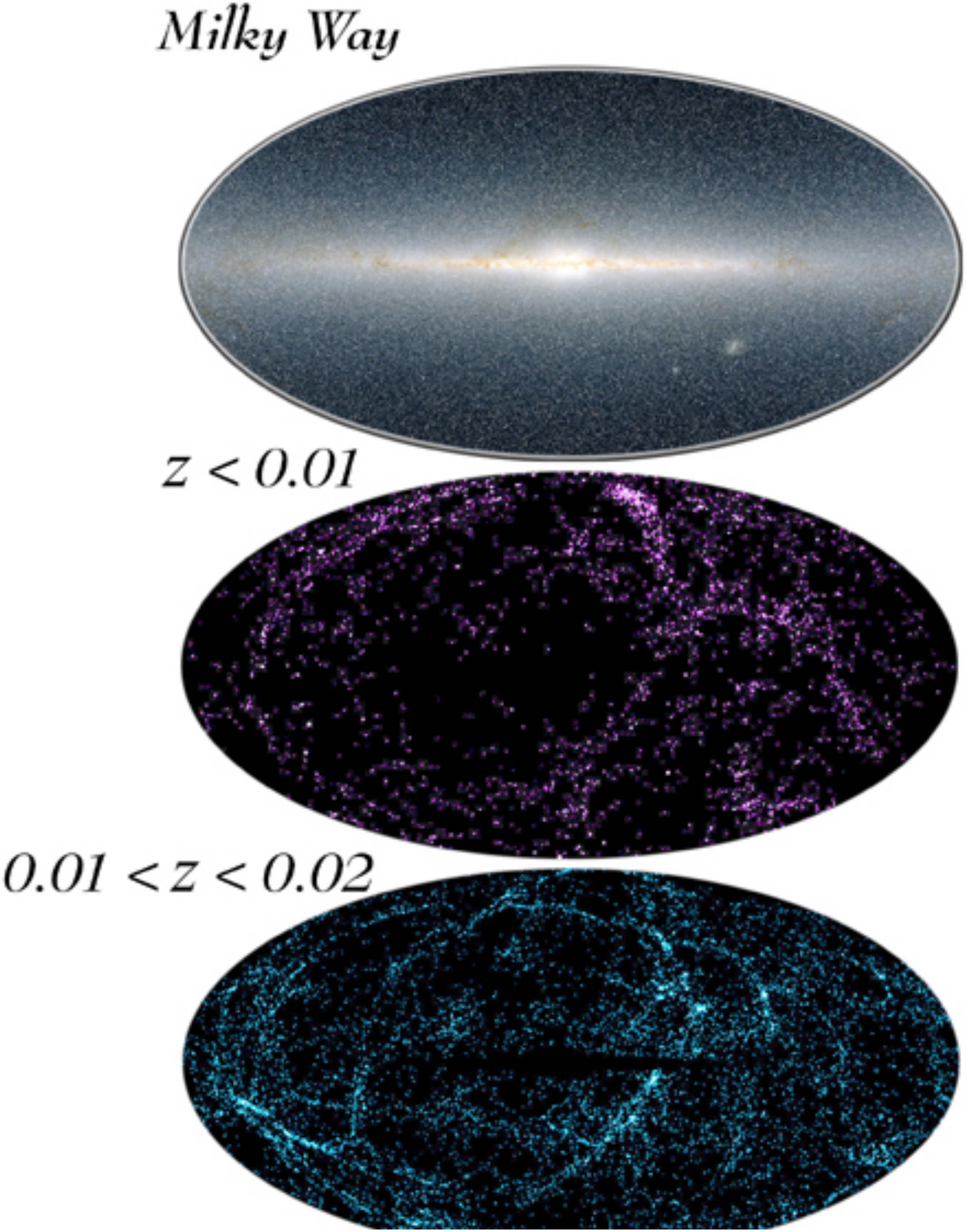

Hogg et al, ApJ 64 (2005) 54

FIG. 2.—Average comoving number density (i.e., number counted divided by expected number from a homogeneous random catalog) of LRGs inside comoving spheres centered on the 3658 LRGs shown in Fig. 1, as a function of comoving sphere radius $R$. The average over all 3658 spheres is shown with squares, and the averages of each of the five R.A. quantiles are shown as separate lines. At small scales, the number density drops with radius, because the LRGs are clustered; at large scales, the number density approaches a constant, because the sample is homogeneous. (for a critical view see Sylos-Labini et al, arXiv:0805.1132)

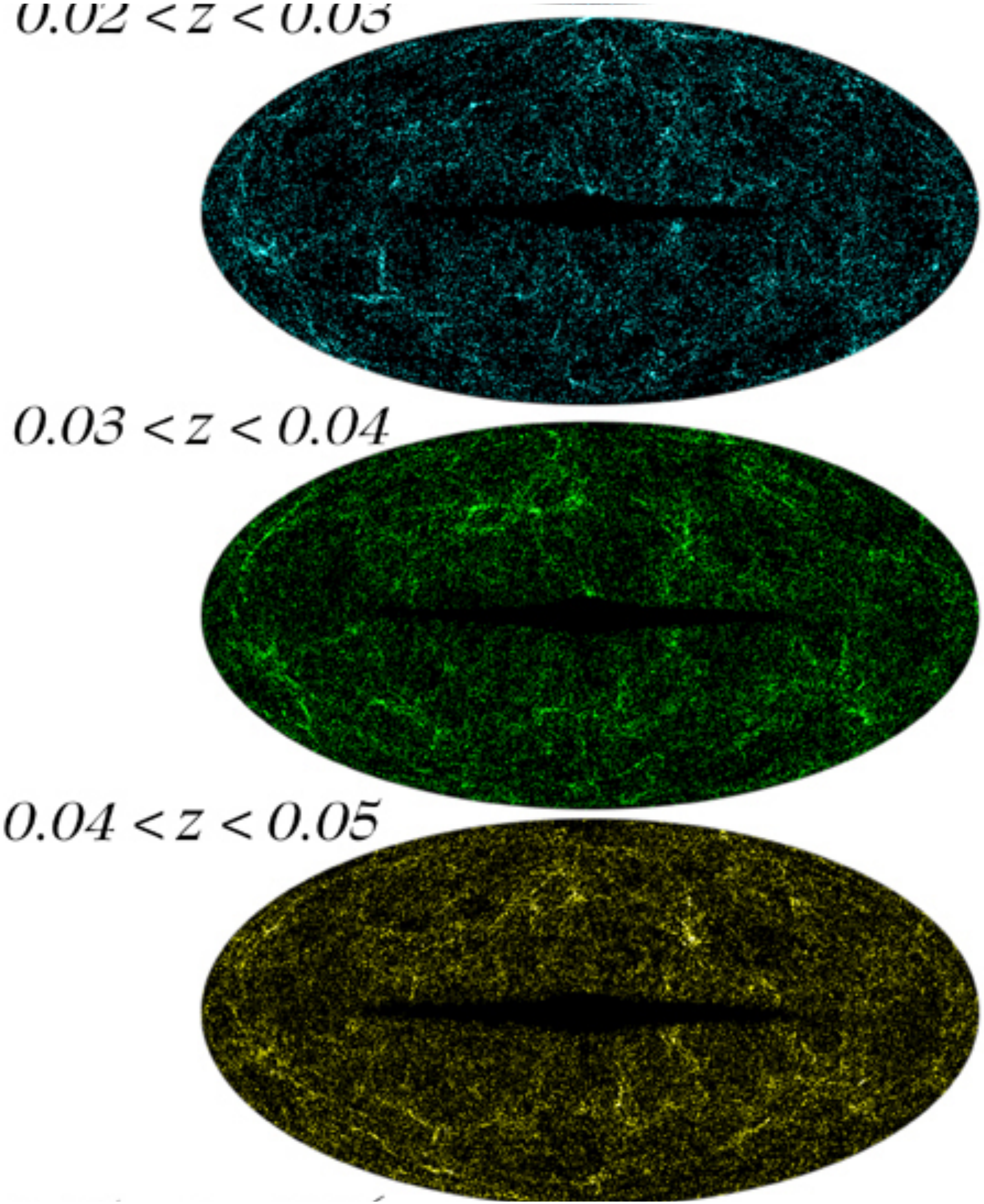
Our local neighbourhood is however rather inhomogeneous



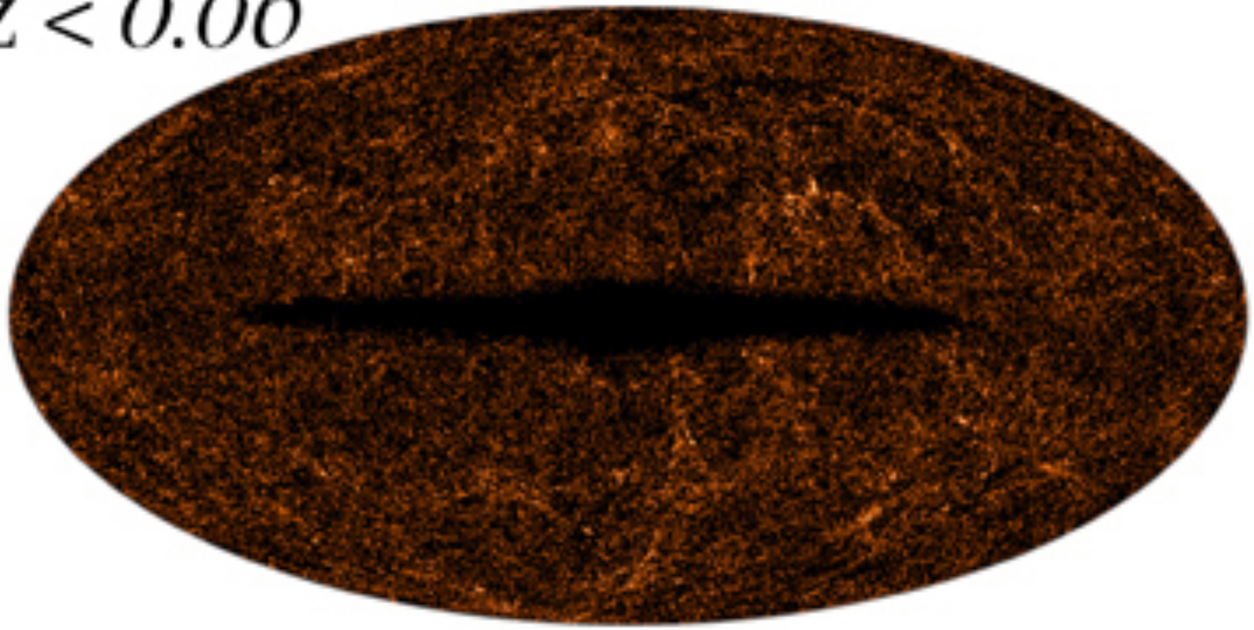

This is important since the measured local Hubble expansion rate is generally assumed to be the universal expansion rate



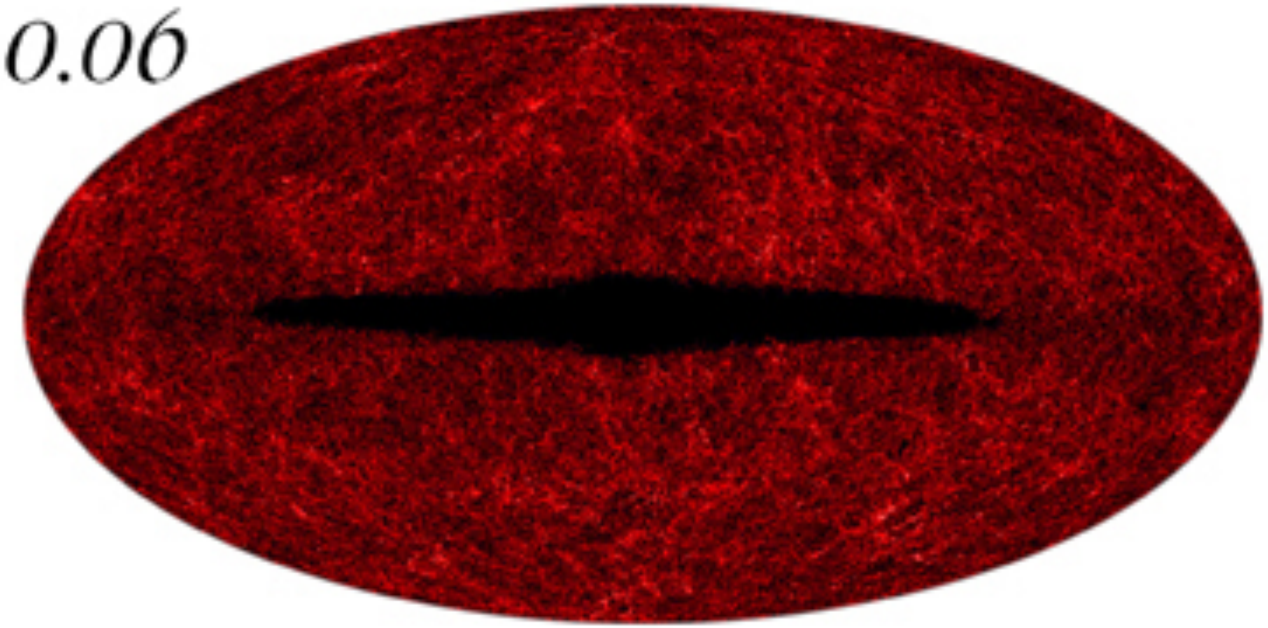
Milky Way
z < 0.01
0.01 < z < 0.02


0.02 < z < 0.03
0.03 < z < 0.04
0.04 < z < 0.05

$0.05 < z < 0.06$

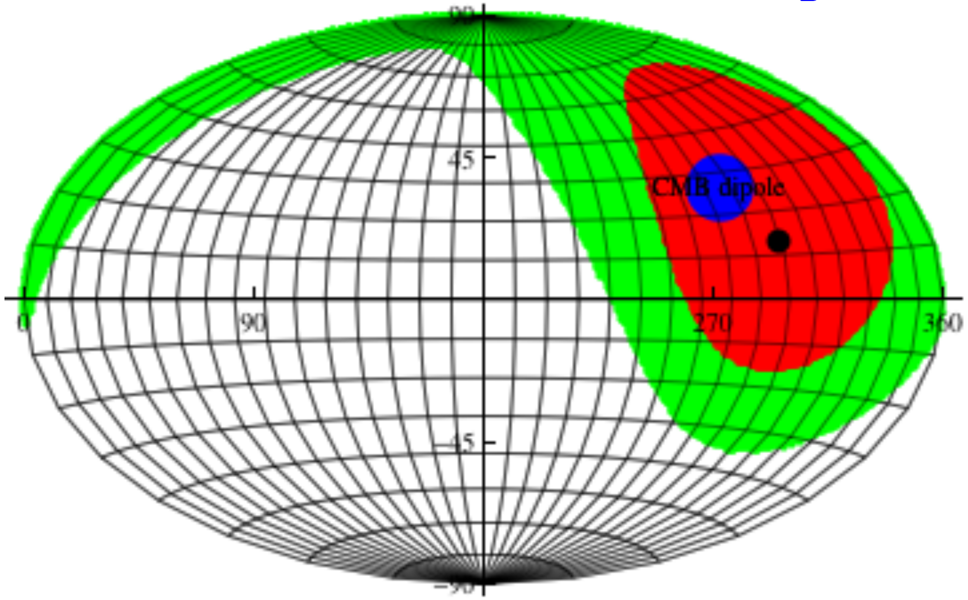


$z > 0.06$

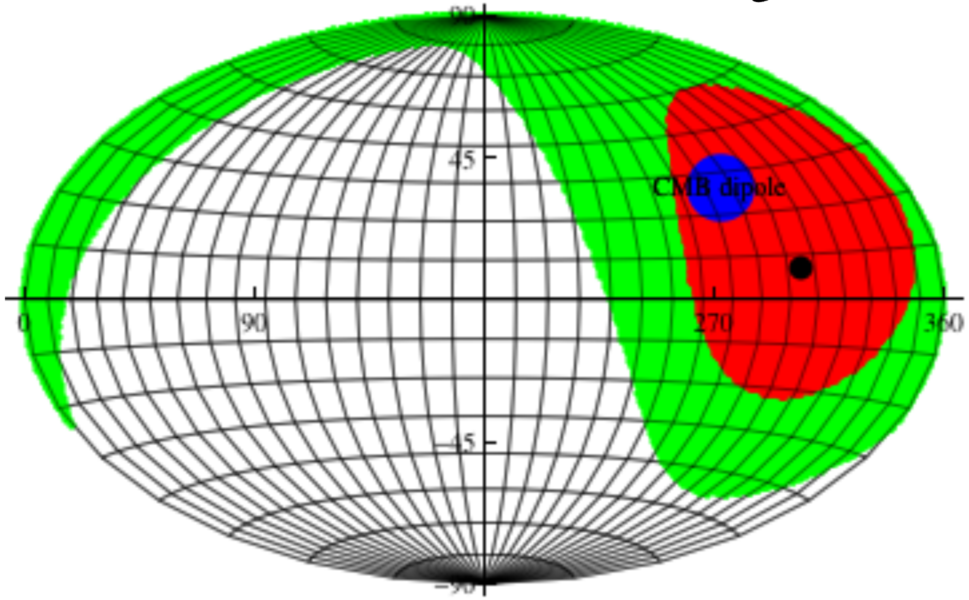


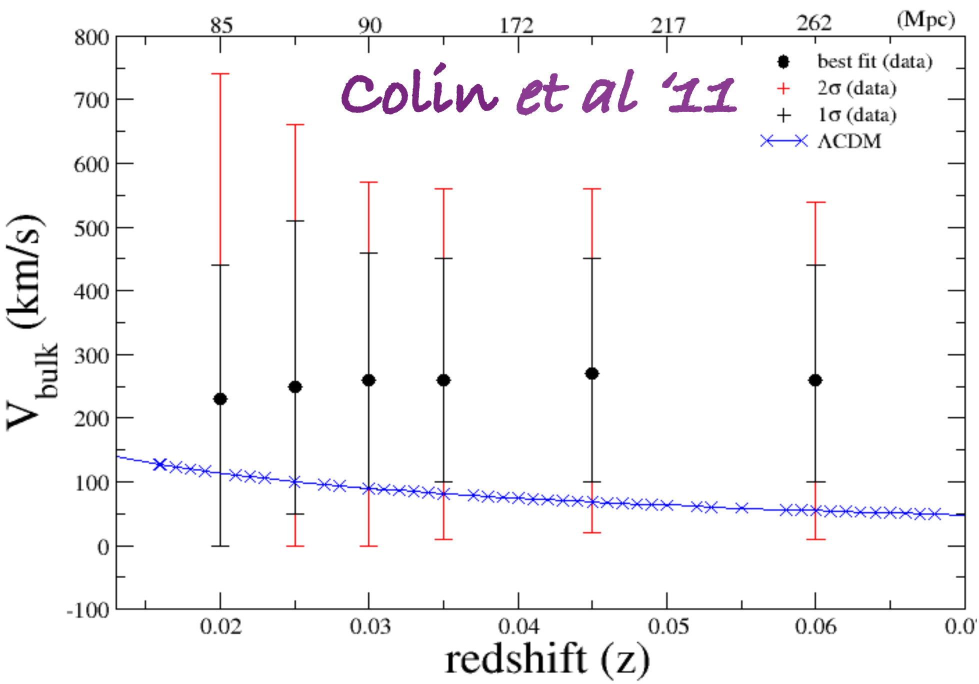
But convergence to the CMB frame has not yet occurred!

$0.015 < z < 0.045$, $v = 270$ km/s, $l = 291$, $b = 15$




$0.015 < z < 0.06$, $v = 260$ km/s, $l = 298$, $b = 8$




The bulk flow is higher than expected for a gaussian random field (standard $\Lambda$CDM model)

... consistent with bulk flow of $v = 416 \pm 78$ km/s towards $b = 60 \pm 6$, $l = 282 \pm 11$ at a scale of $100h^{-1}$ Mpc (Watkins et al '09)

# Excess Clustering on Large Scales in the MegaZ DR7 Photometric Redshift Survey

Shaun A. Thomas,[1] Filipe B. Abdalla,[1] and Ofer Lahav[1]

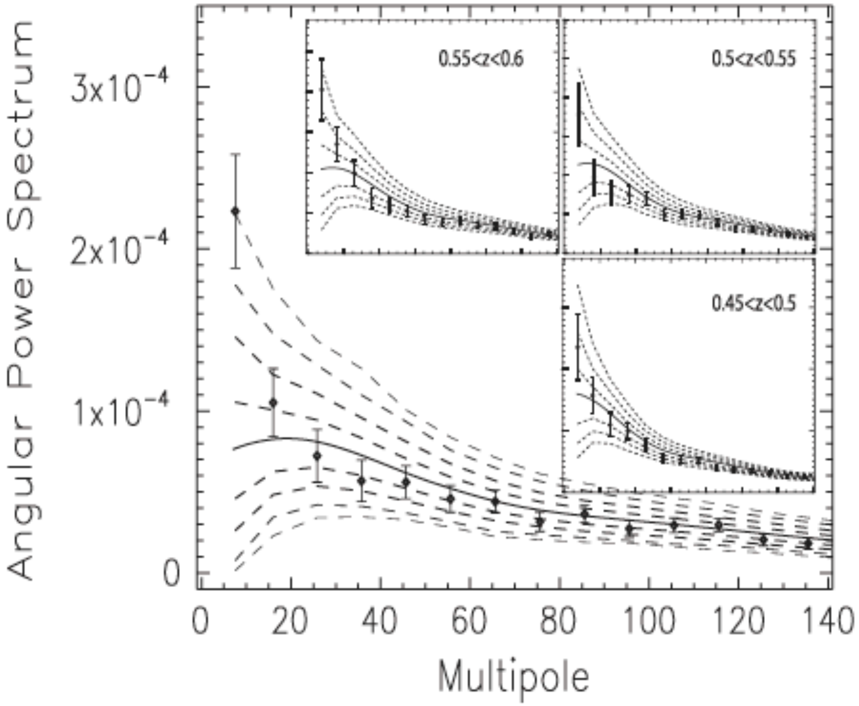



FIG. 1. The angular power spectra $C_\ell$ measured in the SDSS photometric MegaZ DR7 luminous red galaxy survey. The panels relate to four redshift bins with width $\Delta z = 0.05$ from $z = 0.45$ to $z = 0.65$. The best fit theoretical spectra (solid lines) are excellent matches to the data including multipoles up to $\ell \sim 500$. However, the largest angular scales are observed to be anomalous; the dashed lines correspond to $1 \rightarrow 4\sigma$ derived from simulations. This is particularly severe in the highest redshift bin (main panel), which is $\sim 4\sigma$.

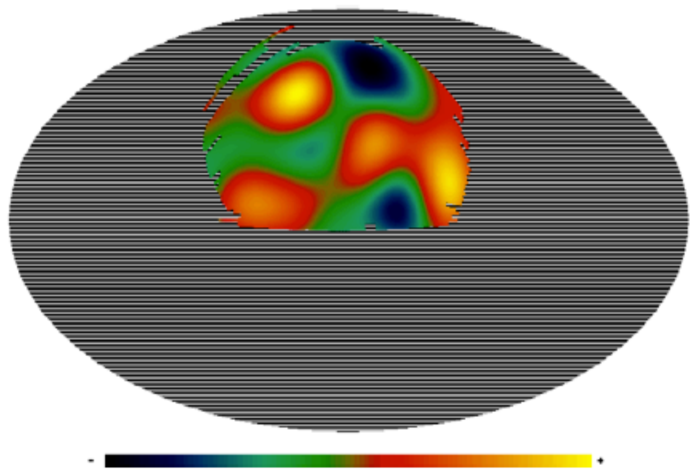


FIG. 2 (color online). A visualization of the measured underlying field within the surveyed region on the sky. Only contributions from $\ell = 4 \rightarrow 10$ in the most anomalous multipole and redshift band are included.

In the largest galaxy survey to date (~3.3 $Gpc^3$) there is $4\sigma$ evidence of excess clustering on scales larger than $700h^{-1}$ Mpc … this poses a challenge to the usual idea of large-scale homogeneity!

## Special relativity

$$ds^2 = \sum g_{ij} dx^i dx^j \ \ldots \ \text{interval between events } x^i \text{ and } x^j \, (i, j = 0, 1, 2, 3)$$

$$g_{ij}(x) \equiv g_{ji}(x) \rightarrow 10 \text{ independent functions}$$

## Minkowski metric

$$\eta_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \frac{\delta g_{ij}}{\delta x^k} = 0 \quad \Rightarrow ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

... invariant under **Lorentz velocity transformations**, *i.e.* *equivalent* to **local inertial coordinates** of **Newtonian mechanics**

## General relativity

Now $g_{ij}$ is related to the **distribution of matter** ... but $g_{ij} = \eta_{ij}$ is a solution in the *absence* of matter – contrary to **Mach's principle*** !

* inertial frames are determined relative to the motion of the matter ('distant stars') in the universe

Einstein (1919) saw two ways out:

* add suitable boundary conditions to eliminate anti-Machian solution, *viz.* let $g_{ij}$ take some pathlogical form (rather than becoming $\eta_{ij}$) when far away from all matter
... however de Sitter pointed out pheonomenological problems with this idea! ✗

* Postulate that the matter distribution is **homogeneous** (in the average) and that matter causes space to curve so as to close in on itself (3D analogue of a 2D balloon)
➔ Spatial volume finite but *no boundaries* and a non-singular metric everywhere ✔

## Einstein's world model

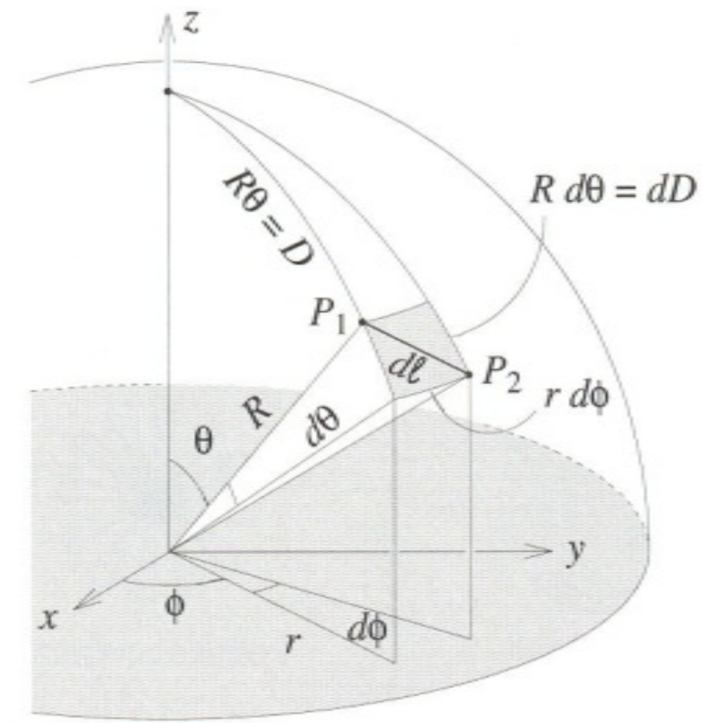
$$\text{Homogeneity} \;\Rightarrow \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}m} \propto 10^{0.6m} \quad \ldots \text{ as observed later (Hubble 1926)}$$

## ... incorporating Milne's Cosmological Principle

$$\mathrm{d}s^2 = \mathrm{d}t^2 + g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta \ldots \text{synchronous gauge (dense set of comoving observers)}$$

This is the 'standard model' we are still using to interpret observational data!

Picture the spatial part as $\mathbf{S}^3$ (3D analogue of balloon, embedded in flat 4D space)




Set of points defining $\mathbf{S}^3$: $R^2 = x^2 + y^2 + z^2 + w^2$

where: $r^2 = x^2 + y^2 + z^2$

Line element: $\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + \mathrm{d}w^2$

i.e. $\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + r^2\mathrm{d}r^2/(R^2\text{-}r^2)$

Polar coordinates ($z$=$r\cos\theta$, $x$=$r\sin\theta\cos\phi$, $y$=$r\sin\theta\sin\phi$):

$$\mathrm{d}l^2 = \mathrm{d}r^2 + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2) + r^2\mathrm{d}r^2/(R^2\text{-}r^2)$$
$$= \mathrm{d}r^2/(1 - r^2/R^2) + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$$

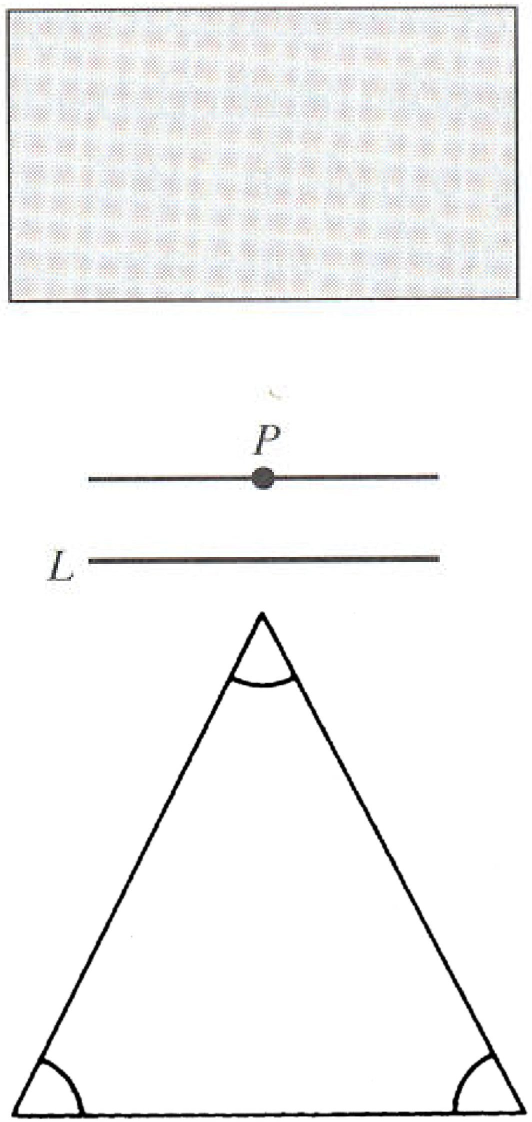
or, $\mathrm{d}s^2 = \mathrm{d}t^2 - R^2\,[\mathrm{d}\chi^2 + \sin^2\chi\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)]$,

where, $r = R\sin\chi$, $\chi \Rightarrow$ polar angle of hypersphere

Note interesting visual effects in curved space (when $r \sim R$), e.g. the angular size $\delta = D/R\sin\chi$ reaches minimum at $\chi = \pi/2$ and diverges to fill the entire sky when $\chi = \pi$ (this point is the just the 'Big Bang' – the antipodal point of the hypersphere)

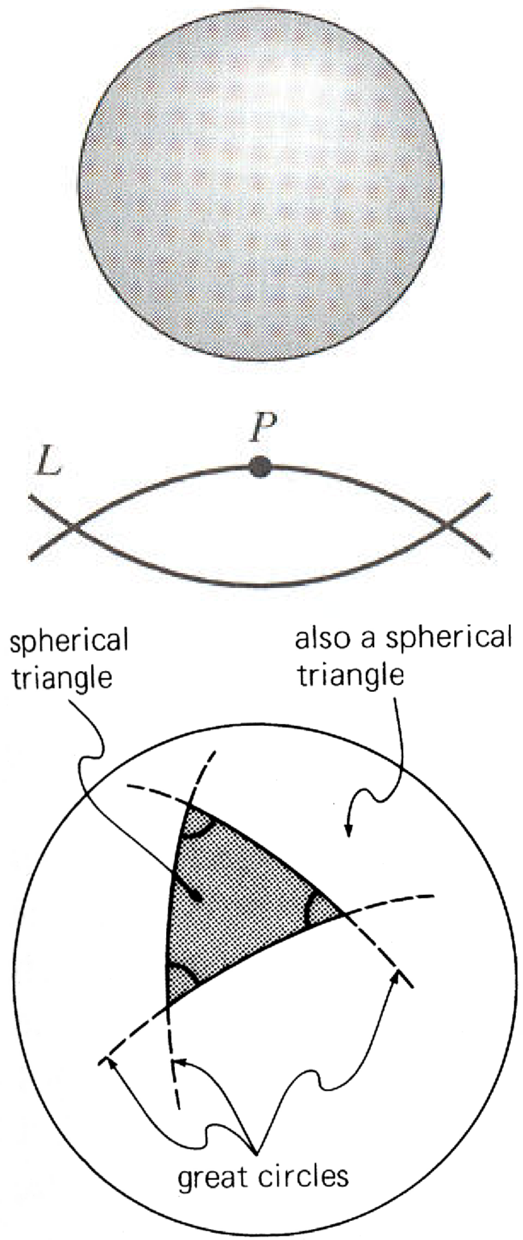
Also the parallax, $\varepsilon = A\cot\phi/R$, vanishes at $\chi = \pi/2$

# The 3 possible geometries of maximally-symmetric space

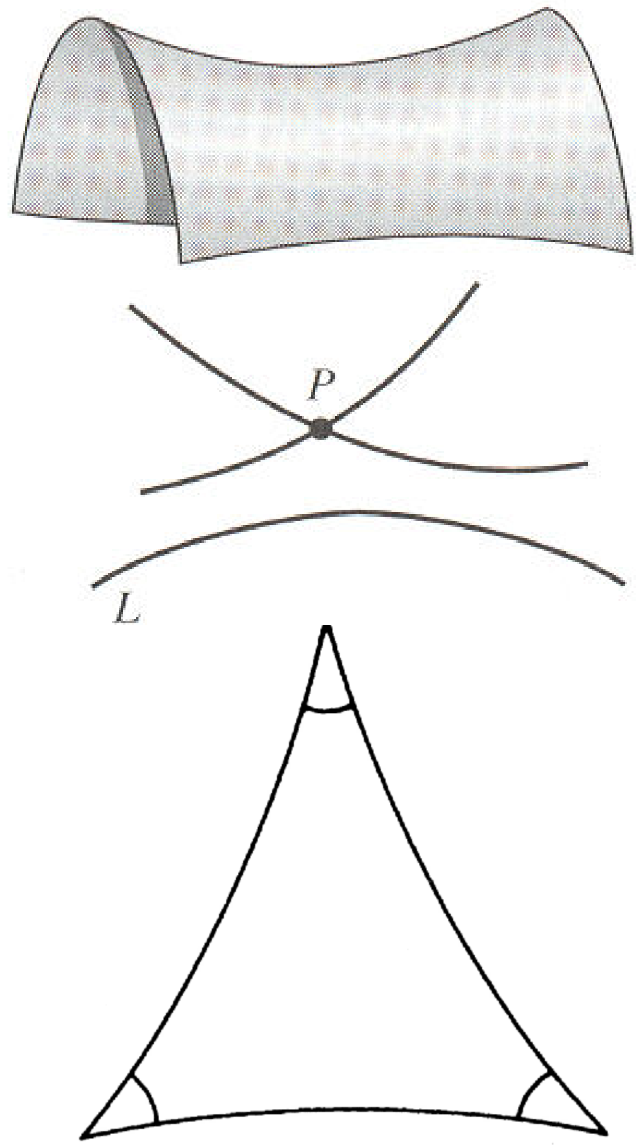



flat space

$= 180°$




$> 180°$




hyperbolic space

$< 180°$

# Could the universe have non-trivial topology?

(... as has been suggested e.g. to explain observed anomalies in the CMB)

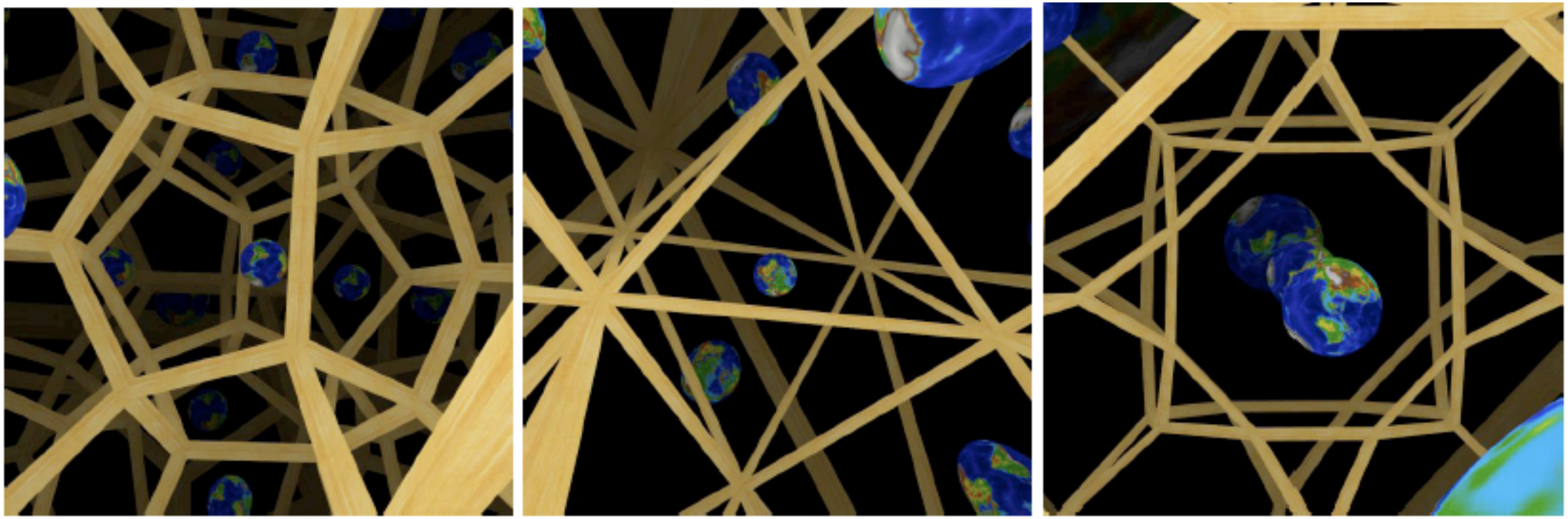
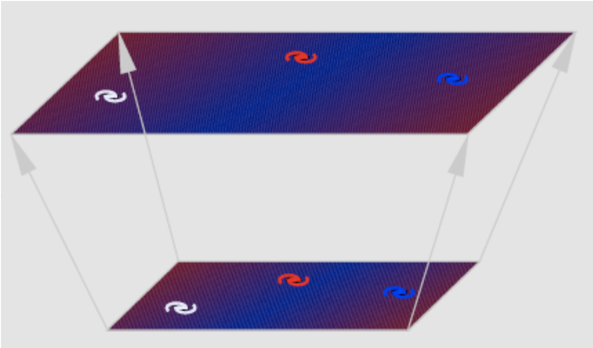


**Figure 3: The many shapes of the universe**

The Poincaré dodecahedral space (left) can be described as the interior of a "sphere" made from 12 slightly curved pentagons. However, this shape has a big difference compared with a football because when one goes out from a pentagonal face, one comes back immediately inside the ball from the opposite face after a 36° rotation. Such a multiply connected space can therefore generate multiple images of the same object, such as a planet or a photon. Other such spaces that fit the WMAP data are the tetrahedron (middle) and octahedron (right). [Credit: Jeff Weeks]

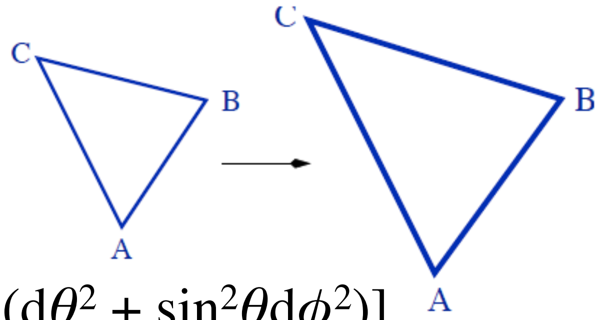
see: Luminet, arXiv:0802.2236, Phys. Rep. 254 (1995) 135

# The expanding universe (Friedmann 1922, Lemaître 1931)



Generalise line element:

$R(t) = R_0\, a(t)$




$$ds^2 = dt^2 - a^2(t)\, R_0^2\, [d\chi^2 + \sin^2\chi\, (d\theta^2 + \sin^2\theta d\phi^2)]$$

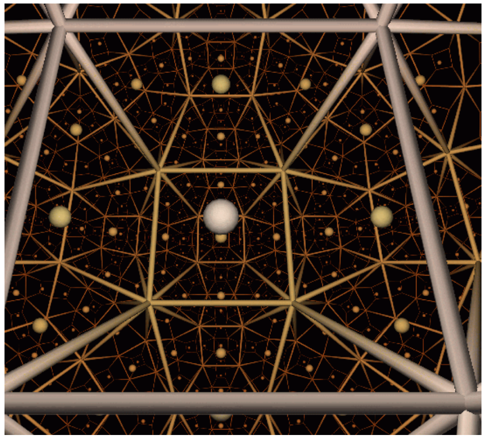
… a spatially **closed** expanding universe

To describe a spatially **open** expanding universe, change: $\chi \to i\chi$, $R_0 \to iR_0$, so

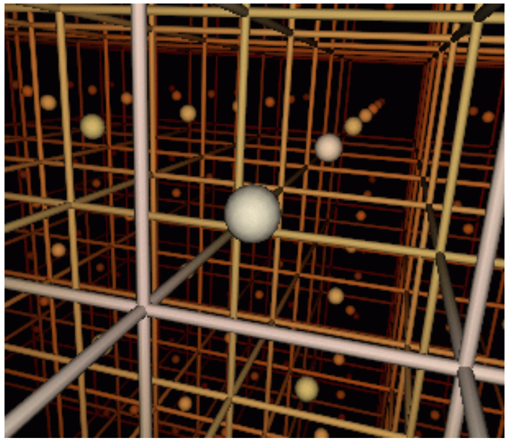
$$ds^2 = dt^2 - a^2(t)\, R_0^2\, [d\chi^2 + \sinh^2\chi\, (d\theta^2 + \sin^2\theta d\phi^2)]$$

This is the Robertson-Walker line element (*maximally-symmetric space-time*):

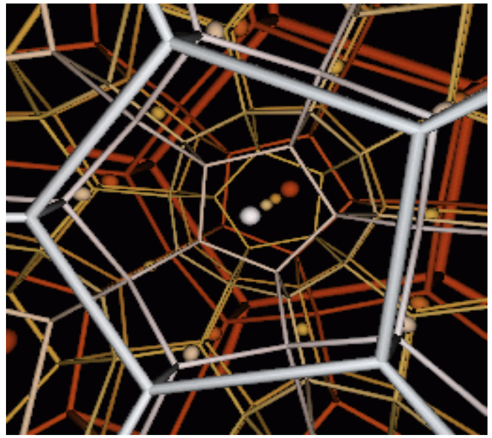
$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$



$k = -1$

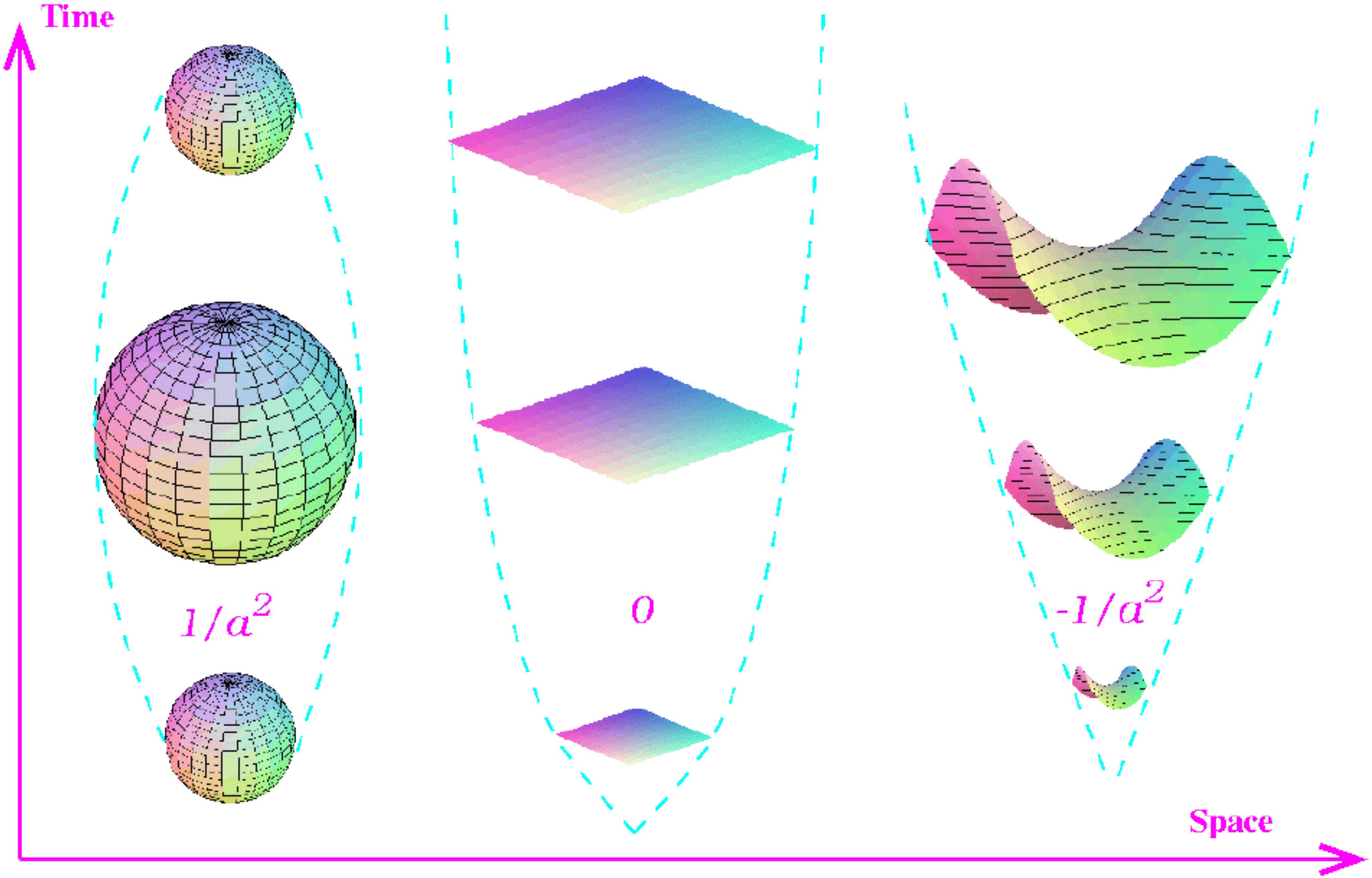


$k = 0$



$k = +1$

# Homogeneous and isotropic world models



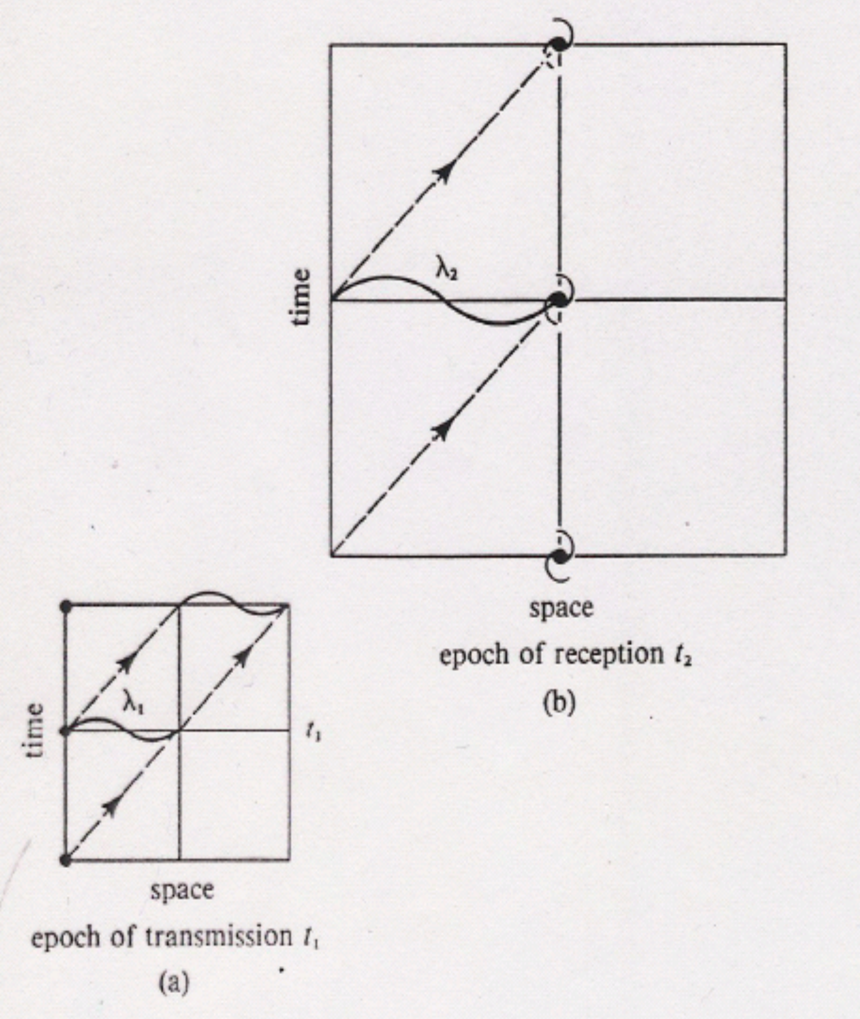
The redshift happens because, for null geodesics:

$$\int_t^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_0^r \frac{\mathrm{d}r}{\sqrt{1-kr^2}} = \text{const}$$

... for a galaxy (in co-moving coordinates), so crests of adjacent waves, separated by $\Delta t$ at emission, will be received with separation, $\Delta t_0$:

$$\frac{\Delta t_0}{\Delta t} = 1 + \frac{\Delta\lambda}{\lambda_0} \equiv 1 + z = \frac{a(t_0)}{a(t)}$$

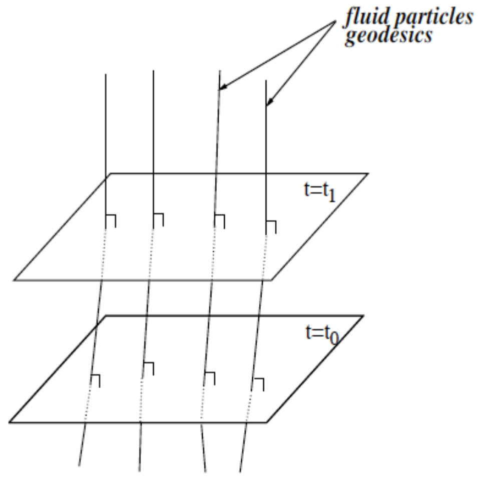
This is the cosmological time dilation or redshift - $z = \infty$ is the 'Big Bang' at $t = 0$ (the antipodal point of the hypersphere) ... the furthest we can look back in principle




Everything is not expanding (how would we know?) ... certainly not bound structures like atoms or planets or galaxies – it is only the large-scale *smoothed* space-time metric which is stretching with cosmic time (and there is no restriction on the rate!)

The 'expansion' is in a sense *illusory* ... because we can always transform to a "comoving" coordinate system where *galaxies are at rest* wrt each other

**Ideal fluid:** $T_{ij} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$




**Poisson's equation:** $\nabla . g = -4\pi G_{\mathrm{N}}(\rho + 3p)$

**Birkhoff's theorem:** If $T_{ij} = 0$ in some region within a spherically symmetric distribution of matter, then the solution in the hole $\Rightarrow$ flat space-time

## Einstein's field equations

$$R_{ij} + \tfrac{1}{2} g_{ij} R_{\mathrm{c}} = 8\pi G_{\mathrm{N}} T_{ij}, \ \text{where } R_{ij} \equiv g^{\lambda k} R_{\mu\nu\lambda k} \text{ and } R_{\mathrm{c}} \equiv g^{\mu\nu} R_{\mu\nu}$$

For the RW metric, the 00 and 11 components simplify to the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\mathrm{N}}}{3}\rho - \frac{k}{a^2}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_{\mathrm{N}}}{3}(\rho + 3p)$$

# 'Newtonian' Cosmology

Consider sphere of radius $l$ embedded in homogeneous background (McCrea & Milne 1934):

$$\ddot{\ell} = -G_{\rm N}M/r^2 = -\tfrac{4\pi}{3}G_{\rm N}(\rho + 3p)\ell; \text{ also } {\rm d}U \equiv \rho {\rm d}V + V{\rm d}\rho = -p{\rm d}V$$

$$\Rightarrow \dot{\rho} = -(\rho + p)\frac{\dot{V}}{V} = -3(\rho + p)\frac{\dot{\ell}}{\ell} \text{ ... energy eq. for ideal fluid}$$

$$\text{So, } \ddot{\ell} = \tfrac{8\pi}{3}G_{\rm N}\rho\ell + \tfrac{4\pi}{3}G_{\rm N}\dot{\rho}\frac{\ell^2}{\dot{\ell}} \Rightarrow \dot{\ell}^2 = \tfrac{8\pi}{3}G_{\rm N}\rho\ell^2 + K$$

To obtain a *static* solution (Einstein's "greatest blunder") we have to set:

$$\rho + 3p = 0 \text{ i.e. } p = -\tfrac{\rho}{3} \text{ (!)} \Rightarrow \text{ universe of radius: } \mathcal{R}^2 = -\frac{\ell^2}{k} = [\tfrac{8\pi}{3}G_{\rm N}\rho]^{-1}$$

The static solution is in fact *unstable* (metric perturbations grow exponentially fast) but we do *not* have the freedom, as Einstein said, to "do away with the cosmological constant" ... it is a *necessary* consequence of **general coordinate invariance** which allows any *arbitrary* constant multiplying the metric tensor to be added to the l.h.s.

So must modify the field equations to: $$R_{ij} + \tfrac{1}{2}g_{ij}R_{\rm c} - \Lambda g_{ij} = 8\pi G_{\rm N}T_{ij}$$

... which can be *interpreted* (when moved to r.h.s.) as a fluid with: $\varrho_\Lambda = -p_\Lambda = \Lambda/8\pi G_{\rm N}$

# FRW Dynamics

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\rm N}}{3}(\rho + 3p) \pm \frac{1}{a^2\mathcal{R}^2} \to -\frac{4\pi G_{\rm N}}{3}(\rho_{\rm b} + 3p_{\rm b}) \pm \frac{1}{a^2\mathcal{R}^2} + \frac{\Lambda}{3}$$

b ⇒ 'background' (i.e. "ordinary" matter/radiation)

Conservation of energy-momentum: $\dot{\rho}_{\rm b} = -3(\rho_{\rm b} + p_{\rm b})\frac{\dot{a}}{a}$

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G_{\rm N}}{3}\rho_{\rm b} \pm \frac{1}{a^2\mathcal{R}^2} + \frac{\Lambda}{3}, \quad \text{where + is open/- is closed universe}$$

Two interesting solutions describing an expanding universe:

## Einstein – de Sitter:

$$p_{\rm b} \ll \rho_{\rm b}, \Lambda = \frac{1}{a^2\mathcal{R}^2} = 0 \Rightarrow a(t) \propto t^{2/3}, t = \frac{2}{3H} = \frac{1}{\sqrt{6\pi G_{\rm N}\rho}}$$

## de Sitter:

$$\rho_{\rm b} = p_{\rm b} = 0 \Rightarrow a(t) = \exp\left(H_\Lambda t\right), \text{ where } H_\Lambda = \sqrt{\frac{\Lambda}{3}}$$

The de Sitter universe was "motion without matter" (violating Mach's Principle!) as opposed to Einstein's static universe which was "matter without motion" de Sitter (1917) presented a third *static* solution:

$$\mathrm{d}s^2 = \left(1 - \frac{r^2}{\mathcal{R}^2}\right)\mathrm{d}t^2 - \mathrm{d}r^2 / \left(1 - \frac{r^2}{\mathcal{R}^2}\right) - r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)$$

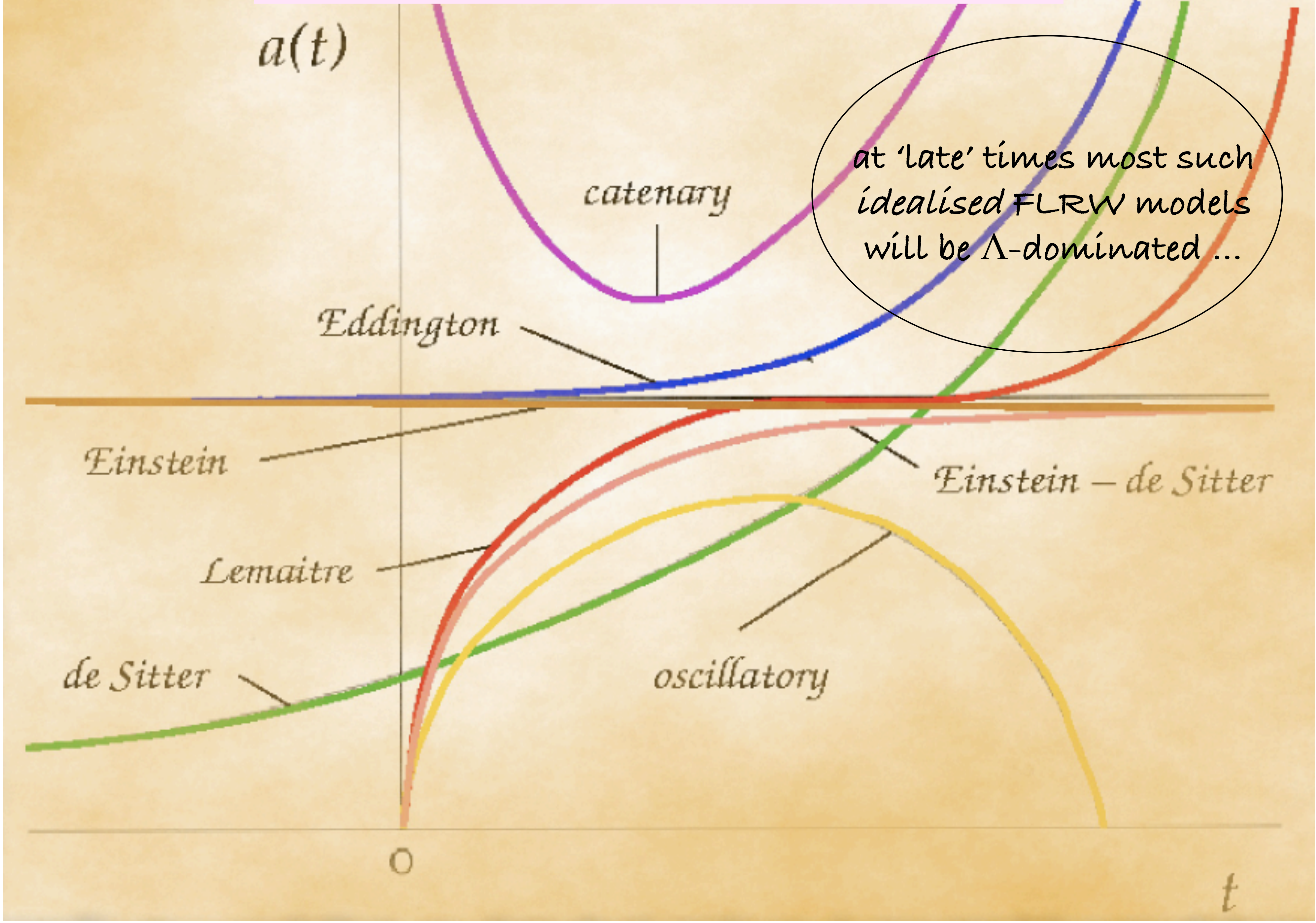
For a clock at rest at a particular point ($\mathrm{d}r = \mathrm{d}\theta = \mathrm{d}\phi = 0$), the time-like interval, $\mathrm{d}s^2 = \mathrm{d}t^2 (1 - r^2/R^2)$, now depends on the radial distance … becoming smaller as $r$ increases ⇒ redshift of light from distant sources, but with:

$$\frac{\mathrm{d}t}{\mathrm{d}t_0} = \sqrt{1 - \frac{r^2}{\mathcal{R}^2}} = \frac{\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0} \Rightarrow z \simeq \frac{1}{2}\frac{r^2}{\mathcal{R}^2}, \text{ for } r \ll \mathcal{R}$$

But Sitter showed later (1933) that the redshift-distance relationship is in fact linear (as it *should* be for any *homogeneous* space-time) since observers in this space are *accelerating* … but meanwhile observers (Stromberg, Lundmark, Wirtz, Silberstein etc) were misled into looking for the "de Sitter effect" and even Hubble (1929) tried to fit the redshift-distance data to a *quadratic* relationship!

Later Hubble (1931) wrote: *"The interpretation, we feel, should be left to you and the very few others who are competent to discuss the matter with authority"* … today's observers would be wise to take note of this when they talk freely about indications from the data for 'phantom energy', 'quintessence' and other such phenomena which have no natural basis in fundamental physics!

# A child's garden of cosmological models



The R-W metric does *not* reduce to the Minkowski form when $r \rightarrow \infty$ (*cf.* the Schwarzchild metric), however when written in terms of the **conformal time** $\mathrm{d}\eta = \mathrm{d}t/a(t)$, **it is globally conformal to the Minkowski metric** (for $k = 0$):

$$\mathrm{d}s^2 = a^2(\eta)\left[\mathrm{d}\eta^2 - \mathrm{d}r^2/(1-kr^2) - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right]$$

This is (relatively) easy to work with, however should we not consider less symmetric metrics which describe our (inhomogeneous) universe better?

The problem is that *very few* exact cosmological solutions are known … so we tend to use 'toy models' rather than attempt a more realistic description

For example, a less symmetric possibility is the Lemaitre-Tolman-Bondi metric describing an universe that is *inhomogeneous but isotropic* around our position

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\left[\left(1 + \frac{r}{a}\frac{\partial a}{\partial r}\right)^2 \mathrm{d}r^2/(1-k(r)r^2) + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right]$$

This requires us to be in a special position (i.e. exactly at the centre of a radial inhomogeneity as specified by $k(r)$ (e.g. a void) but completely changes the interpretation of the data (e.g. no need then for having $\Lambda \neq 0$)

## Using the RW metric we can define *observational quantities* to be measured

... expand in Taylor series

$$\frac{a(t)}{a(t_0)} = 1 + \underbrace{H_0}_{\equiv \dot{a}(t_0)/a(t_0)} (t-t_0) - \frac{1}{2} \underbrace{q_0}_{\equiv -\ddot{a}(t_0)\, a(t_0)/\dot{a}^2(t_0)} H_0^2 (t-t_0)^2 + \cdots$$

... invert to obtain

$$z = H_0 (t_0 - t) + (1 + q_0/2) H_0^2 (t_0 - t)^2 + \cdots$$

i.e. $$(t_0 - t) = H_0^{-1} \left[ z - (1 + q_0/2) z^2 + \cdots \right]$$

$\Rightarrow$ co-ordinate distance: $$r_e = a^{-1}(t_0) H_0^{-1} \left[ z - \frac{1}{2}(1+q_0) z^2 + \cdots \right]$$

using $$\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_0^{r_e} \frac{dr}{(1-kr^2)^{1/2}} = \begin{cases} \sin^{-1} r_e, & k=+1 \\ r_e, & k=0 \\ \sinh^{-1} r_e, & k=-1 \end{cases}$$

$\Rightarrow$ 'Hubble law': $$H_0 \underbrace{d_L}_{\equiv a^2(t_0)\, r_e^2 (1+z)^2} = z + \frac{1}{2}(1-q_0) z^2 + \cdots$$

In an expanding universe described by the R-W metric, the apparent luminosity $\ell$ of a source of intrinsic luminosity $L$ is :
$$\ell = \frac{L}{4\pi a_0^2 r^2 (1+z)^2}$$

Since $a(t)$ is dynamically determined by the F-L equations, this yields (Mattig 1958):

$$a_0 r = \frac{c}{H_0 q_0^2 (1+z)} \left\{ q_0 z + (q_0 - 1)\left[(1+2q_0 z)^{1/2} - 1\right]\right\}$$

for $q_0 > 0$, where $H_0 \equiv \dot{a}_0/a_0$ and $q_0 \equiv -\ddot{a}_0/a_0 H_0^2$

This gives the intrinsic luminosity as
$$L = 4\pi \ell c^2 H_0^{-2} q_0^{-2} \left\{ q_0 z + (q_0 - 1)\left[(1+2q_0 z)^{1/2} - 1\right]\right\}$$

implying the magnitude-redshift relationship

$$m = 5 \log q_0^{-2} \left[ z q_0 + (q_0 - 1)\left\{ -1 + (2 z q_0 + 1)^{1/2} \right\} \right] + C$$

$$\simeq 5 \log z + 1.086 (1 - q_0 z) + O(z^2) \ldots$$

for $z \lesssim 0.3$

$= 2.5 \log 4\pi + 5 \log (c/H_0)$

Rewriting Friedmann's equation as

$$\left(\frac{da}{d\tau}\right)^2 = 1 + \underbrace{\Omega_m}_{\equiv \frac{8\pi G}{3H_0^2}\rho_{m_0}} (a^{-1} - 1) + \underbrace{\Omega_\Lambda}_{\equiv \frac{\Lambda}{3H_0^2}} (a^2 - 1) \ ; \quad a \equiv \frac{1}{1+z}, \ \tau \equiv H_0 t$$

we see that: $q_0 = \frac{\Omega_m}{2} - \Omega_\Lambda$

... so measurement of the present expansion rate and its rate of change yields, in principle, the dynamical parameters

This programme was however *unsuccessful* because a complete understanding of **evolutionary effects** is essential to determine cosmological parameters

⇒ galaxy counts:
$$\frac{dN_{gal}}{dz\,d\Omega} = \frac{n_c(z)}{H_0^3 a_0^3 (1+z)^3 q_0^4} \frac{\left\{ zq_0 + (q_0-1)\left(\sqrt{2q_0 z+1}-1\right)\right\}^2}{\left[1-2q_0+2q_0(1+z)\right]^{1/2}}$$

⇒ angular diameter:
$$H_0 d_A = \frac{1}{q_0^2(1+z)^2}\left[zq_0 + (q_0-1)\left(\sqrt{2q_0 z+1}-1\right)\right]$$

$$\simeq z - \frac{1}{2}(3+q_0)z^2 + \cdots$$

$$\equiv \frac{D}{\delta} = a(t_e)r_e$$

... many other tests (surface brightness ...)

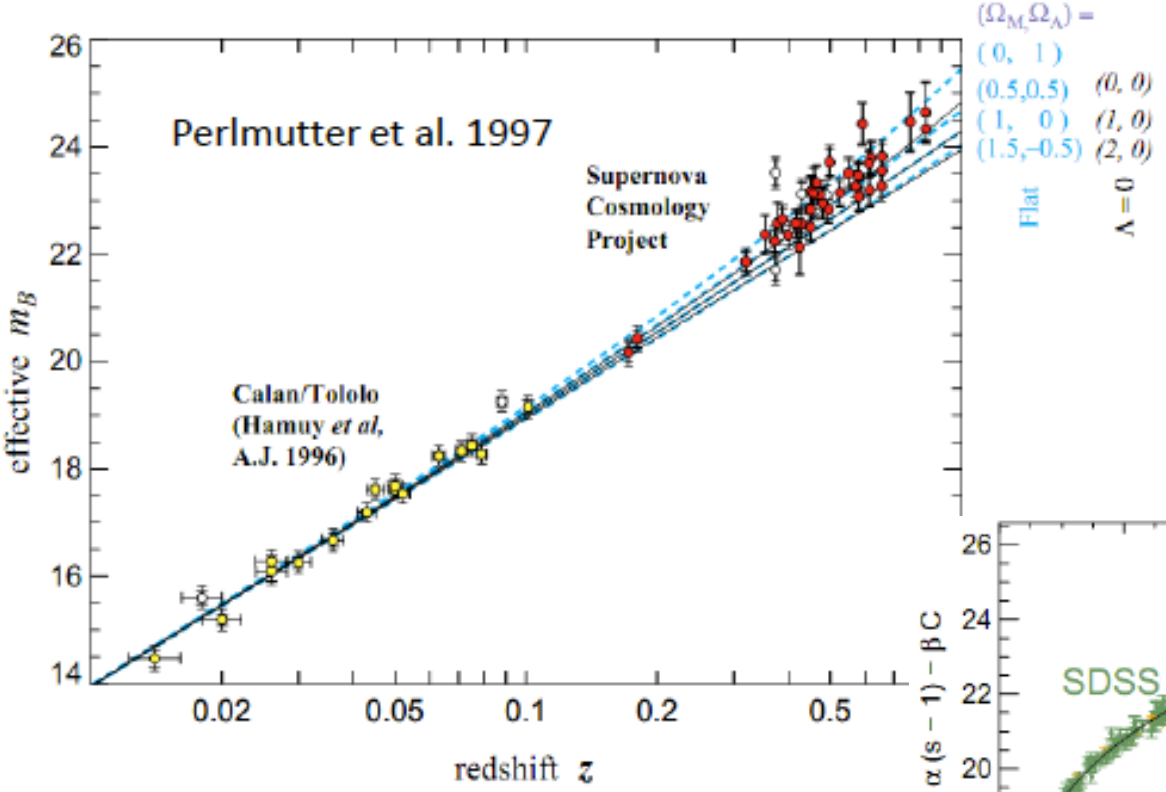
but all subject to bias by evolutionary effects

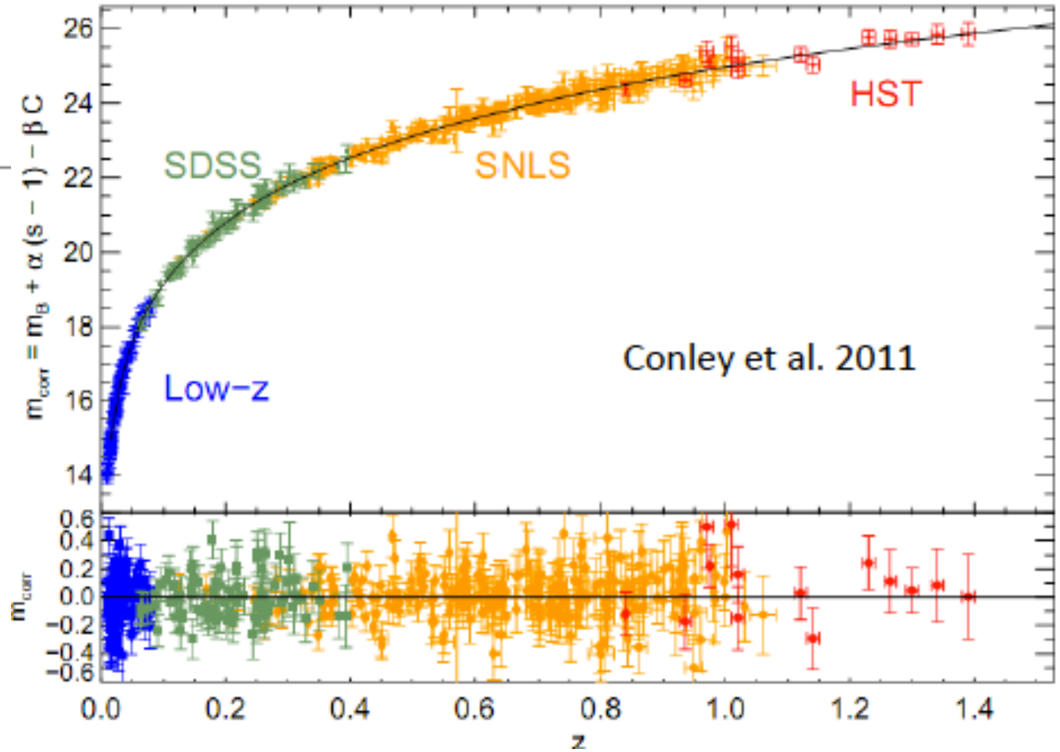
e.g. if $L(t) = L_0[1+\alpha(t-t_0)]$

then $l = \frac{L}{4\pi}\left(\frac{H_0}{z}\right)^2\left[1+(q_0-1)z - \alpha H_0^{-1}z + \cdots\right]$

... so will obtain *wrong* answer for $q_0$ if unaware of possible luminosity evolution

Recently it has become possible to routinely detect SNIa in distant galaxies and use them as 'standard candles' to trace the Hubble expansion out to $z \sim 1$

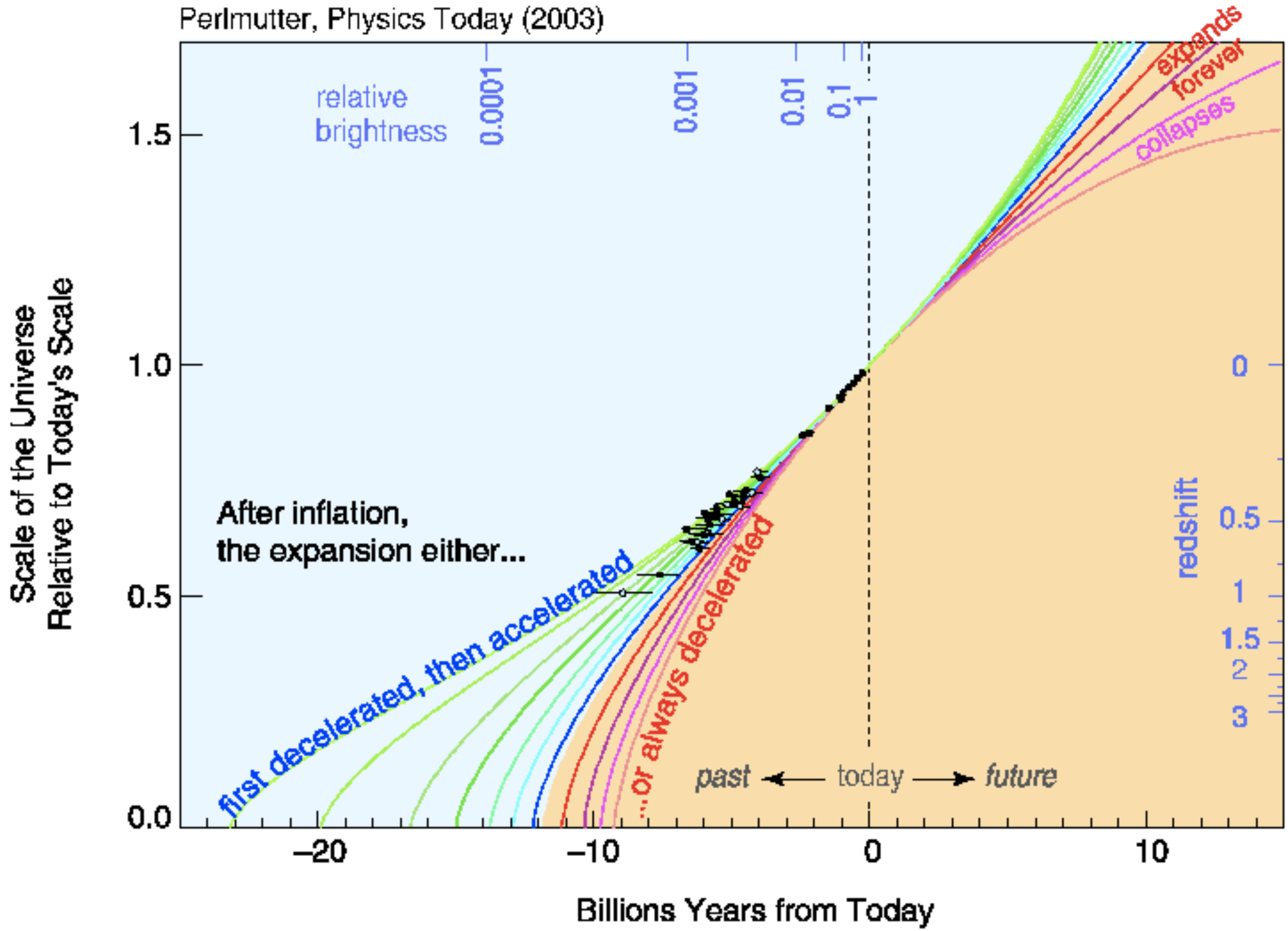


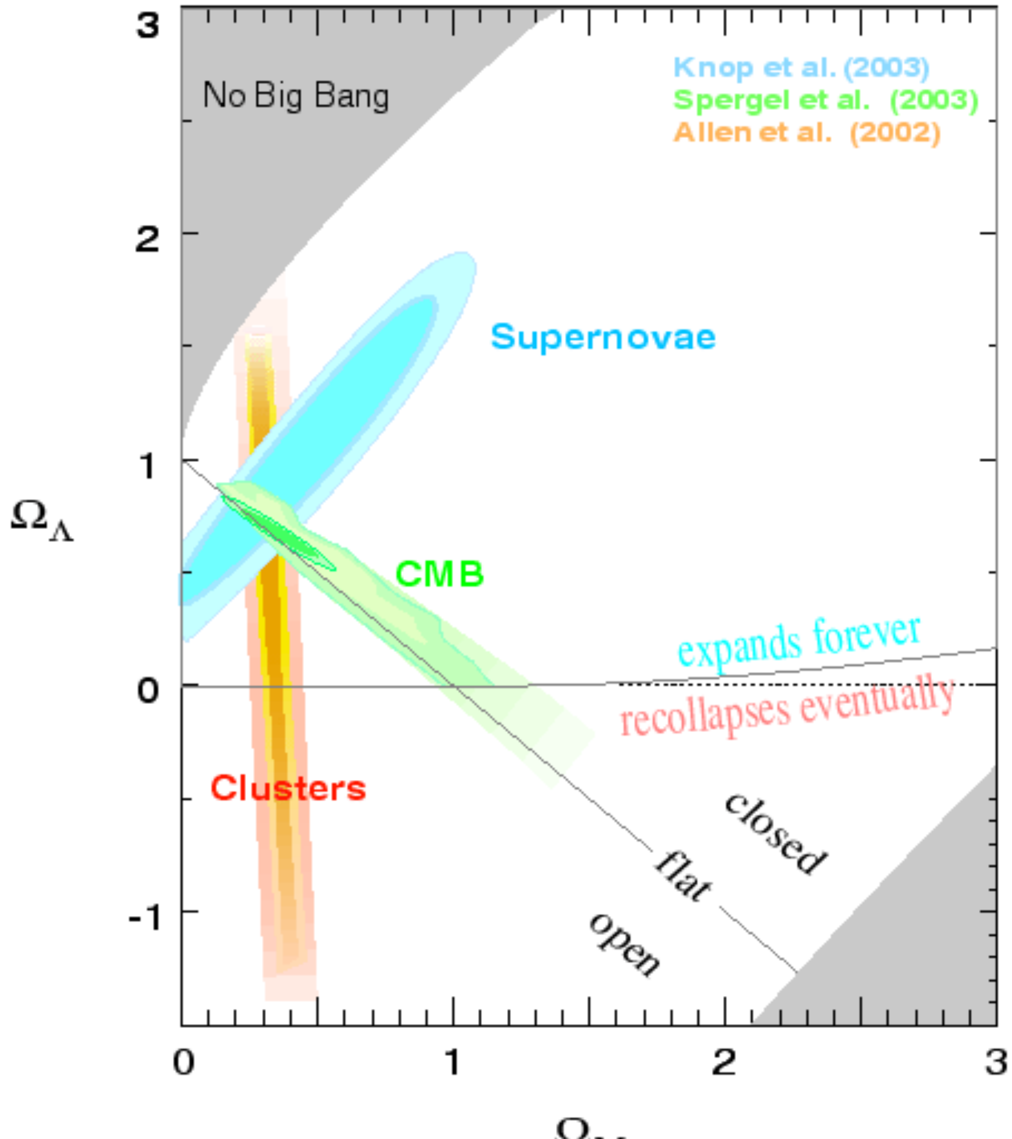

These observations have been interpreted to mean that the expansion rate is *accelerating*

# Expansion History of the Universe

Perlmutter, Physics Today (2003)



All such *geometric* measurements imply a Cosmological Constant today with $\Lambda \sim H_0^2$ … but this has yet to be confirmed by *dynamical* measurements




The Cosmological Constant has been *interpreted* as 'dark energy' but so far there is no definitive evidence for its *negative* pressure

# Conclusions

There has been a renaissance in cosmology but modern data is still interpreted in terms of an *idealised* model whose basic assumptions have not been rigorously tested

**The standard FRW model naturally admits $\Lambda \sim H_0^2$ ...**
**and this is being *interpreted* as dark energy: $\Omega_\Lambda \sim H_0^2 M_P^2$**

More realistic models of our *inhomogeneous* universe may account for the SNIa Hubble diagram without acceleration

(NB: The CMB and LSS data can be equally well fitted if the primordial perturbations are *not* scale-free and $m_\nu \sim 0.5$ eV)

**Dark energy may just be an artifact of an oversimplified cosmological model**