Light Stringy States

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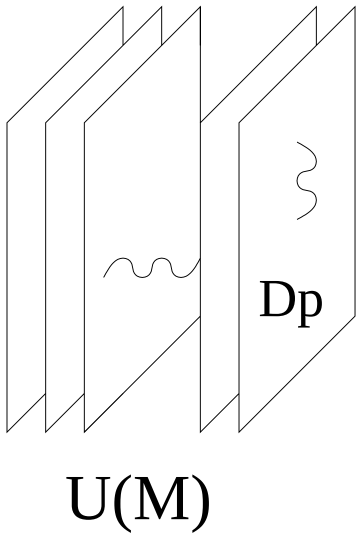
D-branes

- non-perturbative solutions to Type II string theory
- a hypersurface where open strings (gauge bosons) can end
- M D-branes on top of each other give rise to $\mathcal{N} = 4$ U(M) gauge symmetry

$\rightsquigarrow$ non-chiral $\rightsquigarrow$ not realistic



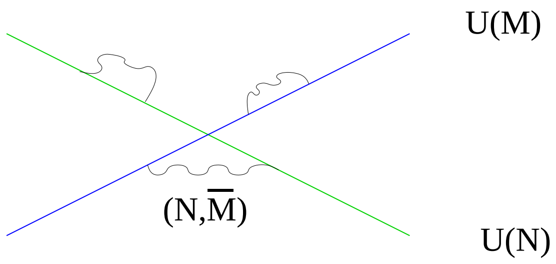
To get chirality:

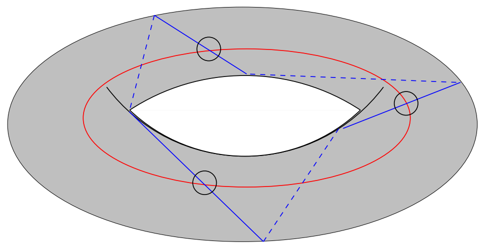
- intersecting D-branes
- magnetized D-branes
- D-branes at singularities

Intersecting D6-branes

D6-branes fill out spacetime and wrap three-cycles Π_a in CY_3

- Gauge group: a stack of M D-branes gives rise to $U(M) = SU(M) \times U(1)$ gauge theory in 4D
- Chirality: at each intersection of two D6-branes appears a massless fermion, transforming as $(N, \overline{M})$
- Family Replication: In the compact space two stacks of D6-branes may intersect multiple times





Bottom-up approach

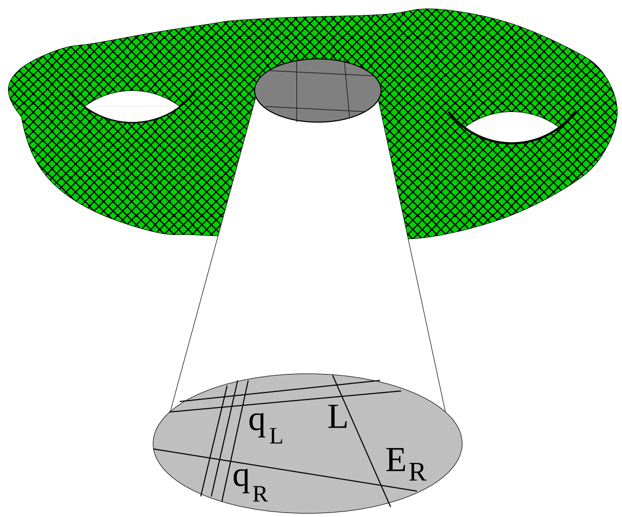
Antoniadis, Kiritsis, Tomaras hep-ph/0004214,

Aldazabal, Ibanez, Quevedo, Uranga hep-th/0005067

D-branes allow for a bottom-up model building approach

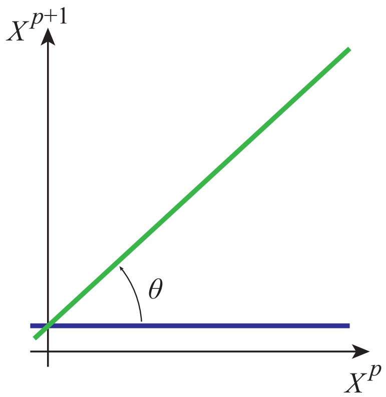
- Local models: considers a local set of D6-branes, which are localized at some region of the CY_3
- does not care about global aspects of compactification and assumes that the local setup can be embedded eventually

- global construction more satisfying, but local setups more efficient and also sufficient to address various phenomenological questions



States localized at intersecting D6-branes

$$\partial_\sigma X^p(\tau,0) = 0 = X^{p+1}(\tau,0)$$
$$\partial_\sigma X^p(\tau,\pi) + \tan(\pi\theta_I)\ \partial_\sigma X^{p+1}(\tau,\pi) = 0$$
$$X^{p+1}(\tau,\pi) - \tan(\pi\theta_I)\ X^p(\tau,\pi) = 0$$



Solution $Z^I = X^p + iX^{p+1}$

$$\partial Z^I(z) = \sum \alpha^I_{n-\theta_I} z^{-n+\theta_I-1} \qquad \partial\overline{Z}(z) = \sum \alpha^I_{n+\theta_I} z^{-n-\theta_I-1}$$

Solution $\Psi^I = \Psi^p + i\Psi^{p+1}$

$$\Psi^I(z) = \sum\ \psi^I_{r-\theta_I}\, z^{-r-\frac{1}{2}+\theta_I} \qquad \overline{\Psi}^I(z) = \sum\ \psi^I_{r+\theta_I}\, \bar{z}^{-r-\frac{1}{2}-\theta_I}$$

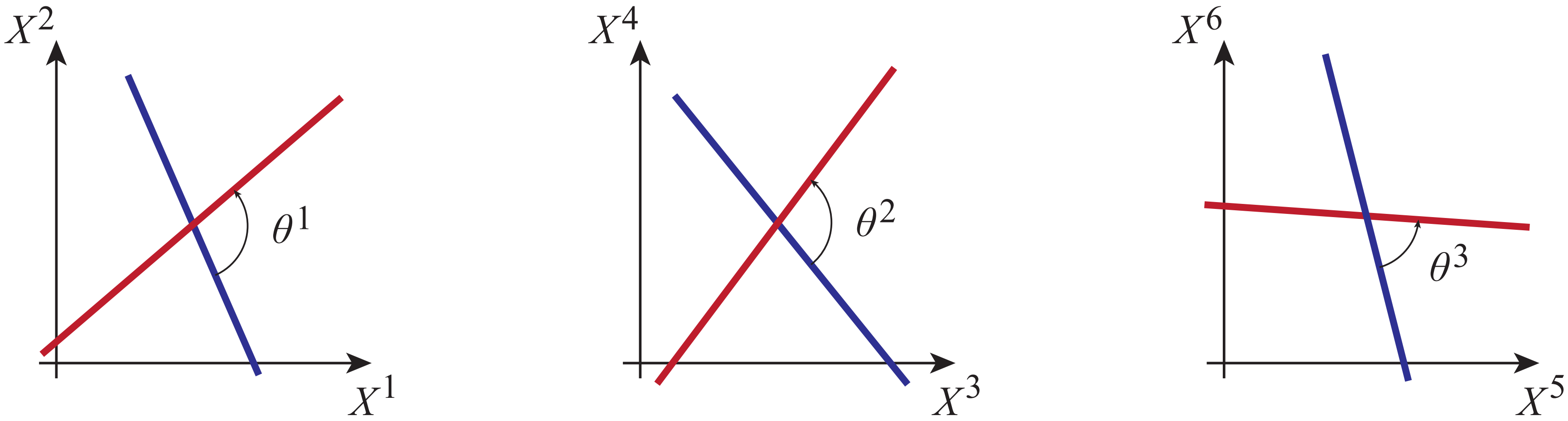
NS-sector: r half-integer R-sector: r integer

Quantization:

$$[\alpha^I_{n\pm\theta}, \alpha^{I'}_{m\mp\theta}] = (m\pm\theta)\,\delta_{n+m}\,\delta^{II'}$$
$$\{\psi^I_{m-\theta_I}, \psi^I_{n+\theta_I}\} = -\delta_{m,n}$$

States localized at intersecting D6-branes

R-sector: fermions in 4 D
NS-sector: bosons in 4 D



all angles positive: $\theta_1 + \theta_2 + \theta_3 = 2$

R-sector

$$\alpha_{m-\theta_I}|0\rangle = 0 \qquad m \geq 1 \qquad \psi_{r-\theta_I}|0\rangle = 0 \qquad r \geq 1$$
$$\alpha_{m+\theta_I}|0\rangle = 0 \qquad m \geq 0 \qquad \psi_{r+\theta_I}|0\rangle = 0 \qquad r \geq 0$$

- massless fermion corresponding to the vacuum $|0\rangle$

States localized at intersecting D6-branes

NS-sector

$$\alpha_{m-\theta_I}|0\rangle = 0 \qquad m \geq 1 \qquad\qquad \psi_{r-\theta_I}|0\rangle = 0 \qquad r \geq \tfrac{1}{2}$$

$$\alpha_{m+\theta_I}|0\rangle = 0 \qquad m \geq 0 \qquad\qquad \psi_{r+\theta_I}|0\rangle = 0 \qquad r \geq \tfrac{1}{2}$$

Lightest states:

$$\psi_{-\frac{1}{2}+\theta_I}\,|\,0\,\rangle \qquad\qquad M^2 = \tfrac{1}{2}\left(-\theta_I + \textstyle\sum_{J\neq I}\theta_J\right) M_s^2$$

$$\textstyle\prod_{I=1}\psi_{-\frac{1}{2}+\theta_I}|\,0\,\rangle \qquad\qquad M^2 = \left(1 - \tfrac{1}{2}\left(\theta_1+\theta_2+\theta_3\right)\right) M_s^2$$

Additional states:

$$\alpha_{\theta_1}\textstyle\prod_{I=1}\psi_{-\frac{1}{2}+\theta_I}|\,0\,\rangle \qquad\qquad M^2 = \left(1 - \tfrac{1}{2}\textstyle\sum_I\theta_I + \theta_1\right) M_s^2$$

$$\left(\alpha_{\theta_1}\right)^2\textstyle\prod_{I=1}\psi_{-\frac{1}{2}+\theta_I}|\,0\,\rangle \qquad\qquad M^2 = \left(1 - \tfrac{1}{2}\textstyle\sum_I\theta_I + 2\theta_1\right) M_s^2$$

$\rightsquigarrow$ potentially fairly light

Any signals of these potential light states?

Amplitude

Compute the amplitude: $\langle\overline{\psi}(0)\psi(x)\chi(1)\overline{\chi}(\infty)\rangle$

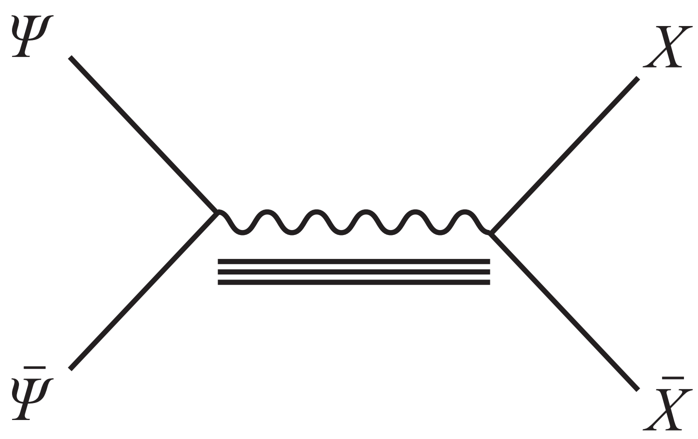
Challenges:

- Knowledge of the vertex operators for arbitrary angles
- bosonic twist field correlator

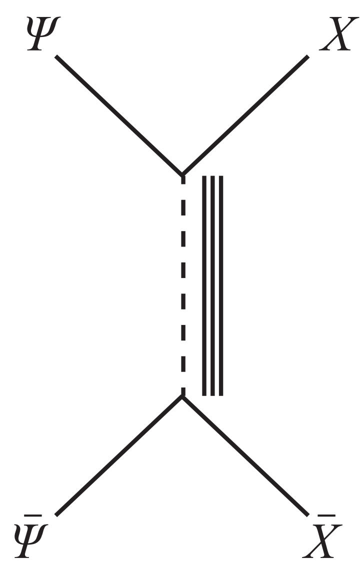
  $\langle\sigma_\theta(0)\sigma_{1-\theta}(x)\sigma_\nu\sigma_{1-\nu}(\infty)\rangle$

Expectations:

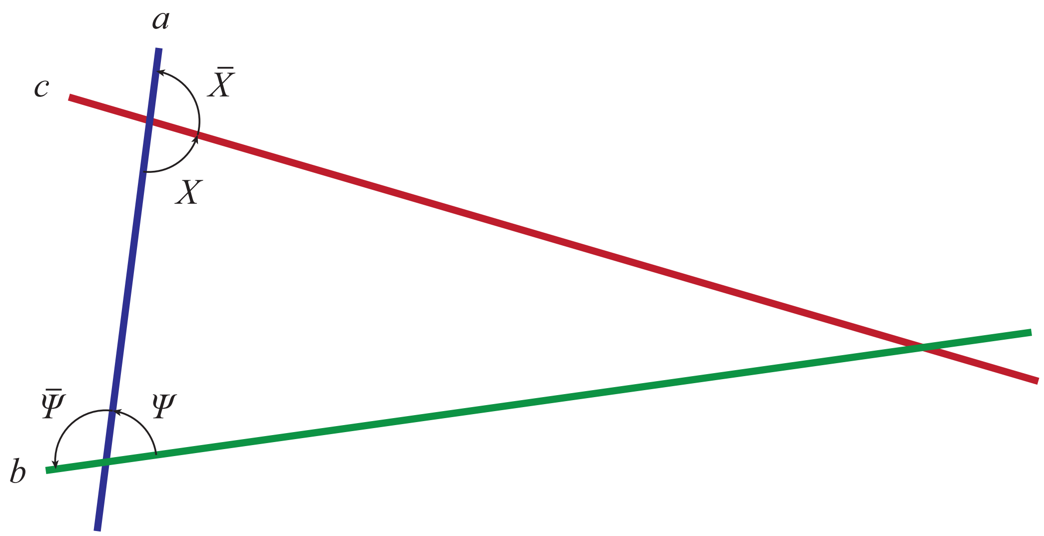
s-channel



t-channel



Concrete Setup



Crucial: interplay of transformation behaviour and angle

$$
\begin{array}{cccc}
\theta^1_{ab} > 0 & \theta^2_{ab} > 0 & \theta^3_{ab} < 0 & \sum_I \theta^I_{ab} = 0 \\
\theta^1_{bc} > 0 & \theta^2_{bc} > 0 & \theta^3_{bc} < 0 & \sum_I \theta^I_{bc} = 0 \\
\theta^1_{ca} < 0 & \theta^2_{ca} < 0 & \theta^3_{ca} < 0 & \sum_I \theta^I_{ca} = -2
\end{array}
$$

Vertex operators

$$V_\psi^{-\frac{1}{2}} = \Omega_{ab} \psi^\alpha \, e^{-\frac{\varphi}{2}} S_\alpha \prod_{I=1}^2 \sigma_{\theta^I_{ab}} e^{i(\theta^I_{ab} - \frac{1}{2}) H_I} \, \sigma_{1+\theta^3_{ab}} e^{i(\theta^3_{ab} + \frac{1}{2}) H_3} \; e^{ikX}$$

$$V_{\overline{\psi}}^{-\frac{1}{2}} = \Omega_{ba} \overline{\psi}_{\dot\alpha} S^{\dot\alpha} \prod_{I=1}^2 \sigma_{1-\theta^I_{ab}} e^{i(-\theta^I_{ab} + \frac{1}{2}) H_I} \, \sigma_{-\theta^3_{ab}} e^{i(-\theta^3_{ab} - \frac{1}{2}) H_3} \; e^{ikX}$$

Four bosonic open twist correlator

1st approach

[Cvetic, Papadimitriou], [Abel, Owen]

- extend via the "doubling trick" the upper half plane to the whole complex plane
- quantum part is computed via energy-momentum tensor method

  $\rightsquigarrow$ analogous to the closed string
- classical part given by the sum over all quadrangles $e^{-\sum \frac{Area}{2\pi\alpha'}}$

  $\rightsquigarrow$ Result is in Lagrangian Form

$$x^{-\theta(1-\theta)}(1-x)^{-\theta(1-\nu)} I^{-\frac{1}{2}}(\theta,\nu,x)$$

$$\times \sum_{\widetilde{p},q} \exp\left[-\pi \frac{\sin(\pi\theta)\, L_a^2}{t(x)}\, \widetilde{p}^2 - \pi \frac{t(x)\, R_1^2\, R_2^2}{\sin(\pi\theta)\, L_a^2}\, q^2\right]$$

Four bosonic open twist correlator

2nd approach

Derive open string result directly from the closed string result

$$\mathcal{A}_{closed} = |K(z)|^2 \sum_{\vec{k},\vec{v}} c_{\vec{k}\vec{v}} w(z)^{\frac{\alpha' p_L^2}{4}} \; \overline{w}(\overline{z})^{\frac{\alpha' p_R^2}{4}}$$

Open string result $\rightsquigarrow$ holomorphic part

$$\mathcal{A}_{open} = K(x) \sum_{p,q} c_{pq} w(x)^{\alpha' p_{open}^2}$$

with $p_{open}^2 = \frac{1}{L^2} p^2 + \frac{1}{\alpha'^2} \frac{R_1^2 R_2^2}{L^2} q^2$

coefficients c_{pq} are fixed in such a way to match known limits

$$\frac{x^{-\theta(1-\theta)}(1-x)^{-\theta(1-\nu)}}{G_1(x)} \sum_{p,q} w(x)^{\left(\frac{\alpha'}{L_a^2} p^2 + \frac{1}{\alpha'} \frac{R_1^2 R_2^2}{L_a^2} q^2\right)}$$

Result in Hamiltonian form

$\rightsquigarrow$ after Poisson resum matching results

Closed string result

[Burwick, Kaiser and Müller], [Stieberger, Jungnickel, Lauer, Spalinski]

$$|z|^{-2\theta(1-\theta)}\,(1-z)^{-\nu(1-\theta)}\,(1-\overline{z})^{-\theta(1-\nu)}\,I^{-1}(\theta,\,\nu,\,z)\;\sum_{v_1,v_2} e^{-S^{cl}}$$

$$I(\theta,\nu,z)=\tfrac{1}{2\pi}\left[B_1(\theta,\nu)\,G_2(z)\overline{H}_1(1-\overline{z})+B_2(\theta,\nu)\,\overline{G}_1(\overline{z})H_2(1-z)\right]$$

$$S^{cl}=V_{11}v_1\overline{v}_1+V_{12}v_1\overline{v}_2+V_{21}^{*}v_2\overline{v}_1+V_{22}v_2\overline{v}_2$$

$$\begin{aligned}V_{11}=\frac{1}{4}\left(\frac{\sin(\pi\theta)}{\pi}\right)^2|I(z,\overline{z})|^{-2}\\
&\Big(\,B_2\big|H_2(1-z)\big|^2\Big[B_1G_1(z)\overline{H}_1(1-\overline{z})+B_1\overline{G}_1(\overline{z})H_1(1-z)\\
&\qquad+\pi\left(\cot(\pi\nu)-\cot(\pi\theta)\right)|G_1(z)|^2\Big]\\
&+B_1\big|H_1(1-z)\big|^2\Big[B_2G_2(z)\overline{H}_2(1-\overline{z})+B_2\overline{G}_2(\overline{z})H_2(1-z)\\
&\qquad+\pi\left(\cot(\pi\theta)-\cot(\pi\nu)\right)|G_2(z)|^2\Big]\Big)\end{aligned}$$

...

Closed string result

$$\frac{1}{\sin(\pi\theta)\tau_2(z,\overline{z})} \sum_{k\in\Lambda^*, v\in\Lambda} e^{i\pi\vec{k}^T B\vec{v}} e^{-2\pi i\vec{f}_{23}\cdot\vec{k}} \; w(z)^{\frac{(k+v/2)^2}{2}} \; \overline{w}(\overline{z})^{\frac{(k-v/2)^2}{2}}$$

$$w(z) = \exp\left[i\frac{\pi\tau(z)}{\sin(\pi\theta)}\right]$$

$$\tau(z) = i\frac{\sin(\pi\theta)}{2\,\pi}\left(\frac{B_1 H_1(1-z)}{G_1(z)} + \frac{B_2 H_2(1-z)}{G_2(z)}\right)$$

Back to the Amplitude

$$\mathcal{A} \sim Tr\,(\Omega_{ba}\,\Omega_{ab}\,\Omega_{bc}\,\Omega_{cb})\,\overline{\psi}\cdot\overline{\chi}\,\psi\cdot\chi\int_0^1 dx x^{-1+k_1\cdot k_2}\,(1-x)^{-\frac{3}{2}+k_2\cdot k_3}$$

$$\times\left[I(\theta^1_{ab},1-\theta^1_{bc},x)\,I(\theta^2_{ab},1-\theta^2_{bc},x)\,I(1+\theta^3_{ab},-\theta^3_{bc},x)\right]^{-\frac{1}{2}}e^{-S_{cl}}$$

in s-channel observes the gauge boson exchange

$\rightsquigarrow$ normalization of the amplitude

also higher string excitations, and KK and Winding states

[Lüst, Stieberger, Taylor]

mass of higher string excitations scale with M_s

mass of KK and Winding states

$\rightsquigarrow$ very model dependent

Back to the Amplitude

in t-channel (θ^1_{ca} is small)

$$\mathcal{A} \sim \overline{\psi} \cdot \overline{\chi}\, \psi \cdot \chi \int_{1-\epsilon}^{1} dx\, (1-x)^{-1+t}\; Y^2_{\psi\chi\Phi} \left(1 + c_1 (1-x)^{2\theta^1_{ca}} + ..\right)$$

$$Y_{\psi\chi\Phi} \sim \Gamma^{-\frac{1}{4}}_{1-\theta^1_{ab},1-\theta^1_{bc},\theta^1_{ab}+\theta^1_{bc}}\; \Gamma^{-\frac{1}{4}}_{1-\theta^2_{ab},1-\theta^2_{bc},\theta^2_{ab}+\theta^2_{bc}}\; \Gamma^{-\frac{1}{4}}_{-\theta^3_{ab},-\theta^3_{bc},2+\theta^3_{ab}+\theta^3_{bc}}$$

$$\times \prod_{I=1}^{2} \exp\left[-2\pi \frac{\sin(\pi\theta^I_{ab})\, \sin(\pi(1-\theta^I_{bc}))}{\sin(\pi(\theta^I_{ab}+\theta^I_{bc}-1))}\; L^2_{a^I} \widetilde{p}^2_I\right] \exp\left[-2\pi \frac{\sin(\pi(1+\theta^3_{ab}))\, \sin(-\pi\theta^3_{bc})}{\sin(\pi(1+\theta^3_{ab}+\theta^3_{bc})}\; L^2_{a^3} \widetilde{p}^2_3\right]$$

[Cvetic, Papadimitriou],[Abel,Owen], [Lüst, Mayr, Stieberger,R. R.]

highest pole: exchange of massless scalar

subdom. pole: exchange of massive scalar $M^2 = -2\theta^1_{ca}$

- this is the state $\left(\alpha_{-\theta^1_{ca}}\right)^2 \prod_{I=1} \psi_{-\frac{1}{2}-\theta^I_{ca}} \,|\, 0\, \rangle$

$$V^{-1}_{\Psi_{\omega_1}} = \Omega_{ac}\, \Psi_{\omega_1}\, e^{-\varphi}\;\; \rho_{1+\theta^1_{ca}}\; e^{i(1+\theta^1_{ca})H_1}\; \prod_{I=2}^{3} \sigma_{1+\theta^I_{ca}}\; e^{i(1+\theta^I_{ca})H_I}\;\; e^{ikX}$$

next leading poles are even graded

$\rightsquigarrow$ massive exchanges $\;M^2 = 2N\theta^I_{ca} M^2_s$

Bosonic twist field

Closed string

$$\sigma_\theta(z,\overline{z})\,\sigma_{1-\nu}(w,\overline{w}) \sim \; C_\sigma \; |z-w|^{-2\nu(1-\theta)}\,\sigma_{\theta-\nu}\,(w,\overline{w})$$
$$+C_\tau \; |z-w|^{-2\nu(2-\theta)+2\theta}\tau_{\theta-\nu}\,(w,\overline{w}) + \ldots$$

Open string

$$\sigma_\theta(z)\,\sigma_{1-\nu}(w) \sim \; \widetilde{C}_\sigma \; (z-w)^{-\nu(1-\theta)}\,\sigma_{\nu-\theta}\,(w,\overline{w})$$
$$+\widetilde{C}_\rho \; (z-w)^{-\nu(3-\theta)+2\theta}\;\rho_{\theta-\nu}(w) + \ldots$$

- in contrast to closed string open string bosonic twist fields couple only to double excited bosonic twist fields

Summary

- quantized system of intersecting D6-branes
  $\rightsquigarrow$ potential light states
- computed the four fermion amplitude
  $\rightsquigarrow$ signals of massive states

observable in the near future?

$\rightsquigarrow$ M_s of TeV scale

$\rightsquigarrow$ small intersection angles $\rightsquigarrow$ $\mathcal{N} = 2$ sector

along the lines

- dictionary between states and vertex operators for arbitrary angles
- interesting behavior of open string bosonic twist correlators
  $\rightsquigarrow$ different to the closed string