A ten-dimensional action for non-geometric fluxes

based on arXiv:1106.4015 in collaboration with
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$\Delta p \cdot \Delta q \geq \frac{1}{2}\hbar$

Basic idea

- String theory has more symmetries than point particles do.
  $\Rightarrow$ Strings probe geometry differently.
- Making use of this can lead to interesting models:
  - De Sitter vacua [de Carlos, Guarino, Moreno: 09]
  - Moduli stabilisation [Shelton, Taylor, Wecht: 06]
- So-called “non-geometric” situations arise in various setups in both 4 and 10 dimensions
- The relation between those is not clear, yet.

Non-geometry

In 10 dimensions:

- Use stringy symmetry (e.g. T-duality) to patch target space fields [Hellerman, McGreevy, Williams: 02] [Flournoy, Wecht, Williams: 04]

In 4 dimensions:

- T-duality covariant superpotential [Shelton, Taylor, Wecht: 05]
- T-duality covariant algebras in gauged supergravity [Dabholkar, Hull: 05, 02]
- T-duality chain: $H_{abc} \xrightarrow{T_a} f^a{}_{bc} \xrightarrow{T_b} Q_c{}^{ab} \xrightarrow{T_c} R^{abc}$

What is the 10-dimensional origin of Q and R?

Generalised geometry [Hitchin: 02] [Gualtieri: 04]

- Generalised tangent bundle $T \oplus T^*$
- Structure group, embedding of the T-duality group
- Generalised metric

$$\mathcal{H} = \begin{pmatrix} \hat{g} - \hat{B}\hat{g}^{-1}\hat{B} & \hat{B}\hat{g}^{-1} \\ -\hat{g}^{-1}\hat{B} & \hat{g}^{-1} \end{pmatrix}$$

- Reproduce T-duality transformations

$$\mathcal{H}' = O^T \mathcal{H} O$$

with $O \in O(d,d)$

Generalised vielbeine

- In analogy to ordinary vielbeine:

$$\mathcal{H} = \mathcal{E}^T \mathbb{1}_{2d} \mathcal{E}$$

- Defined up to $O(2d)$ transformations
- Two particularly important choices:

$$\hat{\mathcal{E}} = \begin{pmatrix} \hat{e} & 0 \\ -\hat{e}^{-T}\hat{B} & \hat{e}^{-T} \end{pmatrix} , \quad \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{e} & \tilde{e}\tilde{\beta} \\ 0 & \tilde{e}^{-T} \end{pmatrix} .$$

with $g = e^T \mathbb{1}_d e$

- $\tilde{\beta}$ has been related to non-geometry

[Grange, Schafer-Nameki: 06, 07] [Graña, Minasian, Petrini, Waldram: 08]

Change of variables

- Use

$$\begin{pmatrix} \hat{g} - \hat{B}\hat{g}^{-1}\hat{B} & \hat{B}\hat{g}^{-1} \\ -\hat{g}^{-1}\hat{B} & \hat{g}^{-1} \end{pmatrix} = \hat{\mathcal{E}}^T \mathbb{1}_{2d}\, \hat{\mathcal{E}}$$

$$= \tilde{\mathcal{E}}^T \mathbb{1}_{2d}\, \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{g} & \tilde{g}\tilde{\beta} \\ -\tilde{\beta}\tilde{g} & \tilde{g}^{-1} - \tilde{\beta}\tilde{g}\tilde{\beta} \end{pmatrix}$$

to replace

$$(\hat{g},\ \hat{B}) \to (\tilde{g},\ \tilde{\beta})$$

- Define

$$e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|} = e^{-2\hat{\phi}}\sqrt{|\hat{g}|}$$

Simplification

- Consider $R^{abc} = 0$ only.
- One possibility to implement this is to assume:

$$\tilde{\beta}^{km}\partial_m \cdot = 0$$

  for all fields.
- Expectation: [Graña, Minasian, Petrini, Waldram: 08]

$$Q_m{}^{np} = \partial_m \tilde{\beta}^{np}$$

The rewritten action

$$e^{-2\hat{\phi}}\sqrt{|\hat{g}|}\left(\widehat{\mathcal{R}}+4|\mathrm{d}\hat{\phi}|^2-\frac{1}{2}|\hat{H}|^2\right)$$

$$\Big\downarrow$$

$$e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\widetilde{\mathcal{R}}+4|\mathrm{d}\tilde{\phi}|^2-\frac{1}{2}|Q|^2\right)$$

The rewritten action

$$e^{-2\hat{\phi}}\sqrt{|\hat{g}|}\left(\widehat{\mathcal{R}} + 4|\mathrm{d}\hat{\phi}|^2 - \frac{1}{2}|\hat{H}|^2\right)$$

$$\Big\downarrow$$

$$e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\widetilde{\mathcal{R}} + 4|\mathrm{d}\tilde{\phi}|^2 - \frac{1}{2}|Q|^2\right) + \partial(\dots)$$

Global aspects and non-geometry

- Non-geometry appears in non-trivial monodromies.
- Both actions may contain ill-defined terms
- Examples suggest: all ill-definedness can be captured in the total derivative.

Prescription:

Use one of these actions, drop the total derivative
$\rightarrow$ "Preferred basis"

Toy example

T-duality frame	Type	Lagrangian
H_{abc}	Three-torus	$\hat{\mathcal{L}}$
$f^a{}_{bc}$	Twisted torus	$\hat{\mathcal{L}} = \tilde{\mathcal{L}}$
$Q_c{}^{ab}$	Non-geometric setup	$\tilde{\mathcal{L}} + \partial(\dots)$

Compactification

- Consider only two moduli: volume and dilaton
- Dimensional reduction:

$$S_E = M_4^2 \int \mathrm{d}^4 x \sqrt{|g_{\mu\nu}^E|} \Big(\mathcal{R}_4^E + \mathrm{kin} + \sigma^{-2} \rho^{-1} \mathcal{R}_6 \\ - \frac{1}{2} \sigma^{-2} \rho |Q|^2 - \frac{1}{2} \sigma^{-2} \rho^{-3} |H|^2 \Big)$$

- Potentials: [Hertzberg, Kachru, Taylor, Tegmark: 07]

$$V_\omega \sim \sigma^{-2} \rho^{-1} \ , \quad V_H \sim \sigma^{-2} \rho^{-3} \ , \quad V_Q \sim \sigma^{-2} \rho$$

Summary & Outlook

What has been done:

- Change of field variables $\rightarrow$ ten-dimensional version of non-geometric Q-flux
- Prescription removes ill-definedness
- Recovered expected potential after dimensional reduction

We found a link between 10d and 4d non-geometric fluxes.

What should be done:

- Obtain R-flux
- Extend to RR-fluxes
- Relate to T-duality invariant formalisms

  [Hohm, Hull, Zwiebach: 10] [Coimbra, Strickland-Constable, Waldram: 11]
- World-sheet studies

Double field theory [Hull, Zwiebach: 09] [Hohm, Hull, Zwiebach: 10]

- Also introduces $\mathcal{H}$, but with doubled coordinates
- T-duality covariant formulation
- Relation to our work:

$$
\begin{array}{ccc}
\mathcal{L}_{DFT}(\mathcal{H}(\hat{g},\hat{B}),\hat{\phi}) & = & \mathcal{L}_{DFT}(\mathcal{H}(\tilde{g},\tilde{\beta}),\tilde{\phi}) \\
\| & & \| \;\; \text{proposal} \\
\hat{\mathcal{L}} + \partial(\dots) & = & \tilde{\mathcal{L}} + \partial(\dots)
\end{array}
$$