

# Discrete symmetries from the heterotic string

Hans Peter Nilles

Physikalisches Institut

Universität Bonn



Bethe Center for  
Theoretical Physics

# 5 Golden Rules (2004)

- Spinors of  $SO(10)$  (for families)
- Incomplete multiplets (for Higgs)
- Repetition of families (from extra dimensions)
- $N = 1$  supersymmetry
- Importance of discrete symmetries

# 5 Golden Rules (2004)

- Spinors of  $SO(10)$  (for families)
- Incomplete multiplets (for Higgs)
- Repetition of families (from extra dimensions)
- $N = 1$  supersymmetry
- Importance of discrete symmetries

These rules have bottom-up and top-down motivation

- grand unification (evolution of couplings)
- quark and lepton (neutrino) masses
- proton stability (R-Parity)

# Rule 1 and 5

- Spinors of  $SO(10)$  might be important even in absence of GUT gauge group
- one can incorporate top-Yukawa coupling and neutrino see-saw mechanism
- discrete symmetries with many applications

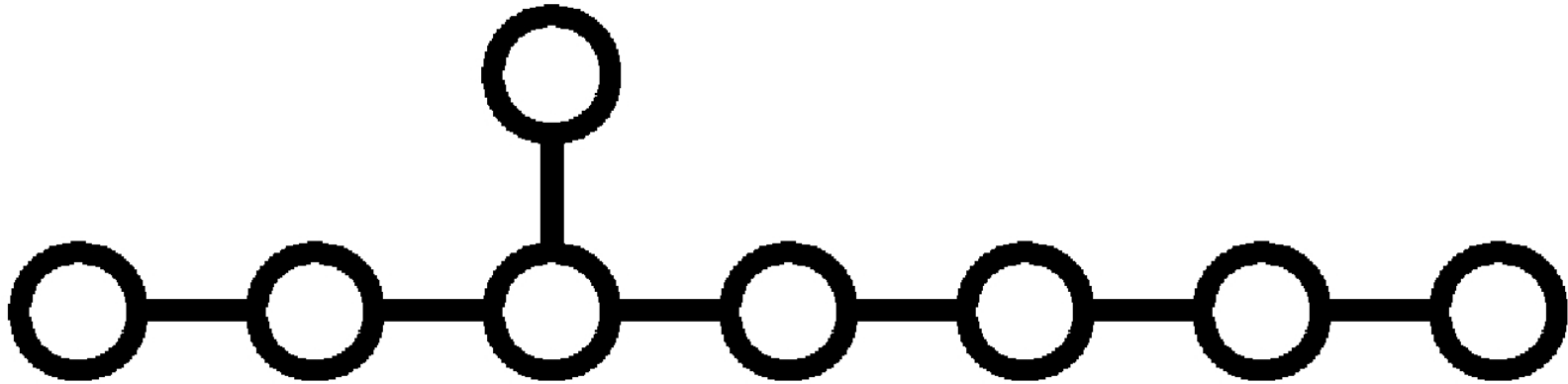
# Rule 1 and 5

- Spinors of  $SO(10)$  might be important even in absence of GUT gauge group
- one can incorporate top-Yukawa coupling and neutrino see-saw mechanism
- discrete symmetries with many applications

From the mathematical structure we would prefer exceptional groups

- There is a maximal group:  $E_8$ ,
- but  $E_8$  and  $E_7$  do not allow chiral fermions in  $d = 4$ .
- How does this fit with our usual picture of unification based on  $SU(5)$  or  $SO(10)$ ?

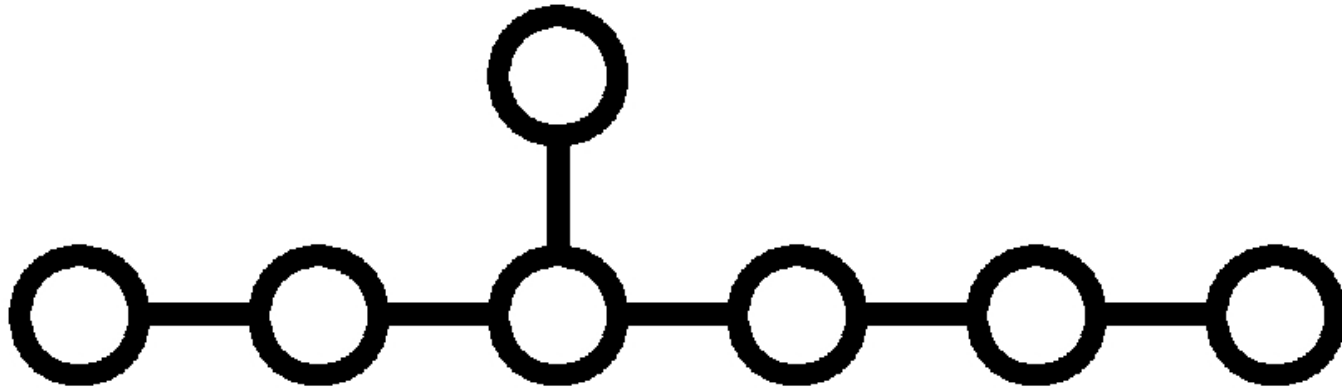
# Maximal Group



$E_8$  is the maximal group.

There are, however, no chiral representations in  $d = 4$ .

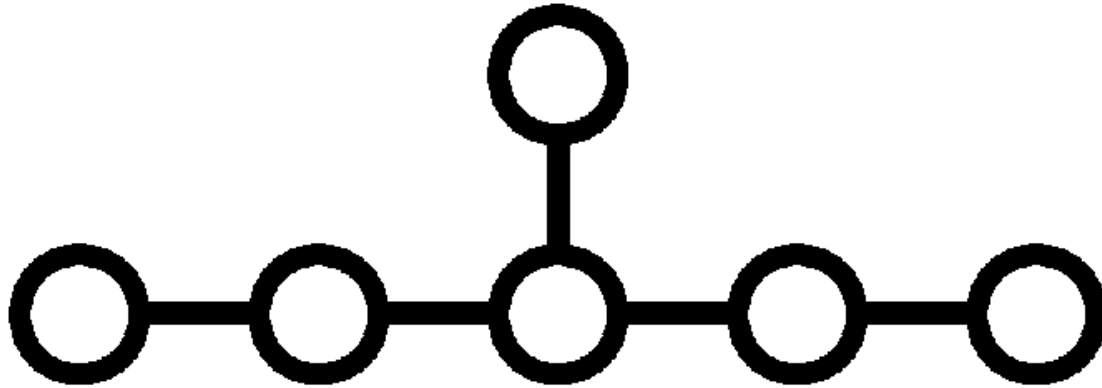
$E_7$



Next smaller is  $E_7$ .

No chiral representations in  $d = 4$  either

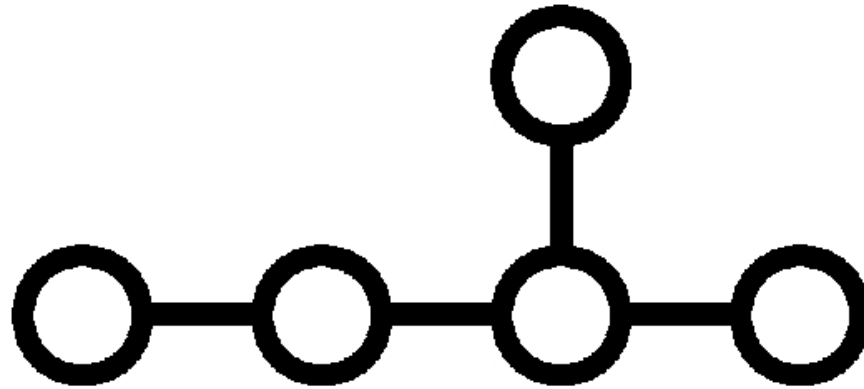
$E_6$



$E_6$  allows for chiral representations even in  $d = 4$ .



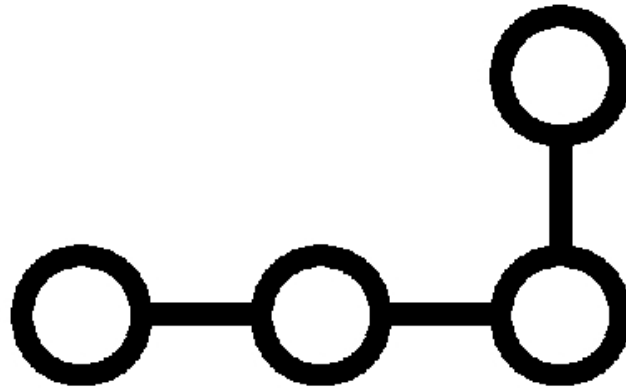
$$E_5 = D_5$$



$E_5$  is usually not called exceptional.

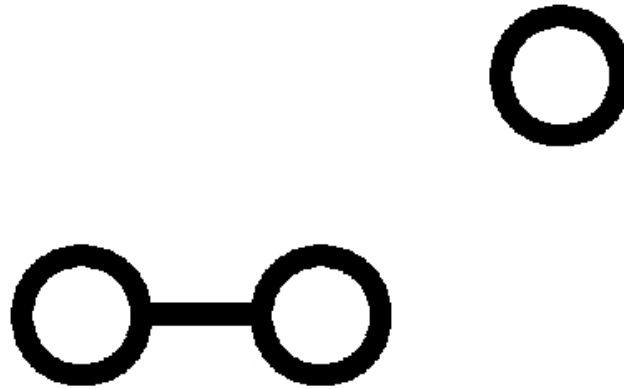
It coincides with  $D_5 = SO(10)$ .

$$E_4 = A_4$$



$E_4$  coincides with  $A_4 = SU(5)$

$E_3$



$E_3$  coincides with  $A_2 \times A_1$  which is  $SU(3) \times SU(2)$ .

# Exceptional groups in string theory

String theory favours  $E_8$

- $E_8 \times E_8$  heterotic string
- $E_8$  enhancement as a nonperturbative effect (M- or F-theory)

# Exceptional groups in string theory

String theory favours  $E_8$

- $E_8 \times E_8$  heterotic string
- $E_8$  enhancement as a nonperturbative effect (M- or F-theory)

Strings live in higher dimensions:

- chiral spectrum possible even with  $E_8$
- $E_8$  broken in process of compactification
- provides source for more **discrete symmetries**
- from  $E_8/SO(10)$  and  $SO(6)$  of the higher dimensional Lorentz group

# The use of additional symmetries

Symmetries are very useful for

- absence of FCNC (solve **flavour problem**)
- **Yukawa textures** à la Frogatt-Nielsen
- solutions to the  **$\mu$  problem**
- creation of hierarchies
- **proton stability**

# The use of additional symmetries

Symmetries are very useful for

- absence of FCNC (solve **flavour problem**)
- **Yukawa textures** à la Frogatt-Nielsen
- solutions to the  **$\mu$  problem**
- creation of hierarchies
- **proton stability**

Continuous global symmetries might be destroyed by gravitational effects. We have to rely on

- **gauge symmetries and**
- **discrete symmetries**

(Banks, Seiberg, 2010)

# Heterotic Braneworld

The heterotic braneworld is based on

- orbifold compactification of the heterotic string
- with calculability from conformal field theory



# Heterotic Braneworld

The heterotic braneworld is based on

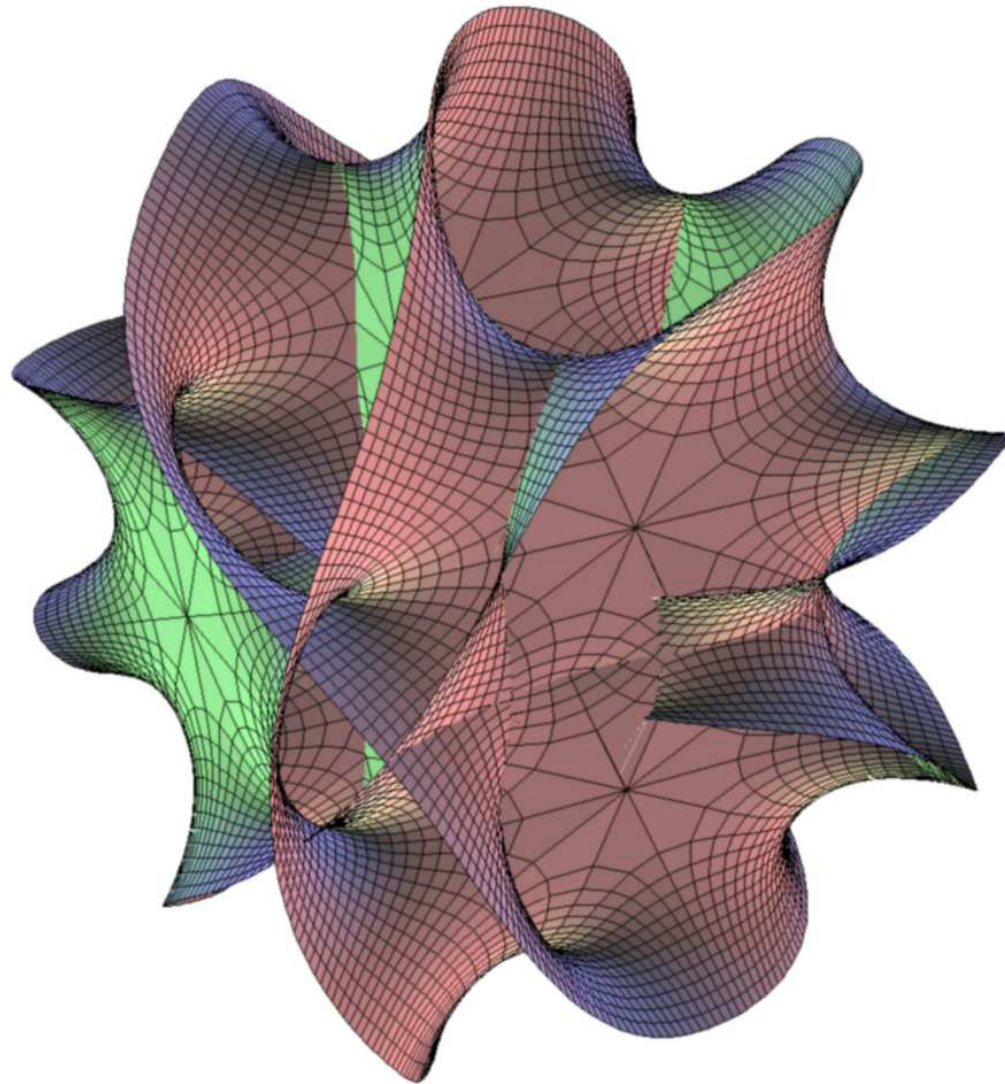
- orbifold compactification of the heterotic string
- with calculability from conformal field theory

Fields can propagate

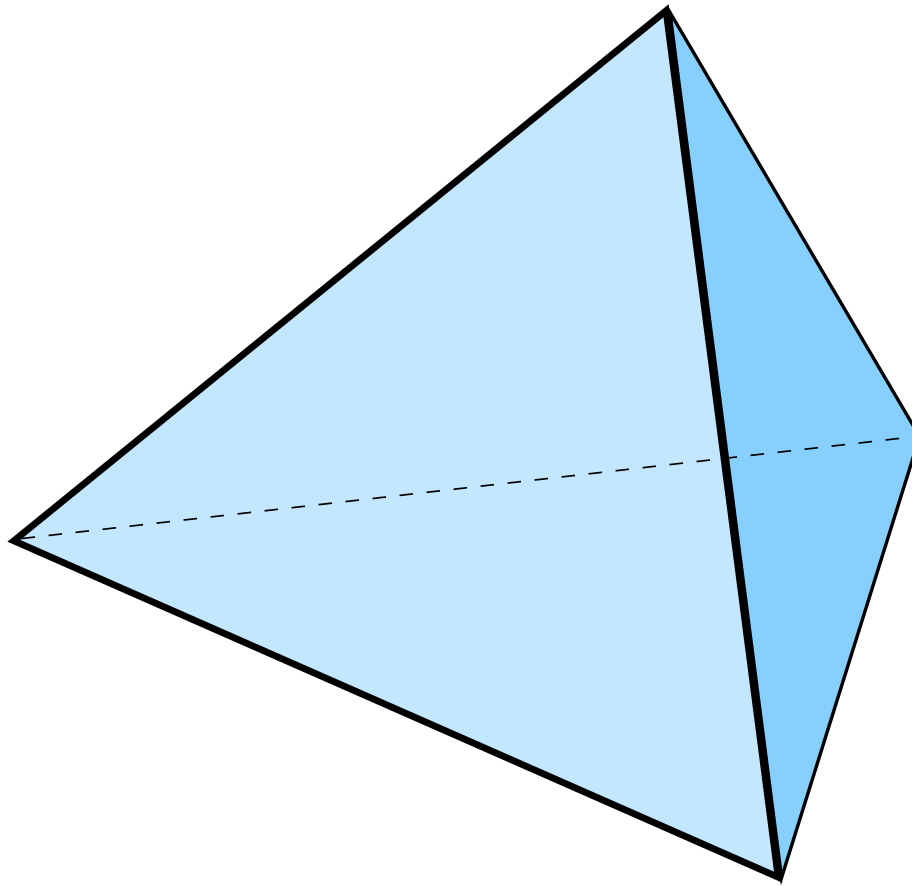
- in the Bulk ( $d = 10$  untwisted sector)
- on 3-Branes ( $d = 4$  twisted sector fixed points)
- on 5-Branes ( $d = 6$  twisted sector fixed tori)

This localization is an important property of the set-up and should be taken seriously (it is not just an approximation to obtain calculability)

# Calabi Yau Manifold



# Orbifold



# Local Grand Unification

String theory gives us a variant of GUTs

- complete (or split) multiplets for fermion families
- split multiplets for gauge- and Higgs-bosons
- partial Yukawa unification

# Local Grand Unification

String theory gives us a variant of GUTs

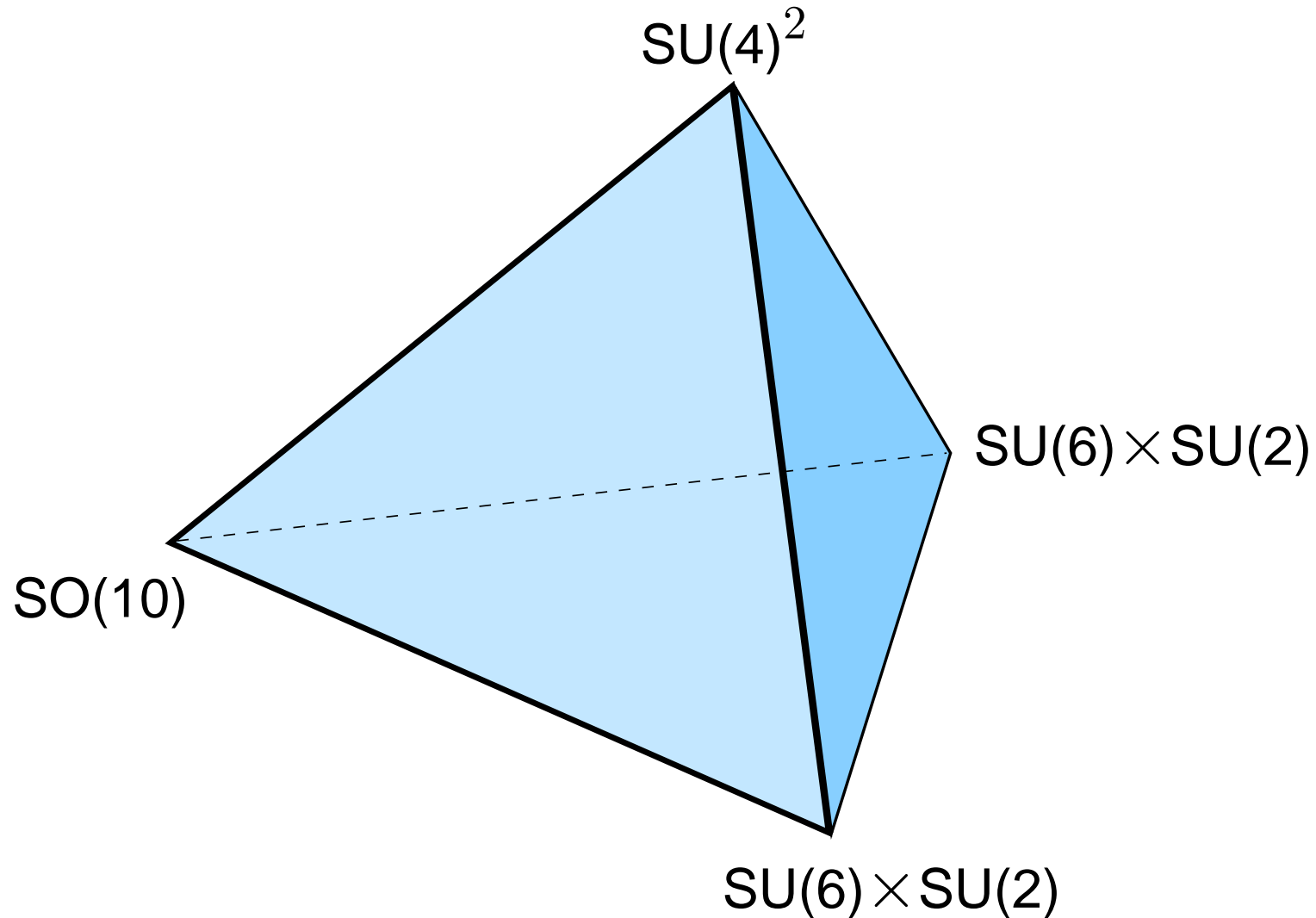
- complete (or split) multiplets for fermion families
- split multiplets for gauge- and Higgs-bosons
- partial Yukawa unification

Key properties of the theory depend on the **geography** of the fields in extra dimensions.

This geometrical set-up is called local grand unification.

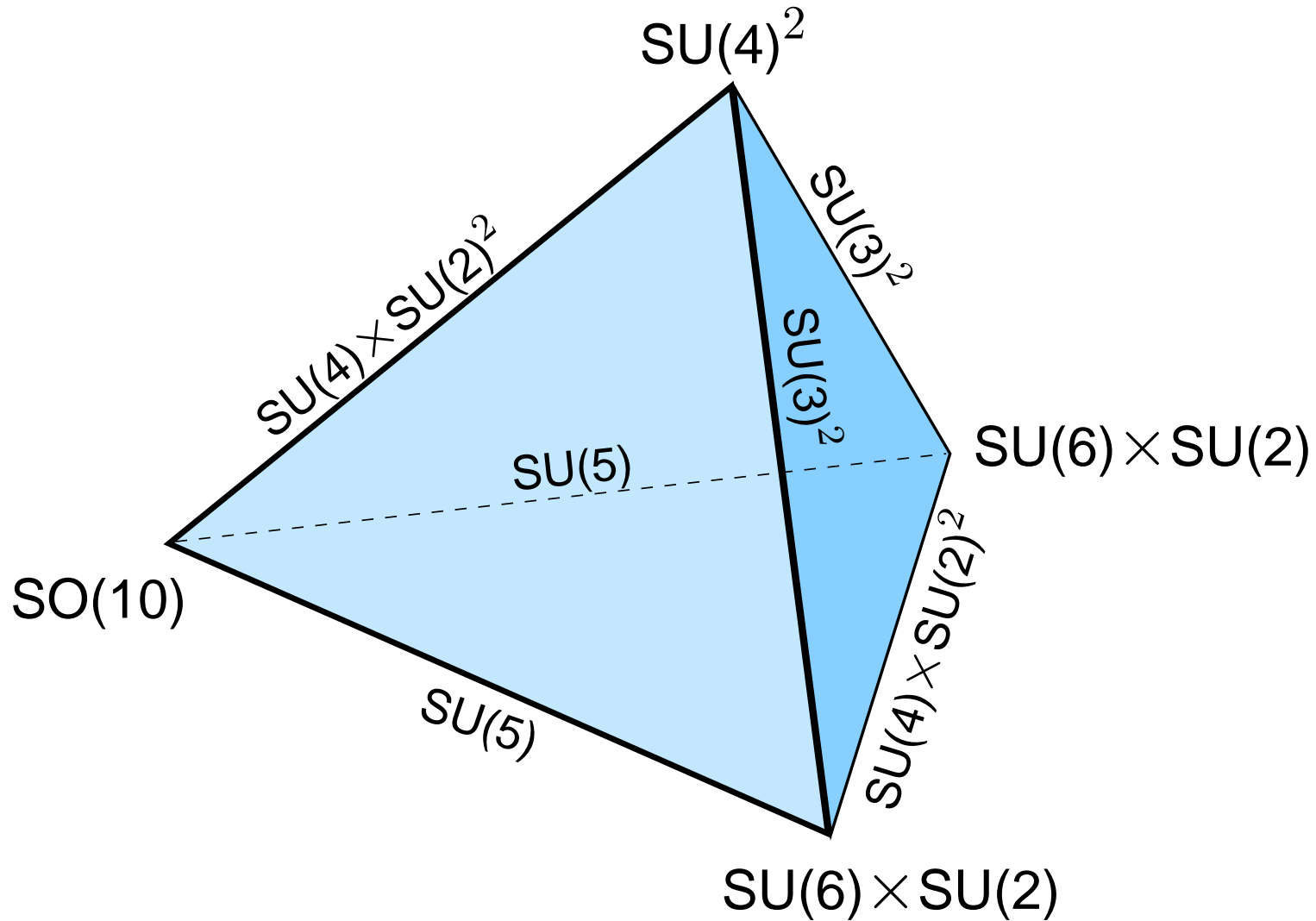
The **localization of matter as well as the local structure of the gauge group determines the properties of the theory.**

# Localized gauge symmetries



(Förste, HPN, Vaudrevange, Wingerter, 2004)

# Standard Model Gauge Group



# Symmetries

In the heterotic braneworld we find

- **gauge** symmetries (no continuous global symmetries)
  - **discrete** symmetries from geometry and stringy selection rules
- (Kobayashi, HPN, Plöger, Raby, Ratz, 2006)

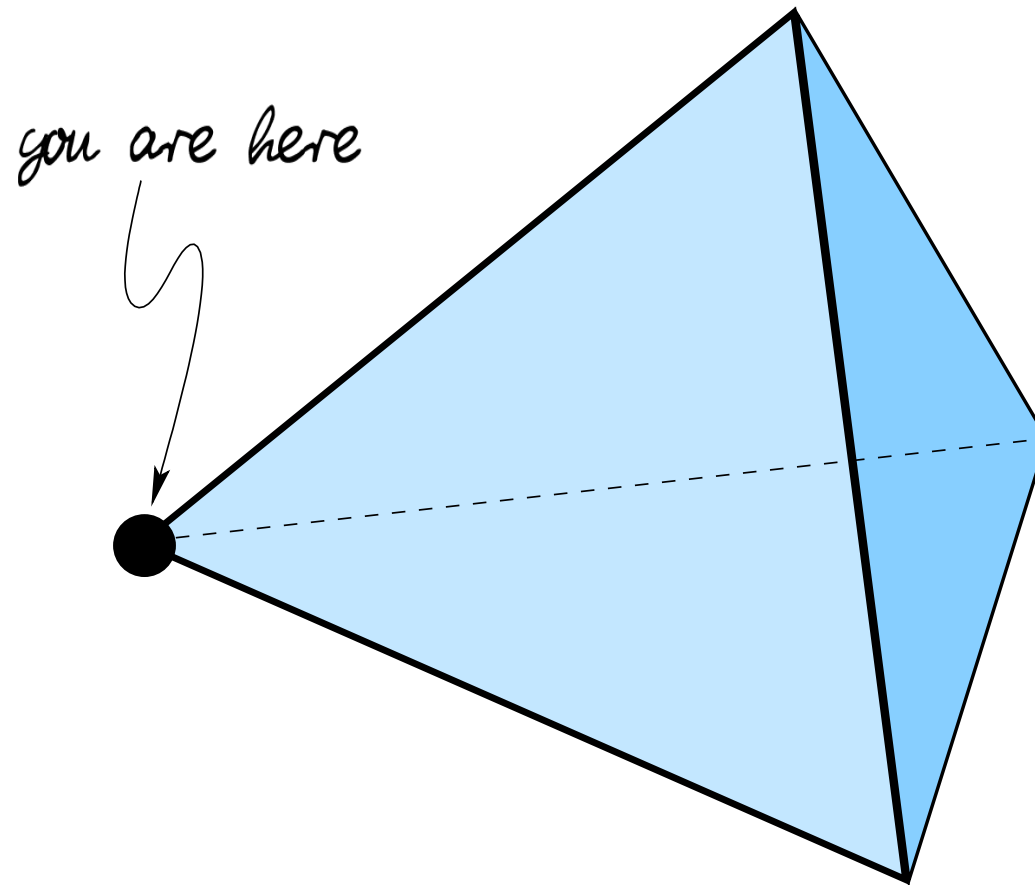
The orbifold point is a **special point in the moduli space of the compact extra dimensions with enhanced symmetries.**

These symmetries might be **slightly broken.** This will introduce small parameters that lead to a creation of hierarchies.

We might live close to the orbifold point.



# Location matters



# Symmetries in heterotic braneworld

## Applications of discrete symmetries:

- (nonabelian) family symmetries (and FCNC)  
(Ko, Kobayashi, Park, Raby, 2007)
- Yukawa textures (via Frogatt-Nielsen mechanism)
- a solution to the  $\mu$ -problem  
(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2007)
- creation of hierarchies  
(Kappl, HPN, Ramos-Sanchez, Ratz, Schmidt-Hoberg, Vaudrevange, 2008)
- proton stability via “Proton Hexality” or  $Z_4^R$   
(Förste et al. 2010; Lee et al. 2011)
- approximate global  $U(1)$  for a QCD action  
(Choi, Kim, Kim, 2006; Choi, HPN, Ramos-Sanchez, Vaudrevange, 2008)

# F-theory

F-theory with enhanced exceptional gauge symmetry is the way to incorporate rule 1 in Type II theories. It allows

- allows spinors of  $SO(10)$
- a non-vanishing top quark Yukawa coupling

# F-theory

F-theory with enhanced exceptional gauge symmetry is the way to **incorporate rule 1** in Type II theories. It allows

- allows spinors of  $SO(10)$
- a non-vanishing top quark Yukawa coupling

Phenomenological constructions are based on the concept of local models, e.g. at the local  $E_8$  point. (Heckman, Vafa, 2010)

- a single gauge group like  $E_8$
- containing other symmetries like R-parity as well
- **there might not be a global completion!**

Local  $E_8$  point does not possess all the ingredients for realistic model building.

(Marsano, Schafer-Namecki, Saulina, 2011; Lüdeling, HPN, Stephan, 2011)

# Clarification

Do not confuse

“Local Grand Unification” with “Local Model Building”.

- **Local Grand Unification** appears in consistent (global) string models where the gauge symmetries are enhanced at special points in extra-dimensional space.
- **Local Model Building** is an attempt to construct models without the incorporation of gravity (these models are potentially inconsistent).

Do not trust the predictions of “Local Models” unless they are confirmed by a global completion!

# Rule 6: Global Models

Sometimes it is said that globally consistent models are only relevant for questions like moduli stabilization.....

- this needs not be correct (as experience shows)
- the really reliable (discrete) symmetries can only be understood within a global approach (e.g. R-parity)

# Rule 6: Global Models

Sometimes it is said that globally consistent models are only relevant for questions like moduli stabilization.....

- this needs not be correct (as experience shows)
- the really reliable (discrete) symmetries can only be understood within a global approach (e.g. R-parity)

Phenomenological analyses of local models typically

- rely on continuous global  $U(1)$ s
- that might be broken in the full theory
- **what are the remaining symmetries?**

We need to answer this question before any predictions can be made!

# Rule 7: Berechenbarkeit

Nowadays we need calculability that goes beyond the effective supergravity field theory approach, e.g. exact conformal field theory

- flat orbifolds, free fermionic constructions (Faraggi et al.)
- tensoring CFTs (Gepner models) (Schellekens et al.)



# Rule 7: Berechenbarkeit

Nowadays we need calculability that goes beyond the effective supergravity field theory approach, e.g. exact conformal field theory

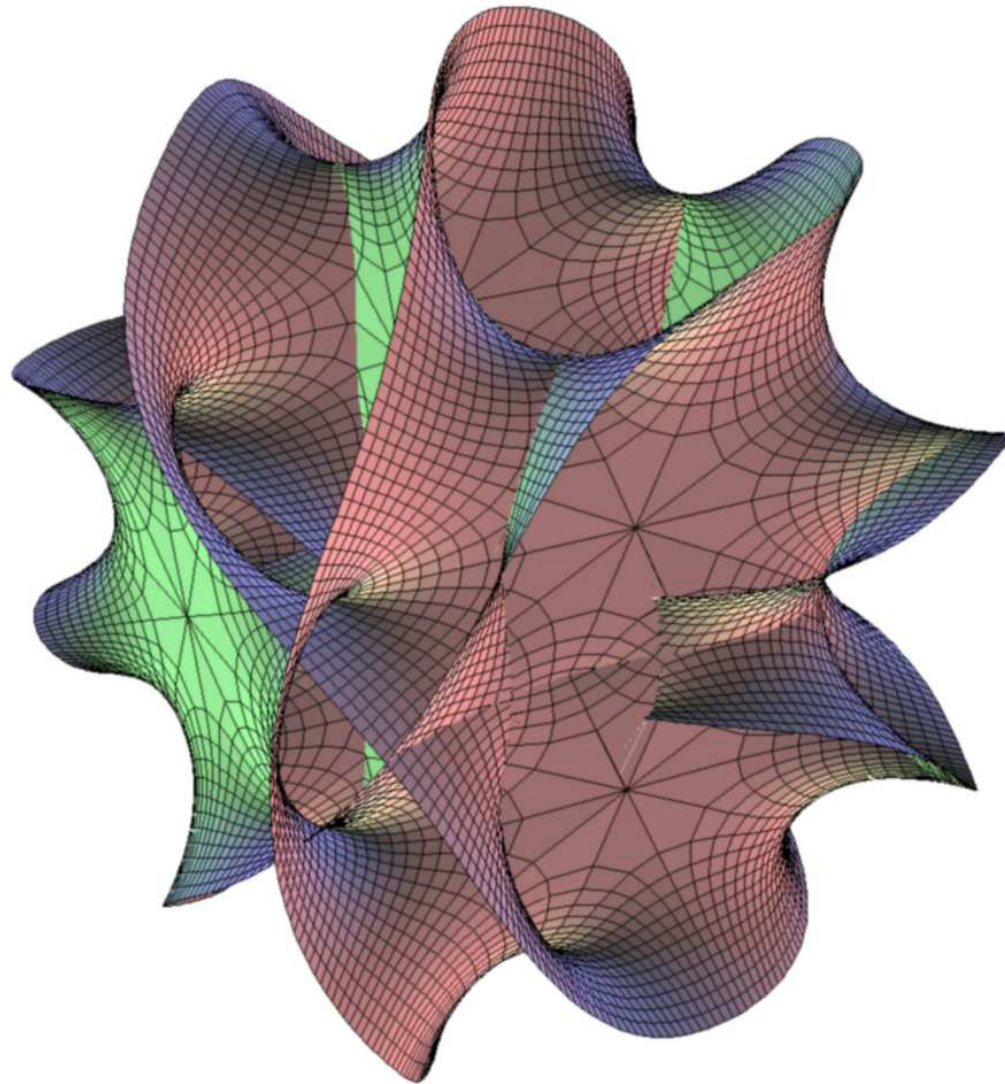
- flat orbifolds, free fermionic constructions (Faraggi et al.)
- tensoring CFTs (Gepner models) (Schellekens et al.)

We have to analyze points of enhanced symmetries and enhanced particle spectra

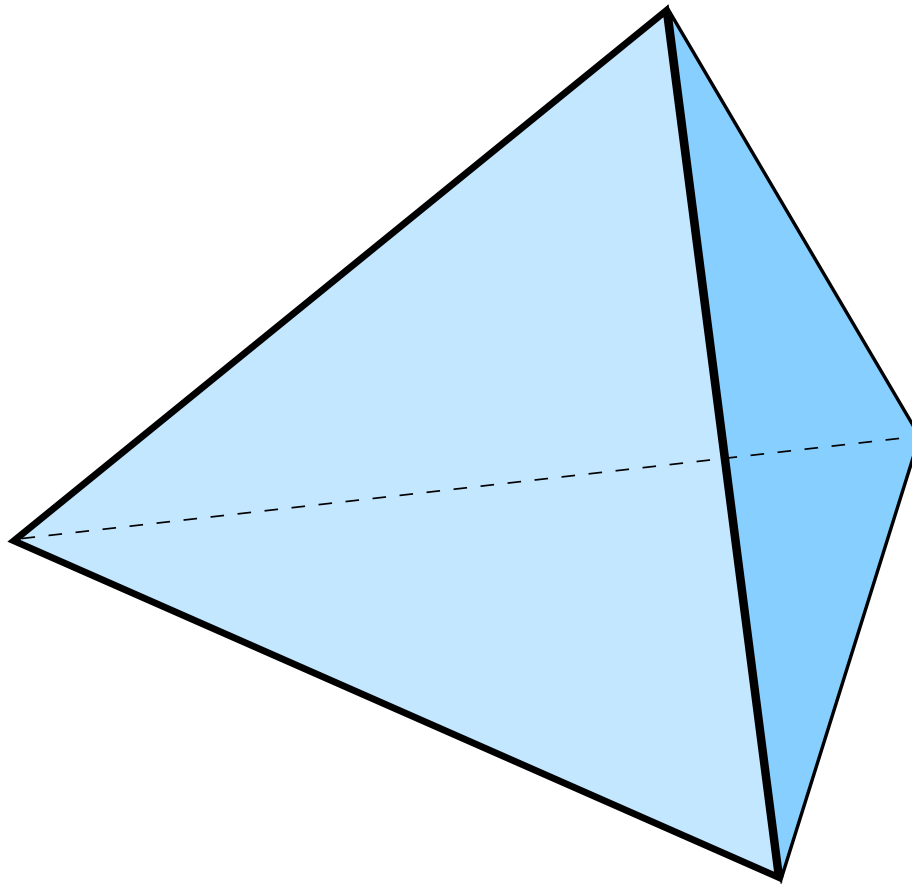
- slightly broken symmetries (Frogatt-Nielsen)
- small parameters to create hierarchies

Hopefully nature is close to points of enhanced calculability.

# Calabi Yau Manifold



# Orbifold



# The fate of smooth compactification

Models on smooth manifolds describe generic points in moduli space

- limited calculability in practice (not full CFT)
- do not see locally enhanced symmetries and spectra
- **but location of fields is of physical relevance**

# The fate of smooth compactification

Models on smooth manifolds describe generic points in moduli space

- limited calculability in practice (not full CFT)
- do not see locally enhanced symmetries and spectra
- **but location of fields is of physical relevance**

As a result, phenomenological analyses of these models often rely on continuous global symmetries

- an approximation is needed for “calculability”
- heterotic Calabi-Yau compactification should be related e.g. to a point with exact CFT

**For F-theory it seems to be a real challenge to find a flat (CFT) approximation.**

# Improve calculability

Have to connect smooth compactification to e.g. flat orbifolds

(Groot Nibbelink et al.; Blaszczyk et al.; 2009-2011)

- resolution of singularities within toric geometry
- is a good approximation in large volume limit

# Improve calculability

Have to connect smooth compactification to e.g. flat orbifolds

(Groot Nibbelink et al.; Blaszczyk et al.; 2009-2011)

- resolution of singularities within toric geometry
- is a good approximation in large volume limit

But there are still some points that have to be clarified

- relation of number of massless states in orbifold and blow-up
- “missing” Yukawa couplings in large volume limit

Local anomalies might play an important role in the attempt to transfer calculability from orbifolds to smooth manifolds.

(Blaszczyk, Cabo Bizet, HPN, Ruehle, 2011)

# The Anomaly Polynomial

The **Green-Schwarz anomaly polynomial** is a useful tool to study the relation between various schemes. The 12-form

$$I_{12}(F_i, R) = I_4 \times I_8$$

contains crucial information on the properties of the model:



# The Anomaly Polynomial

The **Green-Schwarz anomaly polynomial** is a useful tool to study the relation between various schemes. The 12-form

$$I_{12}(F_i, R) = I_4 \times I_8$$

contains crucial information on the properties of the model:

- can be computed independently in the different set-ups
- controls the coupling of “axions” to matter fields
- reveals broken and unbroken (discrete) symmetries.

Relate models of reduced calculability to those where explicit calculations can be done.

(Blaszczyk, Cabo Bizet, HPN, Ruehle, 2011)

# Golden Rules (2011)

- Spinors of  $SO(10)$  (for families)
- Incomplete multiplets (for Higgs)
- Repetition of families (from extra dimensions)
- $N = 1$  supersymmetry
- Importance of discrete symmetries
- globally consistent models
- Berechenbarkeit
- study GS-anomaly polynomial

# Golden Rules (2011)

- Spinors of  $SO(10)$  (for families)
- Incomplete multiplets (for Higgs)
- Repetition of families (from extra dimensions)
- $N = 1$  supersymmetry
- Importance of discrete symmetries
- globally consistent models
- Berechenbarkeit
- study GS-anomaly polynomial

Let us hope that nature sits at a point of enhanced symmetry and calculability.

# This is the place

