Heterotic Standard Models from Calabi-Yau Three-folds

Andre Lukas

University of Oxford

Workshop on Fields and Strings, Corfu, September 2011

arXiv:1106.4804 in collaboration with Lara Anderson, James Gray and Eran Palti

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Overview

- Introduction: Heterotic Calabi-Yau compactifications
- Heterotic line bundle models
- A standard model data base
- A standard model example
- Phenomenological issues
- Conclusion and outlook

Introduction

Data to define a heterotic N=1, d=4 Calabi-Yau compacitification:

- A Calabi-Yau 3-fold X
- A stable holomorphic vector bundle V on X
  with structure group $G \subset SU(n) \subset E_8$
- Anomaly condition: $c_2(V) \leq c_2(TX)$
  (hidden sector with 5-branes and/or bundle)

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- freely acting symmetry Γ on X, so $\hat{X} = X/\Gamma$ is smooth and non simply-connected
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N=1, D=4 standard-like model (hopefully)

Frequently: structure group $G = SU(5) \subset E_8$ with commutant = low energy gauge group $SU(5)$ is used.

$$\mathbf{248}_{E_8} \to [(\mathbf{1},\mathbf{24}) \oplus (\mathbf{5},\bar{\mathbf{10}}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{24},\mathbf{1})]_{SU(5)\times SU(5)}$$

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Here, we study bundles with a "maximal" splitting:

$SU(5) \to S(U(1)^5)$	$SU(5) \to SU(5) \times U(1)^4$

Key features:

- Gauge fields are Abelian and bundle is a sum of line bundles -> much easier to handle technically
- Abelian bundle still carries many of the properties of the generic non-Abelian bundle
- We have to remember that Abelian bundle usually resides in a larger, non-Abelian bundle moduli space.

Heterotic line bundle models

CY manifold X with $h^{1,1}(X)$ Kahler parameters t^i,
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Note: A model is specified by the integer matrix k_a^i.

Recall: 4d gauge group is $SU(5) \times S(U(1)^5) \cong SU(5) \times U(1)^4$

Label $S(U(1)^5)$ representations by integer vectors $\mathbf{q} = (q_1, \ldots, q_5)$ with identification $\mathbf{q} \sim \tilde{\mathbf{q}}$ iff $\mathbf{q} - \tilde{\mathbf{q}} \in \mathbb{Z}(1,1,1,1,1)$.

4d matter: $\mathbf{10}_{\mathbf{e}_a}, \bar{\mathbf{10}}_{-\mathbf{e}_a}, \bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}, \mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}, \mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}, \mathbf{1}_{-\mathbf{e}_a+\mathbf{e}_b}$ for $a < b$.

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$\mathbf{10}_{\mathbf{e}_a}$	$\mathbf{e}_a$	L_a	V
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$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}$	$\mathbf{e}_a+\mathbf{e}_b$	$L_a \otimes L_b$	$\wedge^2 V$
$\mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}$	$-\mathbf{e}_a-\mathbf{e}_b$	$L_a^* \otimes L_b^*$	$\wedge^2 V^*$
$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$	$\mathbf{e}_a-\mathbf{e}_b$	$L_a \otimes L_b^*$	$V \otimes V^*$
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families and mirror families

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C_{ab}^+ bundle
C_{ab}^- moduli

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Number of each multiplet type obtained from $H^1(X,L)$.

bundle supersymmetry: $D_a = k_a^i t_i - \sum_{b>a} \left(|C_{ab}^+|^2 - |C_{ab}^-|^2 \right) \stackrel{!}{=} 0$

At Abelian locus $\langle C_{ab}^{\pm} \rangle = 0$, so slopes $\mu(L_a) = k_a^i t_i \stackrel{!}{=} 0$.

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Bundle moduli and moving away from Abelian locus:

$$0 \to L_a \to U \to L_b \to 0, \quad \mathrm{Ext}^1(L_b, L_a) \cong H^1(X, L_a \otimes L_b^*) \ni C_{ab}^+$$

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$$\langle C_{ab}^\pm \rangle = 0 \implies U = L_a \oplus L_b$$

$$\langle C_{ab}^\pm \rangle \neq 0 \implies U \text{ is } U(2) \text{ bundle}$$

$$L_a \oplus L_b \text{ is replaced by } U$$

What happens with additional U(1) symmetries?

$\longrightarrow$ Basically Green-Schwarz anomalous and massive!

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Models with massless U(1)s are still included. Such U(1)s can be broken spontaneously when moving away from the Abelian locus by switching on VEVs $\langle C_{ab}^{\pm} \rangle$.

A standard model data base

Arena: complete intersection CY manifolds (CICYs)

CICYs defined as common zero locus $X = \{p_i = 0\} \subset \mathcal{A}$ of homogeneous polynomials p_i in ambient space $\mathcal{A} = \bigotimes_{r=1}^{m} \mathbb{P}^{n_r}$.

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for example: quintic $X \sim [\mathbb{P}^4 | 5]$ or bi-cubic $X \sim \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array} \right]$

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Complete classification of about 8000 spaces

(Hubsch, Green, Lutken, Candelas 1987)

Classification of freely-acting discrete symmetries

(Braun, 2010)

Line bundle cohomology can be computed.

(Anderson, He, Lukas, 2008)

The search for heterotic line bundle standard models:

- For a given CICY X with symmetry Γ generate many integer matrices (k^i_a) representing bundles $V = \oplus_{a=1}^5 L_a \; , \;\; L_a = \mathcal{O}_X(\mathbf{k}_a)$.

- Check that $\mu(L_a) = 0$ (bundle supersymmetric) and that V is equivariant under Γ (V descends to X/Γ).

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$\longrightarrow$ "heterotic standard model": SM gauge group (plus U(1)'s, massive or massless), three MSSM families, one or more Higgs doublets, bundle moduli (SM singlets), no exotics.

Have scanned CICYs with symmetries and $h^{1,1}(X) \leq 5$ (60 spaces).

$\left(-9 + h^{1,1} \leq k_a^i \leq 9 - h^{1,1}\right)$ (Anderson, Constantin, Gray, Lukas, Palti, in prep.)

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Find het. standard models on 15 CICYs with $h^{1,1}(X) = 4, 5$:

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no massless U(1)	283
exact MSSM spectrum	217+

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Note: This is counting "upstairs" models. One upstairs model is often valid for more than one symmetry and one choice of Wilson line.

⟶ Total number of downstairs SMs is O(10000).

CICY 6777: $\left(\begin{array}{c\|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 2 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 \\ \mathbb{P}^1 & 2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,1,1,0,-1)(1,0,-2,-1,1)(1,-2,0,-1,1)(-1,1,1,1,-1)(-2,0,0,1,0)	(1,1,0,1,-1)(1,0,-1,0,0)(0,0,1,-1,0)(0,-2,-1,0,1)(-2,1,1,0,0)
(1,0,1,1,-1)(1,-1,0,0,0)(0,1,0,-1,0)(0,-1,-2,0,1)(-2,1,1,0,0)	(1,0,0,-1,0)(1,-1,-2,0,1)(0,1,1,1,-1)(0,-1,1,0,0)(-2,1,0,0,0)
(1,0,0,-1,0)(1,-2,-1,0,1)(0,1,1,1,-1)(0,1,-1,0,0)(-2,0,1,0,0)	(1,1,1,0,-1)(0,1,-1,0,0)(0,0,1,-2,0)(0,-2,-1,1,1)(-1,0,0,1,0)
(1,1,0,1,-1)(0,1,1,-2,0)(0,0,-1,1,0)(0,-2,-1,0,1)(-1,0,1,0,0)	(1,1,-1,-1,0)(0,1,1,1,-1)(0,0,-1,-2,1)(0,-2,1,1,0)(-1,0,0,1,0)
(1,1,1,0,-1)(0,1,0,-2,0)(0,-1,1,0,0)(0,-1,-2,1,1)(-1,0,0,1,0)	(1,0,1,1,-1)(0,1,1,-2,0)(0,-1,0,1,0)(0,-1,-2,0,1)(-1,1,0,0,0)
(1,-1,1,-1,0)(0,1,1,1,-1)(0,1,-2,1,0)(0,-1,0,-2,1)(-1,0,0,1,0)	
CICY 6890: $\left(\begin{array}{c\|ccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 2 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 & 0 \\ \mathbb{P}^4 & 1 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,1,1,0,-1)(1,0,-1,0,0)(1,-2,0,1,0)(-1,1,1,-1,0)(-2,0,-1,0,1)	(1,1,1,0,-1)(1,0,-1,0,0)(0,0,1,-2,0)(0,-1,0,1,0)(-2,0,-1,1,1)
(1,1,0,1,-1)(1,0,1,-2,0)(0,0,-1,1,0)(0,-1,1,0,0)(-2,0,-1,0,1)	(1,1,-1,-1,0)(1,0,1,1,-1)(0,0,-1,-2,1)(0,-1,0,1,0)(-2,0,1,1,0)
(1,1,1,0,-1)(1,-1,1,1,-1)(0,1,-2,-1,1)(0,-2,0,1,0)(-2,1,0,-1,1)	(1,1,0,1,-1)(1,-2,1,0,0)(0,1,-1,0,0)(0,0,1,-1,0)(-2,0,-1,0,1)
(1,0,1,1,-1)(1,0,-1,0,0)(0,1,0,-1,0)(0,-2,1,0,0)(-2,1,-1,0,1)	(1,1,1,0,-1)(1,0,0,-2,0)(0,-1,0,1,0)(-1,0,1,0,0)(-1,0,-2,1,1)
(1,1,1,0,-1)(1,0,-2,1,0)(0,-2,-1,0,1)(-1,1,1,-1,0)(-1,0,1,0,0)	(1,1,1,0,-1)(1,-2,0,1,0)(0,1,0,-1,0)(-1,0,1,0,0)(-1,0,-2,0,1)
(1,0,1,1,-1)(1,0,-2,1,0)(0,-1,0,1,0)(-1,1,1,-1,0)(-1,0,0,-2,1)	(1,0,1,-2,0)(1,-1,0,0,0)(0,1,1,1,-1)(-1,0,0,1,0)(-1,0,-2,0,1)
(1,0,1,1,-1)(1,-2,0,1,0)(0,1,0,-1,0)(-1,1,1,-1,0)(-1,0,-2,0,1)	(1,0,1,1,-1)(1,-2,0,0,0)(0,1,0,-1,0)(-1,1,-2,0,1)(-1,0,1,0,0)
(1,0,0,-1,0)(1,-2,1,0,0)(0,1,1,1,-1)(-1,1,0,0,0)(-1,0,-2,0,1)	(1,1,1,0,-1)(0,1,0,-2,0)(0,0,-1,1,0)(0,-2,-1,1,1)(-1,0,1,0,0)
CICY 7447: $\left(\begin{array}{c\|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \end{array}\right)^{5,45}_{-80}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(0,1,0,-2,1)(0,1,-2,1,0)(0,0,1,1,-2)(0,-1,1,0,0)(0,-1,0,0,1)	(1,-2,0,0,1)(0,1,-2,0,1)(0,0,1,1,-2)(0,0,1,-1,0)(-1,1,0,0,0)
(1,-2,0,0,1)(0,1,0,1,-2)(0,0,1,-2,1)(0,0,-1,0,1)(-1,1,0,1,-1)	(1,-2,-1,1,1)(0,1,1,-2,0)(0,1,-1,0,0)(0,0,1,1,-2)(-1,0,0,0,1)
⋮	⋮

CICY 6777: $\left(\begin{array}{c\|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 2 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 \\ \mathbb{P}^1 & 2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,1,1,0,-1)(1,0,-2,-1,1)(1,-2,0,-1,1)(-1,1,1,1,-1)(-2,0,0,1,0)	(1,1,0,1,-1)(1,0,-1,0,0)(0,0,1,-1,0)(0,-2,-1,0,1)(-2,1,1,0,0)
(1,0,1,1,-1)(1,-1,0,0,0)(0,1,0,-1,0)(0,-1,-2,0,1)(-2,1,1,0,0)	(1,0,0,-1,0)(1,-1,-2,0,1)(0,1,1,1,-1)(0,-1,1,0,0)(-2,1,0,0,0)
(1,0,0,-1,0)(1,-2,-1,0,1)(0,1,1,1,-1)(0,1,-1,0,0)(-2,0,1,0,0)	(1,1,1,0,-1)(0,1,-1,0,0)(0,0,1,-2,0)(0,-2,-1,1,1)(-1,0,0,1,0)
(1,1,0,1,-1)(0,1,1,-2,0)(0,0,-1,1,0)(0,-2,-1,0,1)(-1,0,1,0,0)	(1,1,-1,-1,0)(0,1,1,1,-1)(0,0,-1,-2,1)(0,-2,1,1,0)(-1,0,0,1,0)
(1,1,1,0,-1)(0,1,0,-2,0)(0,-1,1,0,0)(0,-1,-2,1,1)(-1,0,0,1,0)	(1,0,1,1,-1)(0,1,1,-2,0)(0,-1,0,1,0)(0,-1,-2,0,1)(-1,1,0,0,0)
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CICY 6890: $\left(\begin{array}{c\|ccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 2 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 & 0 \\ \mathbb{P}^4 & 1 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,1,1,0,-1)(1,0,-1,0,0)(1,-2,0,1,0)(-1,1,1,-1,0)(-2,0,-1,0,1)	(1,1,1,0,-1)(1,0,-1,0,0)(0,0,1,-2,0)(0,-1,0,1,0)(-2,0,-1,1,1)
(1,1,0,1,-1)(1,0,1,-2,0)(0,0,-1,1,0)(0,-1,1,0,0)(-2,0,-1,0,1)	(1,1,-1,-1,0)(1,0,1,1,-1)(0,0,-1,-2,1)(0,-1,0,1,0)(-2,0,1,1,0)
(1,1,1,0,-1)(1,-1,1,1,-1)(0,1,-2,-1,1)(0,-2,0,1,0)(-2,1,0,-1,1)	(1,1,0,1,-1)(1,-2,1,0,0)(0,1,-1,0,0)(0,0,1,-1,0)(-2,0,-1,0,1)
(1,0,1,1,-1)(1,0,-1,0,0)(0,1,0,-1,0)(0,-2,1,0,0)(-2,1,-1,0,1)	(1,1,1,0,-1)(1,0,0,-2,0)(0,-1,0,1,0)(-1,0,1,0,0)(-1,0,-2,1,1)
(1,1,1,0,-1)(1,0,-2,1,0)(0,-2,-1,0,1)(-1,1,1,-1,0)(-1,0,1,0,0)	(1,1,1,0,-1)(1,-2,0,1,0)(0,1,0,-1,0)(-1,0,1,0,0)(-1,0,-2,0,1)
(1,0,1,1,-1)(1,0,-2,1,0)(0,-1,0,1,0)(-1,1,1,-1,0)(-1,0,0,-2,1)	(1,0,1,-2,0)(1,-1,0,0,0)(0,1,1,1,-1)(-1,0,0,1,0)(-1,0,-2,0,1)
(1,0,1,1,-1)(1,-2,0,1,0)(0,1,0,-1,0)(-1,1,1,-1,0)(-1,0,-2,0,1)	(1,0,1,1,-1)(1,-2,0,0,0)(0,1,0,-1,0)(-1,1,-2,0,1)(-1,0,1,0,0)
(1,0,0,-1,0)(1,-2,1,0,0)(0,1,1,1,-1)(-1,1,0,0,0)(-1,0,-2,0,1)	(1,1,1,0,-1)(0,1,0,-2,0)(0,0,-1,1,0)(0,-2,-1,1,1)(-1,0,1,0,0)
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(0,1,0,-2,1)(0,1,-2,1,0)(0,0,1,1,-2)(0,-1,1,0,0)(0,-1,0,0,1)	(1,-2,0,0,1)(0,1,-2,0,1)(0,0,1,1,-2)(0,0,1,-1,0)(-1,1,0,0,0)
(1,-2,0,0,1)(0,1,0,1,-2)(0,0,1,-2,1)(0,0,-1,0,1)(-1,1,0,1,-1)	(1,-2,-1,1,1)(0,1,1,-2,0)(0,1,-1,0,0)(0,0,1,1,-2)(-1,0,0,0,1)

⋮ ⋮

A standard model example

$L_1 = \mathcal{O}_X(1,0,0,-1,0),\ L_2 = \mathcal{O}_X(1,-1,-2,0,1),\ L_3 = \mathcal{O}_X(0,1,1,1,-1)$

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$$h^\bullet(X, L_2) = (0,4,0,0)$$
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Non-zero cohomologies:

$$\mathbf{10}: \quad \begin{array}{ll} h^\bullet(X, L_2) = (0,4,0,0) & \longrightarrow \mathbf{10_{e_2}},\ \mathbf{10_{e_2}} \\ h^\bullet(X, L_5) = (0,2,0,0) & \longrightarrow \mathbf{10_{e_5}} \end{array}$$

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$\bar{\mathbf{5}} - \mathbf{5}:$

$h^\bullet(X, L_2 \otimes L_4) = (0,4,0,0) \longrightarrow \bar{\mathbf{5}}_{\mathbf{e}_2+\mathbf{e}_4},\ \bar{\mathbf{5}}_{\mathbf{e}_2+\mathbf{e}_4}$

$h^\bullet(X, L_4 \otimes L_5) = (0,2,0,0) \longrightarrow \bar{\mathbf{5}}_{\mathbf{e}_4+\mathbf{e}_5}$

$h^\bullet(X, L_2 \otimes L_5) = (0,1,1,0) \longrightarrow \bar{\mathbf{5}}^{H_d}_{\mathbf{e}_2+\mathbf{e}_5},\ \mathbf{5}^{H_u}_{-\mathbf{e}_2-\mathbf{e}_5}$

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$h^\bullet(X, L_4 \otimes L_5) = (0,2,0,0) \longrightarrow \bar{\mathbf{5}}_{\mathbf{e_4+e_5}}$

$h^\bullet(X, L_2 \otimes L_5) = (0,1,1,0) \longrightarrow \bar{\mathbf{5}}^{H_d}_{\mathbf{e_2+e_5}},\ \mathbf{5}^{H_u}_{\mathbf{-e_2-e_5}}$

$\mathbf{1}:$

$h^\bullet(X, L_1 \otimes L_2^*) = (0,0,4,0)$

$h^\bullet(X, L_1 \otimes L_5^*) = (0,0,8,0)$

$h^\bullet(X, L_2 \otimes L_3^*) = (0,4,0,0)$

$h^\bullet(X, L_2 \otimes L_4^*) = (0,12,0,0)$

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$h^\bullet(X, L_2) = (0,4,0,0)$ $\longrightarrow$ $\mathbf{10_{e_2}},\ \mathbf{10_{e_2}}$

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$h^\bullet(X, L_2 \otimes L_5) = (0,1,1,0)$ $\longrightarrow$ $\bar{\mathbf{5}}^{H_d}_{\mathbf{e_2+e_5}},\ \mathbf{5}^{H_u}_{\mathbf{-e_2-e_5}}$

$\mathbf{1}$:

$h^\bullet(X, L_1 \otimes L_2^*) = (0,0,4,0)$ $C_{1,\mathbf{e_2-e_1}}$

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$h^\bullet(X, L_4 \otimes L_5^*) = (0,6,0,0)$ $C_{7,\mathbf{e_4-e_5}}$

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$h^\bullet(X, L_2 \otimes L_5^*) = (0,11,3,0)$ $\quad C_{5,\mathbf{e}_2-\mathbf{e}_5},\ C_{6,\mathbf{e}_5-\mathbf{e}_2}$

$h^\bullet(X, L_4 \otimes L_5^*) = (0,6,0,0)$ $\quad C_{7,\mathbf{e}_4-\mathbf{e}_5}$

$\epsilon_I = \langle C_I \rangle$

Phenomenological issues

$S(U(1)^5)$ symmetry restricts operators in 4d theory.

Note: $S(U(1)^5)$ non-invariant operators are perturbatively forbidden, but allowed operators are not necessarily present.

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- Dimension four proton decay at Abelian locus

$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}\bar{\mathbf{5}}_{\mathbf{e}_c+\mathbf{e}_d}\mathbf{10}_{\mathbf{e}_f}$ allowed if $\mathbf{e}_a+\mathbf{e}_b+\mathbf{e}_c+\mathbf{e}_d+\mathbf{e}_f=(1,1,1,1,1)$

Forbidden for example.

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- Dimension four proton decay away from Abelian locus

  $p(C_I)\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10}$ can be checked explicitly, since all charges known

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Again, forbidden for example.

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$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}\mathbf{10}_{\mathbf{e}_c}\mathbf{10}_{\mathbf{e}_d}\mathbf{10}_{\mathbf{e}_f}$ allowed if $\mathbf{e}_a+\mathbf{e}_b+\mathbf{e}_c+\mathbf{e}_d+\mathbf{e}_f=(1,1,1,1,1)$

Ok for example (also with singlet insertions).

μ - term

$H_u H_d$ is singlet, but term absent at Abelian locus.

Away from Abelian locus: $C_I H_u H_d$ forbidden

$C_I C_J H_u H_d$ may be allowed

For example: $C_5 C_6 H_u H_d$ allowed, so we need $\epsilon_5 \epsilon_6 \ll 1$.

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$H_u H_d$ is singlet, but term absent at Abelian locus.

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$\mathbf{5}^{H_u}_{-\mathbf{e}_a-\mathbf{e}_b}\mathbf{10}_{\mathbf{e}_c}\mathbf{10}_{\mathbf{e}_d}$ allowed if $\mathbf{e}_a+\mathbf{e}_b=\mathbf{e}_c+\mathbf{e}_d$

$\bar{\mathbf{5}}^{H_d}_{\mathbf{e}_a+\mathbf{e}_b}\bar{\mathbf{5}}_{\mathbf{e}_c+\mathbf{e}_d}\mathbf{10}_{\mathbf{e}_f}$ allowed if $\mathbf{e}_a+\mathbf{e}_b+\mathbf{e}_c+\mathbf{e}_d+\mathbf{e}_f=(1,1,1,1,1)$

Away from Abelian locus add singlets -> Froggatt-Nielson

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For example: $Y^u = \begin{pmatrix} \epsilon_5 & 1 & 1 \\ 1 & \epsilon_6 & \epsilon_6 \\ 1 & \epsilon_6 & \epsilon_6 \end{pmatrix}$, $Y^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(needs non-perturbative effects)

Conclusion and outlook

- Heterotic line bundle models on CY manifolds are a useful and technically accessible arena for string model building.
- We have found 1000+ (upstairs) heterotic standard models on CICYs with $h^{1,1}(X) \leq 5$.
- The $S(U(1)^5)$ symmetry restricts the 4d theory and facilitates phenomenological analysis beyond the computation of the spectrum.

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Thanks