Moduli stabilization in early superstring cosmology

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Introduction

Moduli stabilization in superstring cosmology

- Giving mass to moduli arising from compactification.
- Cosmological moduli problem: coherent oscillation of moduli dominates the energy of the universe $\rightarrow$ late time decay may spoil big bang nucleosynthesis
  - Can be solved by fabricating super-heavy moduli (KKLT, racetrack, etc)

In our work:

- We propose an alternative approach to moduli stabilization using thermal superstring models.
- Early universe: before EWSB phase transition.
- Lifting flat directions by non supersymmetric compactification
  - SUSY breaking by T: Scherk-Schwarz reduction on $S^1(R_0)$, $\beta = 2\pi R_0 = T^{-1} \Rightarrow M_{\text{string}} \gg T \gg \Lambda_{\text{EW}}$.
  - Nontrivial thermal vacuum energy generated at one-loop level.

Effective action approach

- To take into account both quantum effect and thermal effect: quantum effective action Γ under finite T:

$$e^{-\Gamma[\varphi]} = \int_{1\text{PI}} \mathcal{D}\eta\, e^{-S[\varphi+\eta]} \;\Rightarrow\; \Gamma[\varphi] = S[\varphi] + Z_{\text{1-loop}}[\varphi] + \ldots$$

- In φ: spacetime metric $ds^2 = -N^2(t)dt^2 + a(t)^2 d\vec{x}^2$; moduli $\vec{\Phi}$.
- $Z_{\text{1-loop}} =$  $=$ thermal string partition function.
- $\mathcal{F} := -\dfrac{Z_{\text{1-loop}}}{\beta V_{D-1}} =$ free energy density.
- General form of the effective action :

$$\Gamma[g_{\mu\nu}, \vec{\Phi}] = \int d^D x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} F_{MN} \partial\Phi^M \partial\Phi^N - \mathcal{F}(\vec{\Phi}, T) \right],$$

- $\mathcal{F} = \mathcal{F}_{\text{min}}$: necessary condition for moduli stabilization.

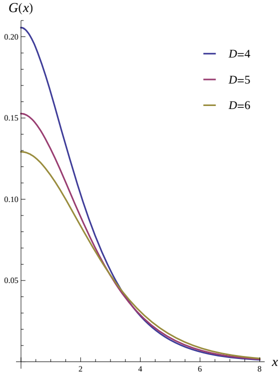
The free energy density

- General form of the thermal free-energy density

$$\mathcal{F}(T,\vec{\Phi}) = -\int_0^\infty \frac{d\ell}{2\ell}\frac{1}{(2\pi\ell)^{\frac{D}{2}}}\sum_{\text{states}} e^{-\frac{1}{2}M_s(\vec{\Phi})^2\ell}\sum_{k_0} e^{-\frac{(2k_0+1)^2}{2T^2\ell}}$$

$$= -T^D\sum_s G\big(M_s(\vec{\Phi})/T\big)$$

  - States of $M_s \ll T$ contribute to $\mathcal{F}$.
  - States of $M_s \gg T$ are suppressed by $e^{-M_s/T}$ (Boltzman factor).



- $\mathcal{F} = \mathcal{F}_{\min}$: $\vec{\Phi}_0$ where $M_s(\vec{\Phi}_0) = 0$ for certain s.
  - Enhancement of gauge symmetry. [P.Ginsparg, C.Vafa, '87]
  - Topology change of the internal CY space.

Heterotic cosmology

- $SO(32)$ heterotic strings on $S^1(R_0)\times\mathbb{R}^{D-1}\times\prod_{i=D}^{9}S^1(R_i)$
- Moduli space: $\{R_D,\ldots,R_9;\phi\}$.
- Effective action:

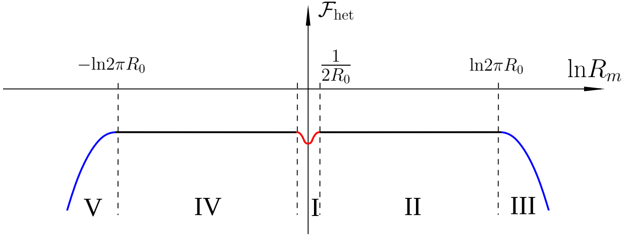
$$\Gamma[g_{\mu\nu},\phi,R_i]=\int d^Dx\sqrt{-g}\,\frac{1}{2}\left[R-\frac{4}{D-2}(\partial\phi)^2-\sum_{i=D}^{9}\left(\frac{\partial R_i}{R_i}\right)^2\right]+Z.$$

- $$Z=\frac{\hat\beta\hat V_{D-1}}{2(2\pi)^D}\int_{\mathcal{F}}\frac{d\tau_1 d\tau_2}{\tau_2^{D/2}}\frac{1}{\eta^8\bar\eta^{24}}\sum_{a,b}(-1)^{a+b+ab}\frac{\theta[^a_b]^4}{\eta^4}\,\Gamma[^a_b](R_0)\prod_{i=D}^{9}\Gamma(R_i)\,\bar\Gamma_{O(32)/\mathbb{Z}_2}.$$
  - $$\Gamma[^a_b](R_0)=\frac{R_0}{\sqrt{\tau_2}}\sum_{\tilde m,n}e^{-\frac{\pi R_0^2}{\tau_2}|\tilde m+\tau n|^2}(-1)^{a\tilde m+bn+\tilde m n}$$
  - $$\Gamma(R_i)=\frac{R_i}{\sqrt{\tau_2}}\sum_{\tilde m,n}e^{-\frac{\pi R_i^2}{\tau_2}|\tilde m+\tau n|^2}$$

Heterotic cosmology

- Focusing on one radius R_m, fixing all other radii, $\mathcal{F} = -\frac{Z}{\beta V_{D-1}}$:

$$\mathcal{F} = -T^D \left[c_D n_0 + n_0 \sum_{m \neq 0} G\left(|m| e^{-|\ln R_m|}/\hat{T}\right) + n_1 G\left(\left|R_m - R_m^{-1}\right|/\hat{T}\right) + \dots \right].$$



- $\hat{T} = e^{-\frac{2\phi}{D-2}} T$: string frame temperature
- Local minimum of $\mathcal{F}$ due to the states of mass $|R_m - R_m^{-1}|$, vanishing at $R_m = 1$.
  - Strings wrapping once $S^1(R_m)$, carrying one unit of momentum.
  - Non Cartan states of the gauge symmetry enhancement $U(1) \to SU(2)$.

Heterotic cosmology

Let $\Phi_m := \ln R_m$, solving equations of motion near $\Phi_m = 0$:

- $a \propto T^{-1} \propto t^{2/D}$, $\rho_{\rm tot} = H^2 \propto a^{-D}$
- $\ddot{\Phi}_m + (D-1)H\dot{\Phi}_m + M_m^2\,\Phi_m = 0$, dynamical mass $M_m \propto T^{\frac{D-2}{2}}$.
  - $\Phi_m \sim t^{-1/2} \sin\left(\lambda_m\, t^{2/D} + \text{phase}\right)$
    $\Phi_m \sim t^{-1/2} \sin\left(M_m\, t + ...\right)$ for $M_m = \text{const.}$
  - $\rho_m \propto \dot{\Phi}_m^2 \sim a^{2-\frac{3D}{2}}$
    $\rho_m \propto \dot{\Phi}_m^2 \sim 1/a^{D-1}$ for $M_m = \text{const.}$

Cosmological moduli problem:

- $D > 4$: $\rho_m \ll \rho_{\rm tot}$: radiation dominated universe. ✓
- $D = 4$: $\rho_m \propto \rho_{\rm tot} \propto a^{-4}$. Other SUSY breaking mechanism required [F.Bourliot, J.Estes, C.Kounnas, H.Partouche '08].

Extension: including all moduli ϕ, G_{ij}, B_{ij}, Y_i^I

- All moduli except ϕ can be stabilized.
- Moduli attractor: gauge symmetry enhancement points.

Type I cosmology by duality

- Type I strings on $S^1(R_0) \times \mathbb{R}^{D-1} \times \prod_{i=D}^{9} S^1(R_i)$
- Dictionary of type I/heterotic duality:
  - $\lambda_{\rm h} = \frac{1}{\lambda_{\rm I}}$, $R_{\rm h} = \frac{R_{\rm I}}{\sqrt{\lambda_{\rm I}}}$, where $\lambda_{\rm h(I)}$ is heterotic (type I) string coupling in 10D
  - $Z_{\rm TI}(T, \lambda_{\rm I}) = Z_{\rm het}(T, 1/\lambda_{\rm I})$ for $\lambda_{\rm h} \ll 1$.
- Type I free energy density by duality, near heterotic self-dual point $R_{\rm h} \sim 1$, or $R_{\rm I} \sim \sqrt{\lambda_{\rm I}}$

$$\mathcal{F}_{\rm het} = -T^D \left[c_D n_0 + n_1 G\left(\frac{1}{\hat{T}_{\rm h}} \left| R_{\rm h} - \frac{1}{R_{\rm h}} \right| \right) + \dots \right] \Longrightarrow$$

$$\mathcal{F}_{\rm TI} = -T^D \left[c_D n_0 + n_1 G\left(\frac{1}{\hat{T}_{\rm I}} \left| \frac{R_{\rm I}}{\lambda_{\rm I}} - \frac{1}{R_{\rm I}} \right| \right) + \dots \right]$$

- $R_{\rm I}$ stabilized at $\sqrt{\lambda_{\rm I}}$ by non perturbative states of mass $\left| \frac{R_{\rm I}}{\lambda_{\rm I}} - \frac{1}{R_{\rm I}} \right|$
  - From $D1$ branes wrapping once the internal circle with one unit of momentum
  - Enhanced gauge symmetry $U(1) \to SU(2)$
  - Also true for $\lambda_{\rm h} \gg 1$: BPS states, will survive strong-coupling extrapolation

Type I cosmology by duality

Extension:

- Direct computation in type I picture?
  - $D1$ brane contribution cannot be inferred from direct perturbative computation in type I picture.
  - However $Z_{\rm TI}$ can be rewritten into the form of $E1$-instanton sum. Technique of first-principle computation of instantonic contribution may be applied.

- Switching on all type I moduli ϕ, G_{ij}, C_{ij}, Y_i^I.
  - All moduli except the dilaton can be stabilized at enhanced gauge symmetry points by either open string perturbative states or non perturbative $D1$ brane states.

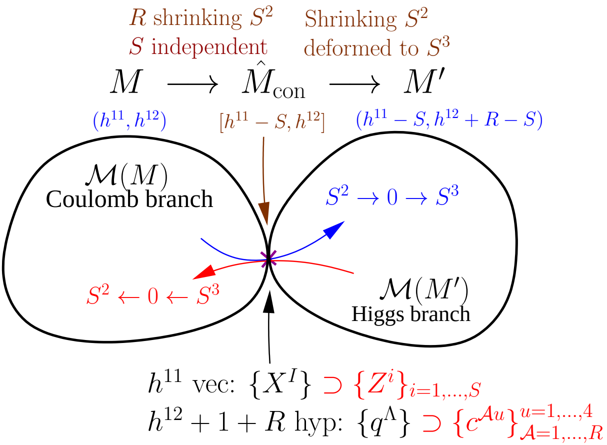
Stabilization of dilaton:

- Maybe 5-brane states can stabilize dilaton when $D \leq 5$.
- Examine type II strings on CY
  - Contains NS5-brane contribution by type II/heterotic duality.

Type II cosmology

- Type II strings compactified on Calabi-Yau (CY) three-folds.
  - Type IIA on CY space M or type IIB on mirror W.
  - Low energy effective theory: $\mathcal{N}_4 = 2$ SUGRA.
- Moduli space: $\mathcal{M} = \mathcal{M}_V \times \mathcal{M}_H$.
  - $\mathcal{M}_V$: scalar components of $h^{1,1}(M)$ vector multiplets
  - $\mathcal{M}_H$: scalar components of $h^{1,2}(M) + 1$ hyper multiplets
- Look for possibility of moduli stabilization: anywhere in $\mathcal{M}$ where extra massless states emerge to minimize $\mathcal{F}$?
- Non perturbative massive states becoming massless when the CY undergoes topology change by extremal transition:
  - Conifold transition [A.Strominger '95]
  - Gauge symmetry enhancement [A.Klemm, P.Mayr '96], [S.H.Katz, D.R.Morrison, M.Ronen Plesser '96]

Type II cosmology — conifold transition



Fields relevant to conifold transition:

- The S vector multiplets $\{Z^i\}$: related to the S independent shrinking S^2.
- The R hypermultiplets $\{c^{\mathcal{A}u}\}$: from the R shrinking S^2.
- Z^i and $c^{\mathcal{A}u}$ vanish at conifold loci.

Type II cosmology — conifold transition

- Resulting $\mathcal{N}_4 = 2$ SUGRA:

$$S = \int d^4x\,\sqrt{-g}\,\Big[\frac{1}{2}R - g_{I\bar{J}}\,\partial X^I \partial \bar{X}^J - h_{\Lambda\Sigma}\,\nabla q^\Lambda \nabla q^\Sigma - V_s(X,q) + \dots\Big].$$

- Expanding $g_{I\bar{J}}$, $h_{\Lambda\Sigma}$ and $V_s(X,q)$ about conifold loci, demanding that the zero-th order metrics have $U(1)^S$ isometry:

$$S = \int d^4x\,\sqrt{-g}\,\Big[\frac{1}{2}R + \mathcal{L}^{\mathcal{N}=2}_{\text{gauge}}(Z^i, c^{\mathcal{A}u}) + \mathcal{L}_{\text{R}} + \dots\Big].$$

$$\mathcal{L}^{\mathcal{N}=2}_{\text{gauge}} = -g_{i\bar{j}}\,\partial Z^i \partial \bar{Z}^j - \nabla c^{\mathcal{A}u} \nabla c^{\mathcal{A}v} - V_s(X,c),$$

$$V_s = g_c^2\, e^{\mathcal{K}} \sum_{i,j} \Big(4 \sum_{\mathcal{A},u} Q_i^{\mathcal{A}} Q_j^{\mathcal{A}}\, \bar{Z}^i Z^j\, c^{\mathcal{A}u} c^{\mathcal{A}u} + g^{i\bar{j}}\, \vec{D}_i \cdot \vec{D}_j\Big),$$

$$\vec{D}_i = \sum_{\mathcal{A}} Q_i^{\mathcal{A}}\, \mathscr{C}^{\mathcal{A}\dagger}\, \vec{\sigma}\, \mathscr{C}^{\mathcal{A}}, \quad \mathscr{C}^{\mathcal{A}\dagger} := \begin{pmatrix} -c^{\mathcal{A}2} + i\,c^{\mathcal{A}1} \\ c^{\mathcal{A}3} - i\,c^{\mathcal{A}4} \end{pmatrix},$$

$$\nabla_\mu c^{\mathcal{A}u} = \partial_\mu c^{\mathcal{A}u} + g_c Q_i^{\mathcal{A}}\, V_\mu^i\, t^u{}_v\, c^{\mathcal{A}v}.$$

  - A sum of gravity action with a rigid $\mathcal{N} = 2\ U(1)^S$ supersymmetry gauge field action formally coupled to gravity.
  - Moduli irrelevant to conifold transition are decoupled, no potential generated.

Type II cosmology — conifold transition

- Focus on relevant fields Z^i and $c^{\mathcal{A}u}$, and compute $\Gamma[\langle Z\rangle, \langle c\rangle]$

<table>
<tr><th>Branch</th><th>$\langle Z^i\rangle$</th><th>$\langle c^{\mathcal{A}u}\rangle$</th></tr>
<tr><td rowspan="2">Coulomb</td><td>$\neq 0$, moduli</td><td>$= 0$, non pert. states of mass $\propto \langle Z^i\rangle$</td></tr>
<tr><td colspan="2">$\Gamma = \int d^4x\sqrt{-g}\left\{\frac{R}{2} - (\partial\langle Z\rangle)^2 - T^4\left[c_4 n_0 + \sum G(\langle Z\rangle/T)\right] + \dots\right\}$
$\Longrightarrow$ $\langle Z\rangle$ attracted to 0</td></tr>
<tr><td rowspan="2">Higgs</td><td>$= 0$, non pert. states of mass $\propto \langle c^{\mathcal{A}u}\rangle$</td><td>$\neq 0$, moduli</td></tr>
<tr><td colspan="2">$\Gamma = \int d^4x\sqrt{-g}\left\{\frac{R}{2} - (\partial\langle c\rangle)^2 - T^4\left[c_4 n_0 + \sum G(\langle c\rangle/T)\right] + \dots\right\}$
$\Longrightarrow$ $\langle c\rangle$ attracted to 0</td></tr>
</table>

- All moduli relevant to conifold transition are stabilized at conifold loci, internal CY attracted to $\hat{M}_{\rm con}$

Type II cosmology — Gauge symmetry enhancement

- Gauge symmetry enhancement $M \to \hat{M}_{\rm enh} \leftarrow \tilde{M}$:

$N-1$ shrinking S^2 along a curve of genus g — Shrinking S^2 deformed to S^3

$$M \longrightarrow \hat{M}_{\rm enh} \longrightarrow \tilde{M}$$

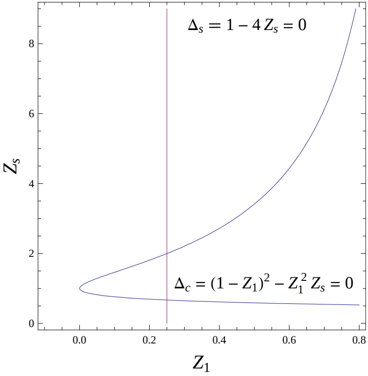
$SU(N)$ with g hypers in the adj

- All moduli relevant to gauge symmetry enhancement are stabilized, the internal CY attracted to $\hat{M}_{\rm enh}$, where we have $SU(N)$ with g hypers in the adjoint.

	M	$\hat{M}_{\rm enh}$	$\tilde{M}$
$\longrightarrow$	$(h^{1,1}, h^{1,2})$	$[h^{1,1}-(N-1), h^{1,2}-g(N-1)]$	$(h^{1,1}-(N-1), h^{1,2}+g(N^2-N)-(N^2-1))$
$\longleftarrow$	$(\tilde{h}^{1,1}+(N-1), \tilde{h}^{1,2})$	$[\tilde{h}^{1,1}, \tilde{h}^{1,2}-(g-1)(N^2-1)]$	$(\tilde{h}^{1,1}, \tilde{h}^{1,2})$

Example: a model with heterotic dual

- Type IIA strings compactified on $M \in \mathbf{P}^4_{(1,1,2,2,6)}[12](2,128)$.
  - Conifold: $\Delta_c = (1-z_1)^2 - z_1^2 z_s = 0$, no Higgs branch.
  - Gauge symmetry enhancement: $\Delta_s = 1 - 4z_s = 0$, with $U(1) \to SU(2)$, $g = 2$ hypers in the adjoint. Higgs branch: $\tilde{M} \in P^4_{(1,1,1,1,4)}[8](1,129)$



- Moduli attracted to the crossing points of $\Delta_c = 0$ and $\Delta_s = 0$.
  - Both of the vector multiplet moduli are stabilized.

    $(2,128) \to [0,126] \leftarrow (1,129)$

- Heterotic dual on $K3 \times T^2$ exists

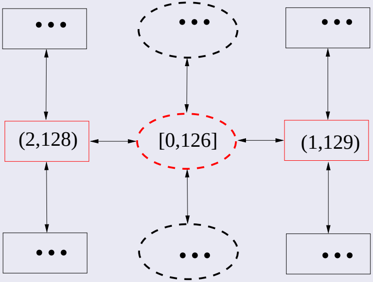
  [S.Kachru, C.Vafa 96']

  - Stabilization of all vec. moduli in the heterotic dual.
  - Stabilization of heterotic dilaton.

Type II cosmology

Perspective

- Possibility of stabilize also all hypermultiplet moduli in the web of CYs.



- Move in the web of Calabi-Yau spaces to reach smaller Hodge number $h^{1,2}$.
- Expect to stabilize all the hypers except the universal hypermultiplet.
- Consider generalized CY with flux to stabilize the universal hypermultiplet.