Asymptotic safety and Higgsless scenario

Roberto Percacci

SISSA Trieste

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Higgsless Higgs mechanism

Higgs doublet Φ has values in $\mathbf{C}^2 = \mathbf{R}^4$. Parametrize as ρ (the Higgs particle) and three angles φ^α (Goldstone bosons).
If $p < m_\rho$ can freeze $\rho = v = 246\text{GeV}$.
Goldstone bosons carry nonlinear realization of $SU(2)_L$, with algebra generators R_i^α.

$$D_\mu \varphi^\alpha = \partial_\mu \varphi^\alpha + g A_\mu^i R_i^\alpha(\varphi)$$

$$\frac{1}{2} v^2 h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta \approx \frac{1}{2} v^2 g^2 A_\mu^i A^{\mu i}$$

Higgs d.o.f. not necessary to give mass to W, Z.
"only" there for renormalizability, unitarity.

T. Appelquist, C. Bernard, Phys. Rev. **D22** 200 (1980);
A.C. Longhitano, Phys. Rev. **D22** 1166 (1980)

Low energy EFTs

χPT (strong and weak interactions)

$$S(U) = \int dx \, \left[g_2 (U^{-1}\partial U)^2 + g_4 (U^{-1}\partial U)^4 + g_6 (U^{-1}\partial U)^6 + \ldots \right]$$

gravity

$$S(g_{\mu\nu}) = \int dx \sqrt{g} \, \left[g_0 + g_2 R + g_4 R^2 + g_6 R^3 + \ldots \right]$$

$R \sim \partial A + AA \sim (g^{-1}\partial g)^2$

Low energy EFTs

all perturbatively nonrenormalizable:
perturbative expansion parameter $p/\sqrt{g_2}$
break down at $p \approx 4\pi\sqrt{g_2}$

- pion χPT: $4\pi\sqrt{g_2} \approx 4\pi f_\pi \approx 1\,GeV$
- EW χPT: $4\pi\sqrt{g_2} \approx 4\pi v \approx 3\,TeV$
- gravity: $4\pi\sqrt{g_2} \approx m_P \approx 10^{16}\,TeV$

UV completions

- pion $\chi\mathrm{PT} \longrightarrow$ QCD
- EW $\chi\mathrm{PT} \longrightarrow$ SM Higgs? technicolor?
- gravity:$\longrightarrow$ strings?

Nonperturbative renormalizability (a.k.a. *asymptotic safety*) is a possibility in both cases.

Asymptotic safety

- for all (essential) couplings we must have $\tilde{g}_i \to \tilde{g}_{i*}$ where $\tilde{g}_i = k^{-d_i} g_i$ or equivalently $g_i \sim k^{d_i}$.
- QFT description OK if we are on trajectory that hits the FP for $k \to \infty$. Such trajectories are said to be *renormalizable* or *asymptotically safe*
- the renormalizable trajectories span the UV critical surface $\mathcal{S}$.
- theory is predictive if $\mathcal{S}$ is finite dimensional

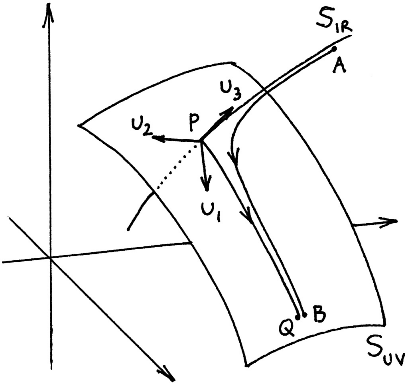
The UV Critical Surface

- In practice can determine $T_*\mathcal{S}$
- $\beta_i = \partial_t g_i$
- $\tilde{\beta}_i = \partial_t \tilde{g}_i = -d_i \tilde{g}_i + k^{-d_i}\beta_i$
- FP at $\tilde{g}_{i*}$
- Linearize flow around FP: $\tilde{\beta}_i(\tilde{g}) = M_{ij}(\tilde{g}_j - \tilde{g}_{j*})$, $M_{ij} = \frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\Big|_*$
- go to principal axes of matrix M: $\partial_t z_i = \lambda_i z_i$
- $z_i(t) = e^{\lambda_i t} z_i$
- positive scaling exponent = negative eigenvalue = UV attractive = relevant
- negative scaling exponent = positive eigenvalue = UV repulsive = irrelevant
- $\mathcal{S}$=span{relevant directions}.

Example: QCD

- Gaußian Fixed Point at $\tilde{g}_{i*} = 0$.
- $M_{ij} = -d_i \delta_{ij}$
- critical exponents$=d_i$
- $\mathcal{S}$=span{renormalizable couplings}.

General picture



Two possibilities for AS theory of all known interactions:

- AS is due to gravity. All interactions reach the FP at the Planck scale.
- Each interaction is AS by itself and reaches the FP at its characteristic scale.

Towards an AS Higgsless SM

work with M. Fabbrichesi, A. Codello, F. Bazzocchi, A. Tonero, L. Vecchi and O. Zanusso

The nonlinear σ model I

$$S = \frac{1}{2f^2}\int d^dx\, h_{\alpha\beta}\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta \qquad\qquad f^2 \sim m^{2-d}$$

$$k\frac{d}{dk}\frac{1}{f^2} = ck^{d-2}$$

$$k\frac{df^2}{dk} = -cf^4k^{d-2}$$

defining $\tilde{f}^2 = k^{d-2}f^2$

$$k\frac{d\tilde{f}^2}{dk} = (d-2)\tilde{f}^2 - c\tilde{f}^4$$

if $c > 0$ fixed point at $\tilde{f}^2 = \frac{d-2}{c} > 0$

The nonlinear σ model II

in general the flow of $\frac{1}{f^2}h_{\alpha\beta}$ is governed by the Ricci tensor:

$$k\frac{d}{dk}\left(\frac{1}{f^2}h_{\alpha\beta}\right) = 2c_d k^{d-2} R_{\alpha\beta} \qquad c_d = \frac{1}{\Gamma(d/2+1)(4\pi)^{d/2}}$$

sometimes G-invariance fixes h up to scale. Then, for Einstein metric

$$k\frac{d}{dk}\left(\frac{1}{f^2}\right) = \frac{2c_d\, R}{D} k^{d-2}\,.$$

If $d > 2$, FP at:

$$\tilde{f}^2{}_* = \frac{(d-2)D}{2c_d\, R}.$$

[A. Codello and R. Percacci, PLB **672** (2009) 280]

Unitarization

$$\mathcal{A}(s,t,u) = \frac{s}{v^2} = \frac{1}{4} s f^2$$

for $k^2 = s$, at high energy $f^2 \sim \tilde{f}^2 s^{-1}$

$$\mathcal{A}(s,t,u) \to \frac{1}{4} \tilde{f}_*^2$$

Related work

- same behavior from holographic RG (ask L. Rachwal)
- similarities to classicalization scenario (Dvali, Giudice, Gomez, Kehagias...)

Gauging the NLσM

Gauged $SU(N)$ chiral NLσM Euclidean action (the left part of the isometry group $SU(N)_L \times SU(N)_R$ is gauged):

$$\mathcal{S} = \frac{1}{2f^2}\int d^d x\, h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta + \frac{1}{4g^2}\int d^d x\, F^i_{\mu\nu} F^{\mu\nu}_i$$

- $D_\mu \varphi^\alpha = \partial_\mu \varphi^\alpha + A^i_\mu R^\alpha_i(\varphi)$ is the gauge covariant derivative, R^α_i-right invariant Killing vectors $(\alpha\,, i = 1, .., N^2 - 1)$.
- $F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + f^i{}_{jl} A^j_\mu A^l_\nu$ is the gauge field strength.
- the action is invariant under local $SU(N)_L$ infinitesimal transformations

$$\delta_\epsilon \varphi^\alpha = -\epsilon^i_L R^\alpha_i(\varphi) \qquad\qquad \delta_\epsilon A^i_\mu = \partial_\mu \epsilon^i_L + f_{jl}{}^i A^j_\mu \epsilon^l_L\,.$$

β-functions in $d = 4$

$$\frac{d}{dt}\frac{1}{g^2} = \frac{N}{(4\pi)^2}\left[\frac{8}{(1+\frac{g^2}{\tilde{f}^2})^3}\left(1+\frac{\eta_a}{6}\right) - \frac{1}{3}\left(\frac{9}{4}+2\eta_a+\frac{1}{8}\eta_\xi\right)\frac{1}{1+\frac{g^2}{\tilde{f}^2}}\right]$$

$$\frac{d}{dt}\frac{1}{f^2} = \frac{N}{(4\pi)^2}\frac{k^2}{4}\frac{1}{(1+\frac{g^2}{\tilde{f}^2})^2}\left[1+\frac{\eta_\xi}{6}+\frac{4\frac{g^2}{\tilde{f}^2}}{1+\frac{g^2}{\tilde{f}^2}}\left(2+\frac{\eta_\xi+\eta_a}{6}\right)\right].$$

One loop RGE for $g^2 \ll \tilde{f}^2$:

$$\frac{d}{dt}g^2 = -\frac{N}{(4\pi)^2}\frac{29}{4}g^4$$

$$\frac{d}{dt}\tilde{f}^2 = 2\tilde{f}^2 - \frac{3N}{2(4\pi)^2}g^2\tilde{f}^2 - \frac{1}{(4\pi)^2}\frac{N}{4}\tilde{f}^4$$

Fixed point - numerical result

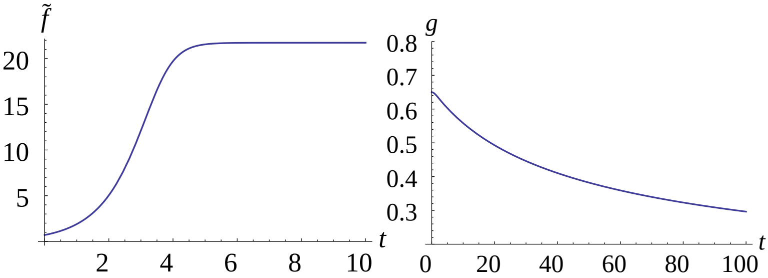


Figure: Running of $\tilde{f}$ and g for $N = 2$ in $d = 4$, $\tilde{f}$ goes to its fixed point $\tilde{f}_* \simeq 21.7$ after 4-5 e-foldings.

$SU(2) \times U(1)$ gauged NLσM

In absence of an Higgs field, the spontaneous breaking of $SU(2) \times U(1) \to U(1)$ would be implemented by coupling the gauge bosons to the NLσM Nambu-Goldstone bosons:

$$S = \frac{1}{2f^2} \int d^4x \, h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta + \frac{1}{4g^2} \int d^4x \, W^i_{\mu\nu} W_i^{\mu\nu} + \frac{1}{4g'^2} \int d^4x \, B_{\mu\nu} B^{\mu\nu}$$

- $1/f^2 = v^2/4$ (v is the EW VEV), g and g' are the gauge couplings.
- $D_\mu \varphi^\alpha = \partial_\mu \varphi^\alpha + W^i_\mu R^\alpha_i - B_\mu L^\alpha_3 \qquad \alpha \, , i = 1, 2, 3$
- R/L are right/left invariant Killing vectors.

β-functions

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{2}\frac{\tilde{f}^2}{(4\pi)^2}\left(\tilde{f}^2 + 6g^2 + 3g'^2\right)$$

$$\frac{dg^2}{dt} = -\frac{g^4}{(4\pi)^2}\frac{29}{2}$$

$$\frac{dg'^2}{dt} = \frac{1}{6}\frac{g'^4}{(4\pi)^2}$$

If we treat gauge couplings as constant ($g = g_* = 0.65$, $g' = g'_* = 0.35$)

$$\tilde{f}_* = \sqrt{64\pi^2 - 6g_*^2 - 3g_*'^2} \simeq 25.06$$

Approximate solution for $f^2(k)$:

$$f^2(k) = \frac{\tilde{f}_*^2 f_0^2}{\tilde{f}_*^2 + (k^2 - k_0^2) f_0^2}\,.$$

Fermions and Goldstone bosons

$SU(N)_L \times SU(N)_R$ NLσM coupled to fermions:

$$\mathcal{L} = -\frac{1}{f^2}\mathrm{Tr}\left(U^\dagger \partial_\mu U U^\dagger \partial^\mu U\right) + \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R$$
$$- \frac{2h}{f}\left(\bar{\psi}_L^{ia} U^{ij} \psi_R^{ja} + \mathrm{h.c.}\right). \qquad (1/f = v/2)$$

$U = e^{if\pi^a T_a}$ is $SU(N)$ valued scalar field, π^a Goldstone bosons.
$\psi^{ia}_{L/R}$ in the fundamental of $SU(N)_{L/R}$ and $SU(N_c)$

Degenerate multiplet of fermions with mass

$$m = 2\frac{h}{f} = hv\,,$$

h is the Yukawa coupling.

Beta functions

$$\frac{d\tilde{f}}{dt} = \tilde{f} - \frac{N}{64\pi^2}\tilde{f}^3 + \frac{N_c}{4\pi^2}h^2\tilde{f}\ ,$$
$$\frac{dh}{dt} = \frac{1}{16\pi^2}\left(4N_c - 2\frac{N^2-1}{N}\right)h^3 + \frac{1}{64\pi^2}\frac{N^2-2}{N}h\tilde{f}^2\,.$$

Fixed Points:

FPI $(h_* = 0, \tilde{f}_* = 0) \Rightarrow$ trivial

FPII $(h_* = 0, \tilde{f}_* = 8\pi/\sqrt{N}) \Rightarrow h = 0$ at all scales

FPIII $(h_* \neq 0, \tilde{f}_* \neq 0) \Rightarrow N > 2N_c$ (not true for the most phenomenologically important case $N = 2,\ N_c = 3$)

Four-fermion interactions

Fix $N = 2$, we add to the lagrangian a complete set of $SU(2)_L \times SU(2)_R$-invariant four fermion operators:

$$
\begin{aligned}
\mathcal{L}_{\psi^4} &= \lambda_1 \left(\bar{\psi}_L^{ia} \psi_R^{ja} \bar{\psi}_R^{jb} \psi_L^{ib} \right) + \lambda_2 \left(\bar{\psi}_L^{ia} \psi_R^{jb} \bar{\psi}_R^{jb} \psi_L^{ia} \right) \\
&+ \lambda_3 \left(\bar{\psi}_L^{ia} \gamma_\mu \psi_L^{ia} \bar{\psi}_L^{jb} \gamma^\mu \psi_L^{jb} + \bar{\psi}_R^{ia} \gamma_\mu \psi_R^{ia} \bar{\psi}_R^{jb} \gamma^\mu \psi_R^{jb} \right) \\
&+ \lambda_4 \left(\bar{\psi}_L^{ia} \gamma_\mu \psi_L^{ib} \bar{\psi}_L^{jb} \gamma^\mu \psi_L^{ja} + \bar{\psi}_R^{ia} \gamma_\mu \psi_R^{ib} \bar{\psi}_R^{jb} \gamma^\mu \psi_R^{ja} \right) .
\end{aligned}
$$

- In our RG analysis we consider only the third family of quarks, $\psi^t = (t\ b)$;
- In the case of $SU(2) \times U(1)$ there would be 10 operators ;
- Gies, Jaeckel and Wetterich [PRD 69 105008 (2004)];
- but we do not seek to model chiral symmetry breaking.

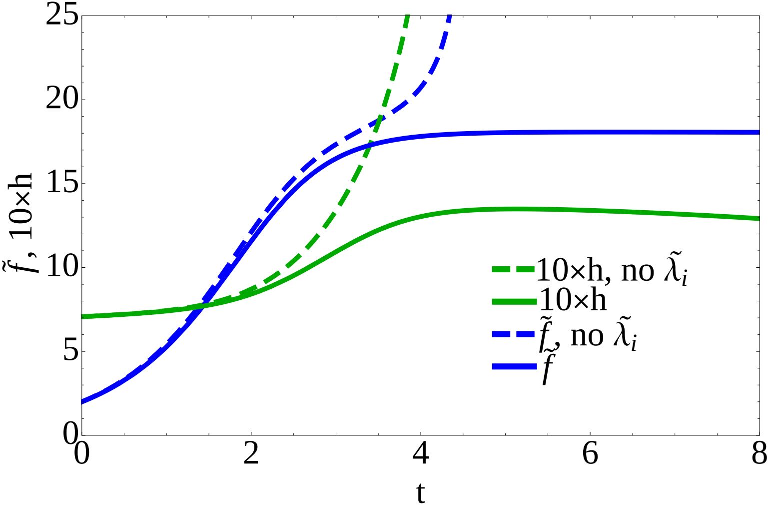
Beta functions

$$
\begin{aligned}
\frac{d\tilde{f}}{dt} &= \tilde{f} - \frac{1}{32\pi^2}\tilde{f}^3 + \frac{N_c}{4\pi^2}h^2\tilde{f} \\
\frac{dh}{dt} &= \frac{1}{16\pi^2}\left[4N_c - 3 + \frac{16}{\tilde{f}^2}(N_c\tilde{\lambda}_1 + \tilde{\lambda}_2)\right]h^3 + \frac{1}{64\pi^2}\left[\tilde{f}^2 - 16(N_c\tilde{\lambda}_1 + \tilde{\lambda}_2)\right]h \\
\frac{d\tilde{\lambda}_1}{dt} &= 2\tilde{\lambda}_1 - \frac{1}{4\pi^2}\left[N_c\tilde{\lambda}_1^2 + \frac{3}{2}\tilde{\lambda}_1\tilde{\lambda}_2 - 2\tilde{\lambda}_1\tilde{\lambda}_3 - 4\tilde{\lambda}_1\tilde{\lambda}_4\right] \\
\frac{d\tilde{\lambda}_2}{dt} &= 2\tilde{\lambda}_2 + \frac{1}{4\pi^2}\left[\frac{1}{4}\tilde{\lambda}_1^2 + 4\tilde{\lambda}_1\tilde{\lambda}_3 + 2\tilde{\lambda}_1\tilde{\lambda}_4 - \frac{3}{4}\tilde{\lambda}_2^2 + 2(2N_c + 1)\tilde{\lambda}_2\tilde{\lambda}_3 + 2(N_c + \right. \\
\frac{d\tilde{\lambda}_3}{dt} &= 2\tilde{\lambda}_3 + \frac{1}{4\pi^2}\left[\frac{1}{4}\tilde{\lambda}_1\tilde{\lambda}_2 + \frac{N_c}{8}\tilde{\lambda}_2^2 + (2N_c - 1)\tilde{\lambda}_3^2 + 2(N_c + 2)\tilde{\lambda}_3\tilde{\lambda}_4 - 2\tilde{\lambda}_4^2\right] \\
\frac{d\tilde{\lambda}_4}{dt} &= 2\tilde{\lambda}_4 + \frac{1}{4\pi^2}\left[\frac{1}{8}\tilde{\lambda}_1^2 - 4\tilde{\lambda}_3\tilde{\lambda}_4 + (N_c + 2)\tilde{\lambda}_4^2\right] .
\end{aligned}
$$

Fixed points table ($\tilde{f}_* = 17.78$, $h_* = 0$)

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	ϵ_h
fp0	0	0	0	0	0.5
fp1a	0	−28.71	−7.18	0	1.22
fp1b	0	0	7.85	−9.51	0.5
fp1c	0	25.61	−4.27	0	−0.15
fp1d	25.80	−1.77	0.19	−1.15	−1.42
fp2a	13.41	20.10	−3.80	−0.24	−1.03
fp2b	20.86	−3.56	7.04	−8.94	−1.00
fp2c	0	−36.55	2.34	−13.92	1.43
fp2d	0	0	−15.79	0	0.5
fp2e	37.17	−37.36	−8.43	−1.65	−1.38
fp2f	−2.92	32.59	4.67	−12.04	−0.10
fp3a	0.	31.67	4.67	−12.06	−0.30
fp3b	19.95	−8.59	−15.27	−0.36	−0.80
fp3c	31.22	−44.52	0.73	−13.38	−0.74
fp3d	−4.87	1.54	−5.42	−20.10	0.83
fp4	0	0	−5.42	−20.13	0.5

Numerical solution, with i.c. $h_0 = m_t/v$ and $\tilde{f}_0 = 2$:



S and T parameters

Add to the effective lagrangian two new operators

$$\mathcal{L} = \frac{1}{2f^2} h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta + \frac{1}{4g^2} W^i_{\mu\nu} W_i^{\mu\nu} + \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu}$$
$$- \frac{a_0}{f^2} D_\mu \varphi^\alpha D^\mu \varphi^\beta L^3_\alpha L^3_\beta - \frac{a_1}{2} B^{\mu\nu} W^i_{\mu\nu} R_{i\alpha} L^\alpha_3$$

a_0 and a_1 are related to the Peskin-Takeuchi parameters S and T

$$S = -16\pi a_1(m_Z) + \frac{1}{6\pi}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right] ,$$
$$T = \frac{2}{\alpha_{em}} a_0(m_Z) - \frac{3}{8\pi \cos^2\theta_W}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right] .$$

β functions I (ungauged case)

$$\mathcal{L} = \frac{1}{2f^2}\hat{h}_{\alpha\beta}\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta \,.$$

New metric $\hat{h}_{\alpha\beta} = L^1_\alpha L^1_\beta + L^2_\alpha L^2_\beta + (1-2a_0)L^3_\alpha L^3_\beta$.

$$\frac{d}{dt}\left(\frac{1}{f^2}\hat{h}_{\alpha\beta}\right) = \frac{1}{(4\pi)^2}k^2\hat{R}_{\alpha\beta}$$

In the basis of the right-invariant vectorfields:

$$\hat{R}_{11} = \hat{R}_{22} = \frac{1}{2} + a_0, \qquad \hat{R}_{33} = \frac{1}{2} - a_0 \,.$$

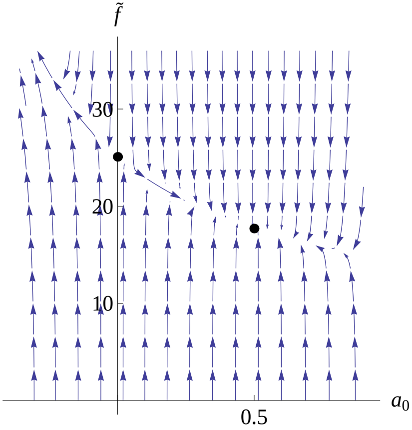
One-loop beta functions of $\tilde{f}^2$ and a_0:

$$\begin{aligned}\frac{d\tilde{f}^2}{dt} &= 2\tilde{f}^2 - \frac{1}{(4\pi)^2}\tilde{f}^4\left(\frac{1}{2}+a_0\right)\\ \frac{da_0}{dt} &= \frac{1}{2}\frac{1}{(4\pi)^2}\tilde{f}^2 a_0(1-2a_0)\,.\end{aligned}$$

Two nontrivial Fixed Points:

FPI: $\tilde{f}_* = 8\pi \qquad a_{0*} = 0 \qquad SU(2)_R$ symmetric

FPII: $\tilde{f}_* = 4\sqrt{2}\pi \qquad a_{0*} = 1/2 \qquad SU(2)_R$ strongly broken



β functions II (gauged case)

$$
\begin{aligned}
\frac{d\tilde{f}^2}{dt} &= 2\tilde{f}^2 - \frac{1}{2}\frac{\tilde{f}^2}{(4\pi)^2}\left(\tilde{f}^2(1+2a_0) + 6g^2 + 3g'^2\right) , \\
\frac{da_0}{dt} &= \frac{1}{2}\frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_0(1-2a_0) + \frac{3}{2}g'^2\right) , \\
\frac{da_1}{dt} &= \frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_1 + \frac{1}{6}\right) .
\end{aligned}
$$

The two nontrivial FPs of the ungauged case are slightly shifted:

FPI: $\tilde{f}_* = 25.1 \qquad a_{0*} = -0.000292 \qquad a_{1*} = -0.000265$
(1 relevant and 2 irrelevant directions)

FPII: $\tilde{f}_* = 17.7 \qquad a_{0*} = 0.501 \qquad a_{1*} = -0.000530$
(2 relevant and 1 irrelevant directions)

Comparison with experimental bounds

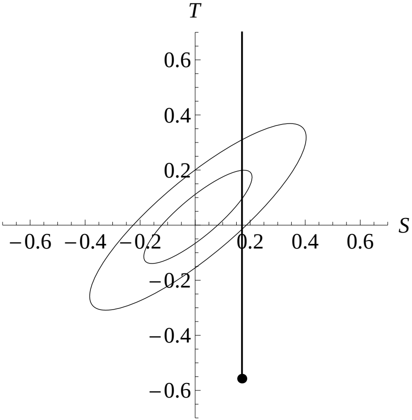


Figure: The half-line (FPII endpoints) and the dot (FPI endpoint) show the values permitted by asymptotic safety. The ellipses show the 1 and 2 σ experimental bounds with m_H=117GeV [PDG, J. Phys. G, 37, 075021 (2010)]

Conclusions

- AS a plausible scenario for all interactions
- most work on gravity
- application to EW theory if no elementary Higgs is found
- predictive (e.g. calculate $\alpha_{\rm e.m.}$)
- AS a "bottom up" approach
- the bare action is *the result* of a calculation
- agreement with low energy EFT guaranteed