Corfu September 2011

Aspects of F-theory Unification

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GREECE

based on work with:

Vlachos, 1105.1858; Phys. Lett. B

Tracas & G.Tsamis, 1102.5244; Eur.Phys.J

Road Map for
F-UNIFICATION

Indications:

▲ Merging of SM gauge couplings at high scales $\Longrightarrow$

$$\mathbf{G_{SM}} = SU(3) \times SU(2) \times U(1) \in SU(5),\ SO(10), \ldots$$

▲ *"See-Saw"* neutrino masses $\Longrightarrow$

$$m_\nu{}^{eff.} = \frac{m_D^2}{M_N}$$

$\star\, m_D \sim m_W \Longrightarrow M_N \sim M_{GUT}.$

▲ Minimal GUT: **SU(5)**

▲ $SU(5)$ Chiral and Higgs Representations:

$$\begin{aligned} 10 &\rightarrow Q + u^c + e^c \\ \bar{5} &\rightarrow d^c + \ell \\ 5 + \bar{5} &\rightarrow (T + h_u) + (\bar{T} + h_d) \end{aligned}$$

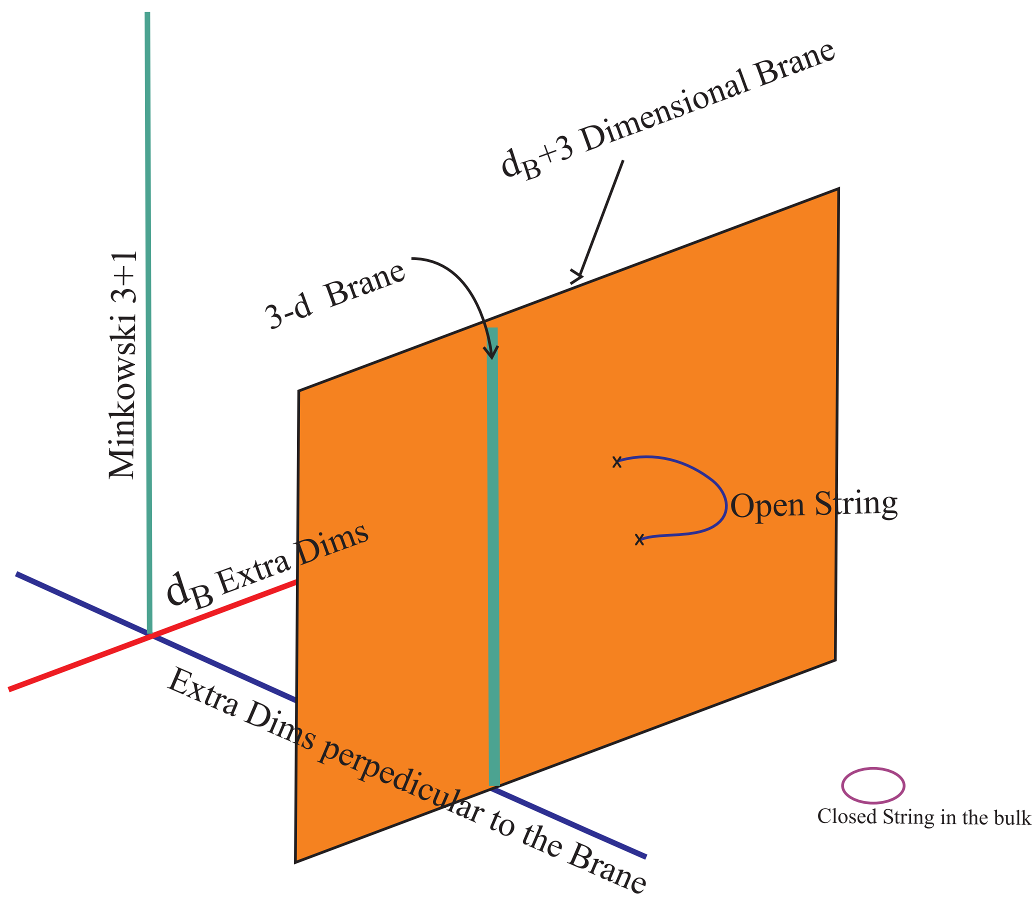
▲ Yukawa Couplings:

$$10 \cdot 10 \cdot 5 \rightarrow m_{top}$$

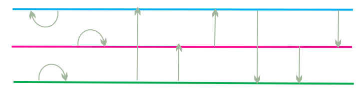
$$10 \cdot \bar{5} \cdot \bar{5} \rightarrow m_b$$

STRING-BRANE Unification Scenario...

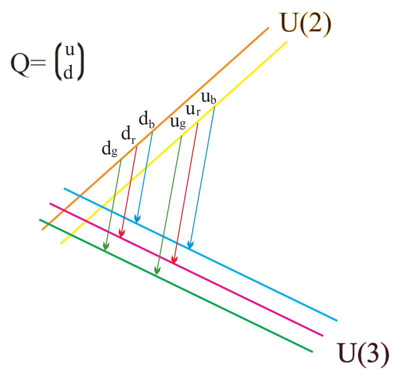
Gravity (closed strings) and QFT (particles ∼ open strings)



U(3) gauge bosons



Quark doublets



A plausible SM scenario... intersecting branes.

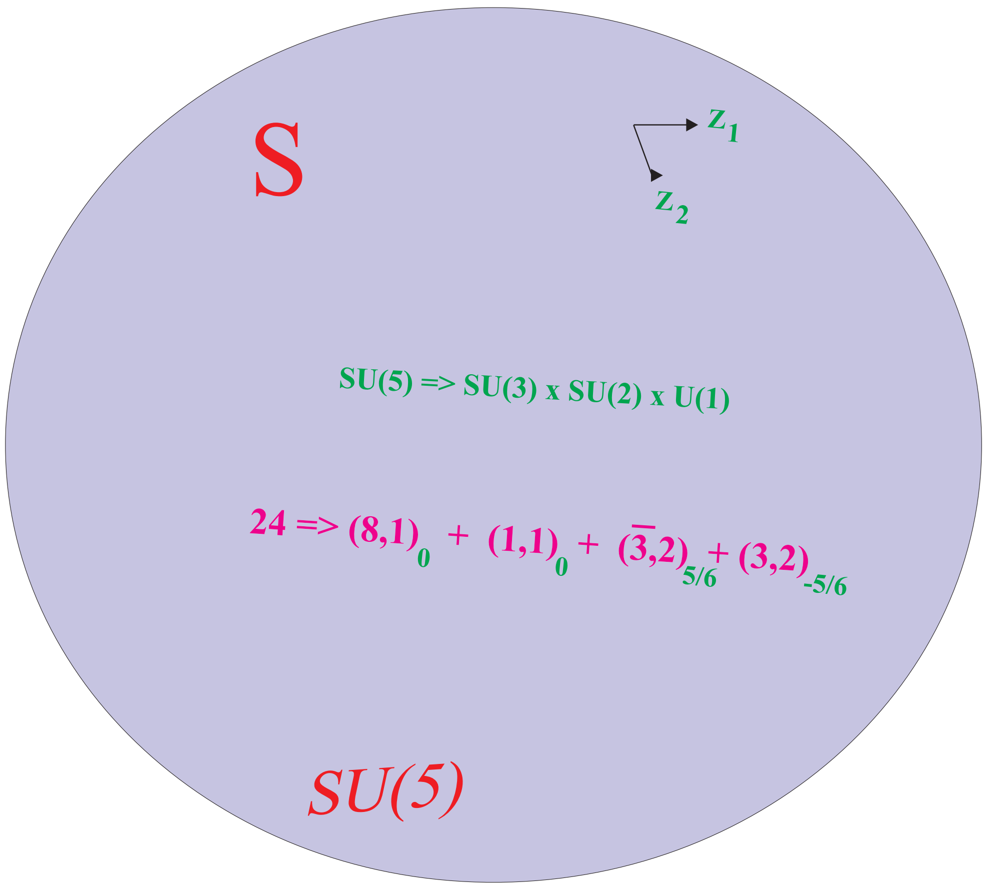
Shortcomings:

No Unification: $g_3(M_U) \neq g_2(M_U)$

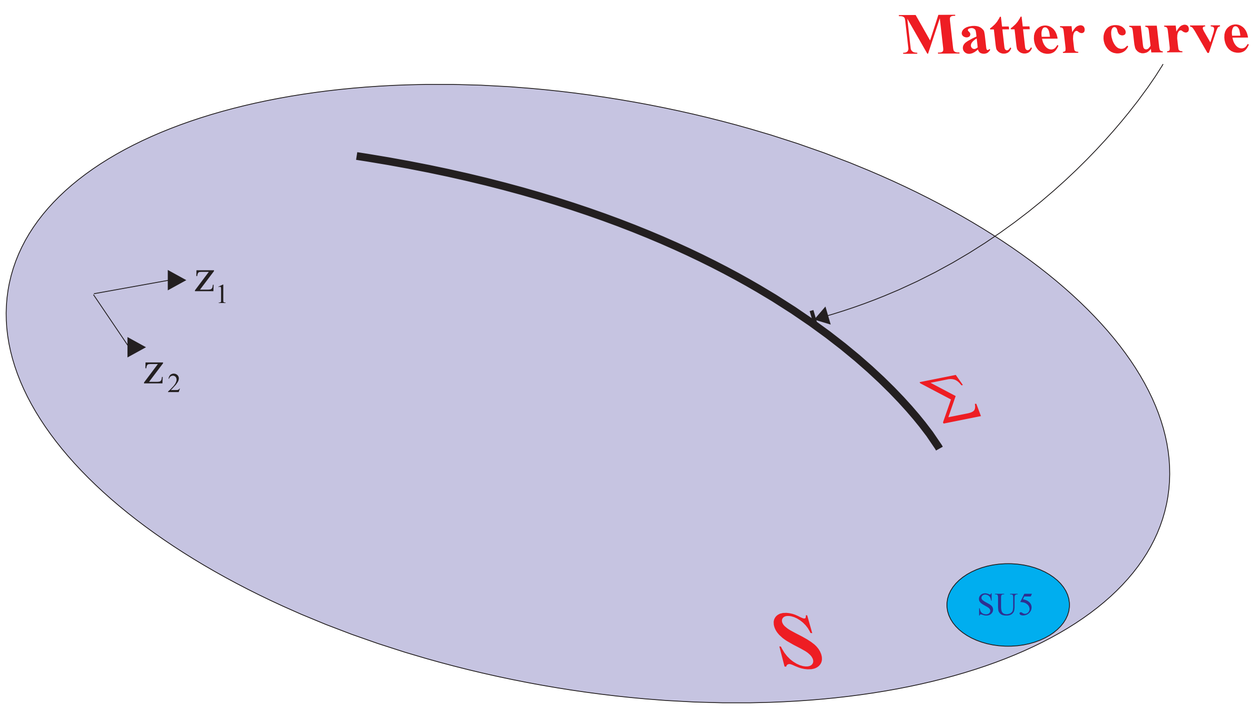
No $SU(5)$-top coupling ~~$10 \cdot 10$~~ $\cdot\, 5$!

$F - SU(5)$

- ▲ S *'internal'* **2-complex dim.** surface
- ▲ S has a singularity associated to gauge group G_S

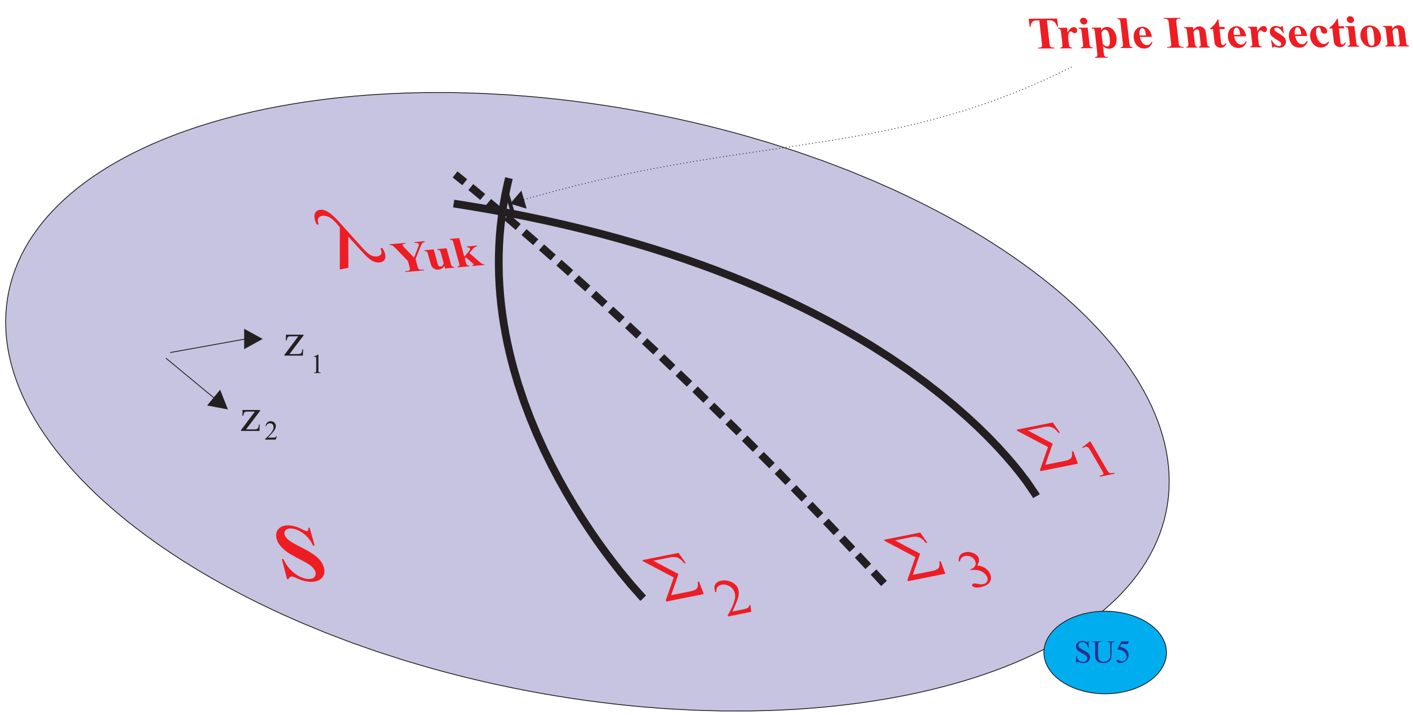


Matter resides along intersections with other 7-branes...

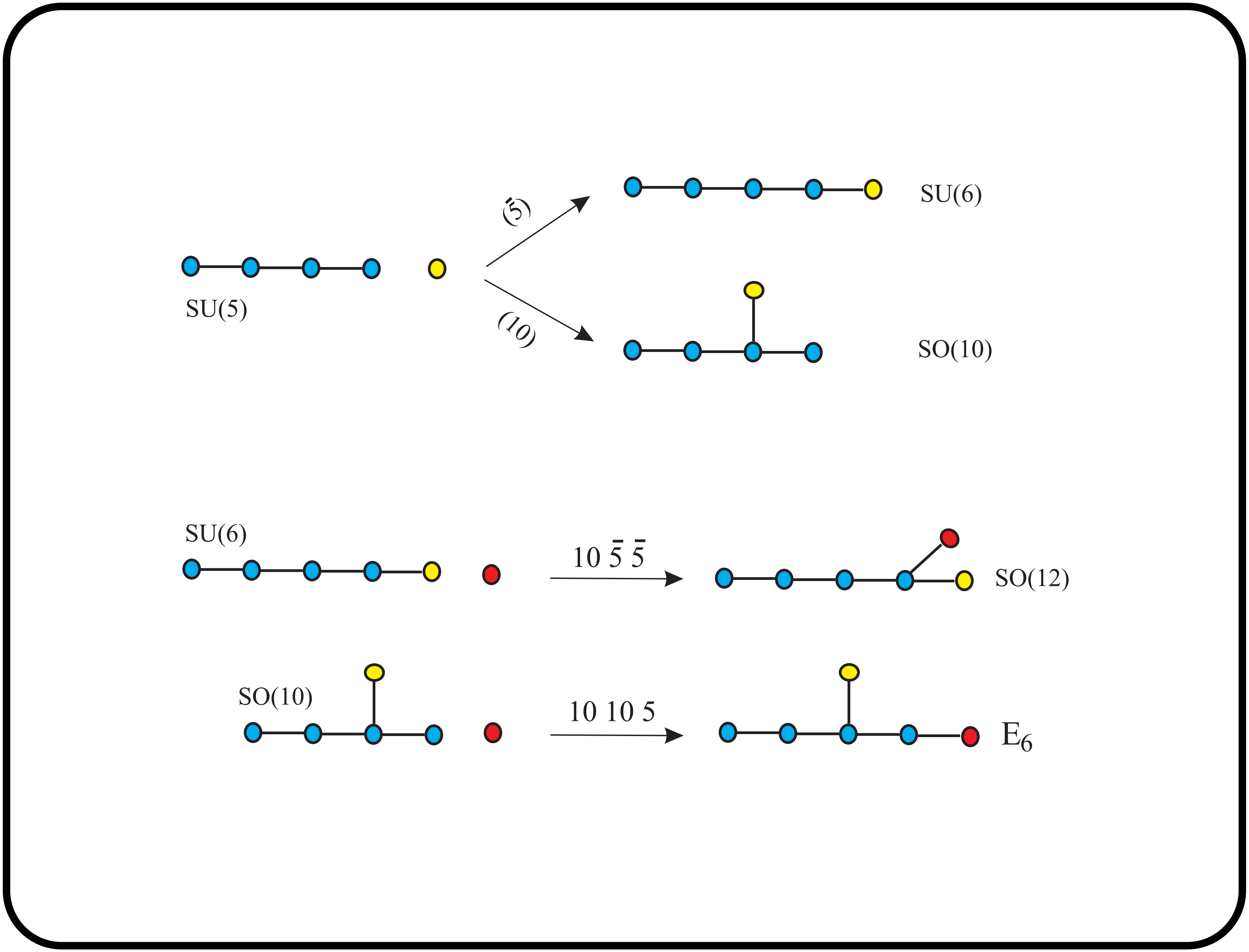


Along a matter curve Σ gauge symmetry is enhanced...

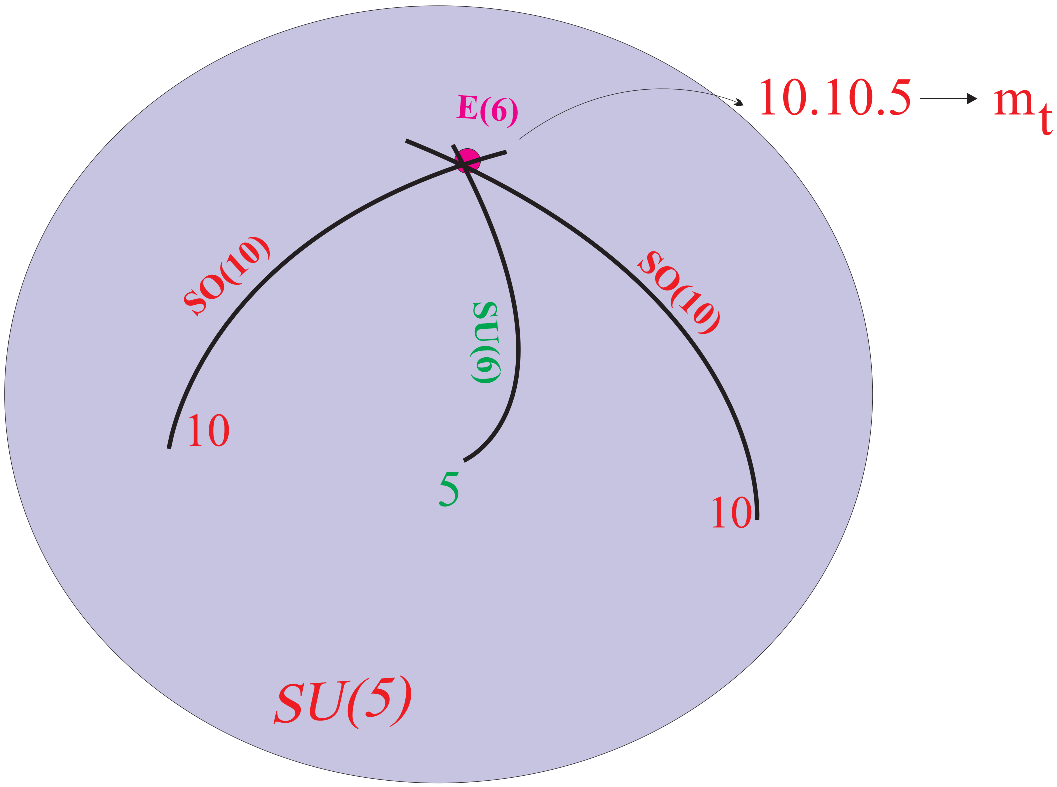
Yukawa couplings



gauge symmetry ... further ... enhanced!


(5̄)
SU(6)
SU(5)
(10)
SO(10)
SU(6)
10 5̄ 5̄
SO(12)
SO(10)
10 10 5
E_6

top Yukawa enhancement ...



Along a matter curve Σ gauge symmetry is enhanced...

★ *for details see talk of E. Dudas*

$\cdots \Rightarrow$ matter resides along a) ‘matter curves’ and b) in the ‘bulk’:

For each of these massless states

$\Downarrow$

$\exists$ massive KK-modes

$\Downarrow$

KK-threshold effects posing a threat to Unification

F-$SU(5)$ THRESHOLDS

1. $U(1)_Y$ Flux Thresholds, *Blumenhagen,* 0812.0248
2. KK-massive Modes, *Donagi et al.*, 0808.222
3. D_3 brane probes, *Heckman et al.*, 1103.3287
4. SUSY

I. Flux thresholds

Correction terms through CS-action $S_{CS} \sim \int C_0 \wedge F^4$:

$$\frac{1}{a_3(M_G)} = \frac{1}{a_G}, \ \frac{1}{a_2(M_G)} = \frac{1}{a_G} + x, \ \frac{1}{a_1(M_G)} = \frac{1}{a_G} + \frac{3}{5}x$$

(*Blumenhagen, Phys.Rev.Lett.102:071601,2009.*)

$$x = -\frac{1}{2}\mathrm{Re}S \int c_1^2(\mathcal{L}_Y)$$

$S = e^{-\phi} + \imath C_0 \ \rightarrow$ axion-dilaton field.

x: *not completely arbitrary*:

$$24 \rightarrow (8,1)_0 + (1,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6}$$

elimination of exotics $(3,2), (\bar{3},2) \ \Rightarrow$

$$\chi(S, \mathcal{L}_{\mathcal{Y}}) = 0 \ \Rightarrow \int c_1^2(\mathcal{L}_Y) = -2$$

At M_{GUT} :

$$\frac{5}{3}\frac{1}{a_1(M_G)} = \frac{1}{a_2(M_G)} + \frac{2}{3}\frac{1}{a_3(M_G)} \tag{1}$$

★ $a_i(M_G)$ relation weaker than unification $a_i(M_G) = a_G$.

Let M_X : decoupling scale of extra matter

$$\underbrace{[5(b_1^x - b_1) - 2(b_3^x - b_3)]}_{\hookrightarrow\ \mathcal{B}} \quad \times \quad \ln\left(\frac{M_G}{M_X}\right) = 0$$

a: **Minimal case:** If extra matter **only** *color* triplets, $[\cdots] \equiv 0 \Rightarrow$

$$\mathcal{B} = 0 \quad \Rightarrow \quad \forall\ x \quad \exists \quad M_X$$

$\Rightarrow M_G \sim 2 \times 10^{16}$GeV is found as the scale where (1) holds.

b: **generalization with various states and decoupling scales**:

(Callaghan, GKL, King, Ross arXiv:1109.11399)

II: Threshold effects from heavy KK-modes

A: The gauge multiplet

One Loop running of gauge couplings:

$$\frac{16\pi^2}{g_a^2(\mu)} = \frac{16\pi^2}{g_s^2} k_a + b_a \log\frac{\Lambda^2}{\mu^2} + \mathcal{S}_a^{(g)}, \quad a = 3, 2, Y \cdot \tag{2}$$

Λ : Cutoff scale.

KK-Thresholds:

$$\mathcal{S}_a^{(g)} = 2\sum_i \mathrm{Tr}_{R_i} Q_a^2 \;\; \mathrm{Str}_{M\neq 0}\left(\frac{1}{12} - \chi^2\right)\log\left(\frac{\Lambda^2}{M^2}\right)$$

χ: helicity operator

R_i: representations from the adjoint decomposition $24 = \bigoplus R_i$

$$24 \to (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6}$$

M: mass of KK-mode $\cdots \Rightarrow \cdots$

Laplacian $\Delta = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial}$ on Riemann surface acting on k-form

$$\Delta_{k,R}\,\psi_n^{(k)} = \lambda_n^{(k)}\,\psi_n^{(k)}$$

In 4-d, eigenvalues → mass squared: $\lambda_n^{(k)} \to M^2$

Thresholds written as (*Friedmann & Witten, hep-th/0211269*)

$$\mathcal{S}_a^{(g)} = 2\sum_i \mathrm{Tr}_{R_i}(Q_a^2)\mathcal{K}_i$$

$\mathcal{K}_i$ found to be equal twice the Ray-Singer analytic torsion

$$\mathcal{T}_{R_i} = \frac{1}{2}\sum_{k=0}^{2}(-1)^{k+1}\log\det{}'\frac{\Delta_{k,R_i}}{\Lambda^2}$$

(*... more about this ... later on in a specific example...*)

Ray-Singer Theorem:

(Annals Math. 98 (1973) 154-177)

Torsion $\mathcal{T}_R$ independent of the metric of the Manifold

independent of cutoff scale Λ

$$\Downarrow$$

$$\mathcal{T}_R = Topological\ Invariant$$

Gauge multiplet thresholds can be cast to the form

$$\mathcal{S}'^{(g)}_a = 20\, k_a \mathcal{T}_{5/6} + \frac{4}{3} b_a{}^{(g)} \left(\mathcal{T}_{5/6} - \mathcal{T}_0\right) \qquad (3)$$

$$k_a \to \{k_3 : k_2 : k_Y\} = \{1 : 1 : \frac{5}{3}\}; \quad b_a^{(g)} = (0, -6, -9)$$

Gauge coupling running becomes:

$$\frac{16\pi^2}{g_a^2(\mu)} = \underbrace{k_a \left(\frac{16\pi^2}{g_s^2} + 20\mathcal{T}_{5/6}\right)} + \underbrace{b_a^{(g)} \log \frac{\exp\left[4/3\left(\mathcal{T}_{5/6} - \mathcal{T}_0\right)\right]}{\mu^2 \mathbf{V}^{1/2}}}$$

$$= k_a \frac{16\pi^2}{g_U^2} + b_a^{(g)} \log \frac{M_{GUT}^2}{\mu^2} \qquad (4)$$

with the GUT scale defined as:

$$M_{GUT} = e^{2/3\left(\mathcal{T}_{5/6} - \mathcal{T}_0\right)}\, \mathbf{M_C}$$

($\mathbf{M_C} = \mathbf{V}^{-1/4}$)

B: The Chiral and Higgs Fields

1) Bulk fields

$$(3,2)_{-5/6} + (\bar{3},2)_{5/6}; \quad \chi(S, L_Y) = 0 \to \delta\mathcal{S} = 0$$

2) $10, \overline{10} \in \Sigma_{10}$, $5, \bar{5} \in \Sigma_5$
(quarks and leptons + possible exotic matter...)

light modes

KK-massive modes

2a)Chiral spectrum:

For complete $SU(5)$ multiplets:

$$b_a \propto k_a \rightarrow (b_3 : b_2 : b_Y) \propto \left\{1 : 1 : \frac{5}{3}\right\}$$

cutoff Λ' can be absorbed in g_U redefinition:

$$\begin{aligned}\frac{16\pi^2}{g_U^2} k_a + b_a \log\left(\frac{\Lambda'^2}{\mu^2}\right) &= \underbrace{k_a \left(\frac{16\pi^2}{g_U^2} + b_0 \log\left(\frac{\Lambda'^2}{M_{GUT}^2}\right)\right)} \\ &+ b_a \log\left(\frac{M_{GUT}^2}{\mu^2}\right) \qquad (5)\end{aligned}$$

$$\Rightarrow \qquad \frac{16\pi^2}{g_{GUT}^2} = \frac{16\pi^2}{g_U^2} + b_0 \log\left(\frac{\Lambda'^2}{M_{GUT}^2}\right)$$

Refinements:

▲ Color triplets should become massive at $M_X \leq M_{GUT}$

$$5_H \to \not{T} + h_u \quad \bar{5}_H \to \not{\bar{T}} + h_d$$

$$b_a \log\left(\frac{M_{GUT}^2}{\mu^2}\right) \to b_a^{MSSM} \log\left(\frac{M_{GUT}^2}{\mu^2}\right) + b_a^T \log\left(\frac{M_{GUT}^2}{M_X^2}\right)$$

▲ $U(1)_Y$ **Flux** (breaking $SU(5)$) splits $10, 5$ multiplets

restrictions on $b_{Y,2,3} = (5/3, 1, 1)$ **imply**

$$\sum_j |M_{10}^j| - \sum_i |M_5^i| = \sum_j |M_{10}^j - N_{10Y}^j| - \sum_i |M_5^i + N_{5Y}^i| \quad (6)$$

$$2\sum_j |M_{10}^j| = \sum_j |M_{10}^j + N_{10Y}^j| + \sum_j |M_{10}^j - N_{10Y}^j| \quad (7)$$

(*GKL*, *N.D. Tracas, G. Tsamis*, *arXiv:1102.5244*)

2b)KK massive spectrum:

$\Delta^R_{i,Y}$: Laplacian acting on chiral/antichiral representations:

$$\mathcal{K}_{\Sigma_R} = -\frac{1}{2}\log\det{}'\frac{\Delta^R_{0,Y}}{\Lambda'^2} - \frac{1}{2}\log\det{}'\frac{\Delta^R_{1,Y}}{\Lambda'^2},\ R = \bar{5},\ 10$$

For a representation r

$$S^r_a = \sum_i 2\mathrm{Tr}(Q^2_{a,r})\mathcal{K}_i.$$

Recall that r refers to

$$10 \to (3,2)_{\frac{1}{6}} + (\bar{3},1)_{-\frac{2}{3}} + (1,1)_1, \quad \bar{5} \to (\bar{3},1)_{\frac{1}{3}} + (1,2)_{-\frac{1}{2}}.$$

Thresholds	$SU(3)$	$SU(2)$	$U(1)$
$S^{\bar{5}}_a$	$\mathcal{K}_{1/3}$	$\mathcal{K}_{-1/2}$	$\mathcal{K}_{-1/2} + 2/3\,\mathcal{K}_{1/3}$
S^{10}_a	$2\mathcal{K}_{1/6} + \mathcal{K}_{-2/3}$	$3\mathcal{K}_{1/6}$	$1/3\mathcal{K}_{1/6} + 2\mathcal{K}_1 + 8/3\mathcal{K}_{-2/3}$

$\mathbf{\Sigma_{10}}$-matter curve: Take first $a = Y$:

$$S_Y^{10} = \frac{1}{3}\mathcal{K}_{1/6} + \frac{8}{3}\mathcal{K}_{-2/3} + 2\mathcal{K}_1$$

$$= \frac{8}{3}\left(\mathcal{K}_{-2/3} - \mathcal{K}_{1/6}\right) - 2\left(\mathcal{K}_{1/6} - \mathcal{K}_1\right) + 3 \cdot \frac{5}{3}\mathcal{K}_{1/6}. \quad (8)$$

★**POSTULATE:**

$$\mathcal{K}_{q_i} - \mathcal{K}_{q_j} = f(q_i - q_j)$$

Then:

$$S_a^{10} = +\frac{4}{3}\,\beta_a^{10}\,(\mathcal{T}_{-2/3} - \mathcal{T}_{1/6}) + k_a \cdot (6\,\mathcal{T}_{1/6})$$

$\mathbf{\Sigma_{\bar{5}}}$-matter curve:

$$S_a^{\bar{5}} = -\frac{4}{3}\,\beta_a^{\bar{5}}\,(\mathcal{T}_{-1/2} - \mathcal{T}_{1/3}) + k_a \cdot (2\,\mathcal{T}_{1/3}) \quad (9)$$

⇒ Because of:

$$\left.\begin{array}{lll} \mathcal{A}: & \mathcal{K}_{-1/2} - \mathcal{K}_{1/3} & = \mathcal{T}_{-2/3} - \mathcal{T}_{1/6} \\ \mathcal{B}: & \beta_a^{10} = \beta_a^{\bar{5}} & = \left\{\frac{3}{2}, 0, 1\right\} \end{array}\right\} \quad (10)$$

⇒ Total correction from KK-massive modes on matter curves:

$$S_a^{\bar{5}} + S_a^{10} = k_a \cdot (2\,\mathcal{T}_{1/3} + 6\,\mathcal{T}_{1/6}) \;\propto\; k_a$$

⇒ **Only effect:**

SHIFT of gauge coupling value at M_{GUT}

$$k_a \frac{16\pi^2}{g_{GUT}^2} = k_a \left(\frac{16\pi^2}{g_U^2} + 2\,\mathcal{T}_{1/3} + 6\,\mathcal{T}_{1/6} \right)$$

Implementation

Postulate $\Leftrightarrow$ existence of non-trivial line bundle with appropriate structure.

Is there such a thing?

YES!

Example:

$g = 1$ Riemann Surfaces

Thresholds (functions of $\mathcal{T}$) expressed in terms of Laplacian Δ_k eigenvalues

$$\Delta_{k,R(V)} = (\bar{\partial} + \bar{\partial}^\dagger)^2 = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial} \cdot$$

$$\Delta_{k,R(V)}\psi_k^n = \lambda_n^k \psi_k^n$$

To compute thresholds we need the associated zeta-function

$$\zeta_{\Delta_k}(s) \quad = \quad \sum_n \frac{1}{\lambda_n^s} \; = \; \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} \mathrm{Tr}\left(e^{-\Delta_k\, t}\right) t \tag{11}$$

Torsion written as

$$\mathcal{T} \quad = \quad \sum_k (-1)^{k+1}\, k \;\left. \frac{d\zeta_{\Delta_k}(s)}{ds} \right|_{s=0} . \tag{12}$$

A laborious calculation gives (*Ray & Singer 1973*):

$$\mathcal{T}_z = \ln \left| \frac{e^{\pi\, i\, v^2 \tau} \vartheta_1(z, \tau)}{\eta(\tau)} \right|, \quad z = u - \tau\, v \tag{13}$$

★ Observe that under $v \to v-1$:

$$\mathcal{T}_{z=u-\tau(v-1)} = \ln \left| \frac{e^{\pi\, i\tau\,(v-1)^2} \vartheta_1(u-\tau(v-1),\tau)}{\eta(\tau)} \right| = \cdots = \mathcal{T}_{z=u-\tau v} \quad (14)$$

★ Assume the hypercharge embedding :

$$v_i \quad = \quad \frac{q_i}{|q_i - q_j|}, \; q_i = hypercharge$$

then:

$$\boxed{\mathcal{T}_{-2/3} - \mathcal{T}_{1/6} = \mathcal{T}_{-1/2} - \mathcal{T}_{1/3} = 0\,!}$$

$$\Downarrow$$

In agreement with our POSTULATE !

OUTCOME

★ KK-thresholds $\propto$ Analytic Torsion $\mathcal{T}$.

★ GUT-scale intimately related to KK-massive modes scale

★ These imply topologically invariant ratio:

$$\frac{M_{GUT}}{M_C} = e^{\frac{2}{3}(\mathcal{T}_{5/6}-\mathcal{T}_0)}$$

★ cutoff scale independence (Λ) $\rightarrow$ constraints on spectrum

★ SHIFT of the GUT value of gauge coupling

$$\frac{16\pi^2}{g^2_{GUT}} = \frac{16\pi^2}{g^2_s} + \sum_i c_i \mathcal{T}_i$$

ADDITIONAL MATERIAL

★ Twisted YM and F-Spectrum

10-d Super YM theory :

$$\begin{cases} 10\,dim \text{ Gauge Field } A \\ \text{Adjoint fermions in } 16_+ \text{ of } SO(9,1) \end{cases}$$

Under Reduction $SO(9,1) \to SO(7,1) \times U(1)_R$ fields decompose to

$$\begin{cases} 8\,dim \text{ Gauge Field } A \\ \text{scalars } \varphi, \bar{\varphi} = A_8 \pm i\, A_9 \\ \text{fermions } \Psi_\pm = (S_\pm, \pm\frac{1}{2}) \end{cases}$$

F-theory described by 8-d YM Compactified on $R^{7,1} = R^{3,1} \times S$.

$$SO(7,1) \times U(1)_R \to SO(3,1) \times SO(4) \times U(1)_R$$

The 8-d spinor Ψ_+ decomposes $(O(4) \sim SU(2) \times SU(2))$

$$\left(S_+, \frac{1}{2}\right) \to \left((2,1), (2,1), \frac{1}{2}\right) \oplus \left((1,2), (1,2), -\frac{1}{2}\right)$$

$\Rightarrow$ globally, NOT well defined!

TWIST:

$J \sim U(1) \in U(2), \;\; J_R \sim U(1)_R \to J_\pm = J \pm 2J_R$

$\Rightarrow$

$$\left(S_+, \frac{1}{2}\right) \to \{(2,1) \otimes 2_1\} \oplus \{(1,2) \otimes (1_2 \oplus 1_0)\}$$

preserving $\mathcal{N} = 1$ SUSY.

(*Beasley, Heckmann, Vafa, 0802.3391*)

- Under twisting, scalars & fermions become **forms:**

$$\text{scalars} : \varphi = \varphi_{mn}\, dz^m \wedge dz^n$$

$$\text{fermions} : = \begin{cases} \eta_\alpha & (0,0) \\ \psi_{\dot\alpha} = \psi_{\dot\alpha m}\, dz^m & (1,0) \\ \chi_\alpha = \chi_{\dot\alpha mn}\, dz^m \wedge dz^n & (2,0) \end{cases}$$

The above can be organised in $\mathcal{N} = 1$ multiplets

$$(\mathbf{A}_\mu, \eta),\ (\mathbf{A}_{\bar{\mathbf{m}}}, \psi_{\bar{\mathbf{m}}}),\ (\phi_{\mathbf{12}}, \chi_{\mathbf{12}})$$

An $F - SU(5)$ fulfilling the requirements
(GKL, GG Ross JHEP 1102:108,2011.)

Chiral Matter

	M	N	Q	u^c	e^c		M	N	d^c	L
$10^{(1)}\ (F_3)$	1	0	1	1	1	$5^{(4)}\ (\bar{f}_1)$	-1	0	-1	-1
$10^{(2)}\ (F_{2,1})$	1	-1	1	2	0	$5^{(1)}\ (\bar{f}_2)$	-1	0	-1	-1
$10^{(3)}\ (F_{1,2})$	1	1	1	0	2	$5^{(2)}\ (\bar{f}_3)$	-1	0	-1	-1
$10^{(4)}\ (-)$	0	0	0	0	0	$5^{(3)}\ (-)$	0	0	0	0

Higgs and Colour Triplets

	M	N	T	$h_{u,d}$
$5^{(0)}\ (h_u, T)$	1	0	1	1
$5^{(5)}\ (h_d)$	0	-1	0	-1
$5^{(6)}\ (\bar{T})$	-1	1	-1	0