AdS/CFT and the cosmological constant problem

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CORFU

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- In QFT vacuum energy not observable
- In Gravity vacuum energy *does* backreact
- Quantum fluctuations $\Rightarrow \rho_{1-loop} \sim M_{cut}^4$
- If we take $M_{cut} \sim M_P$ then

$$\frac{\rho_{observed}}{\rho_{1-loop}} \sim 10^{-120}$$

Cosmological constant fine-tuning

- "Renormalization"

$$\rho_{obs} = \rho_{bare} + \rho_{1-loop}$$

- Two terms must be fine-tuned, one part in 10^{120}

- Will not discuss what fixes specific ρ_{obs} or cosmic coincidence problem etc.

c.c. fine-tuning and effective field theory

- c.c. problem indicates something wrong with effective field theory

- Naturalness $\Rightarrow$

$$S_{eff} \sim \int \left(c_1 \, M_{cut}^{4-\Delta_1} \, \mathcal{O}_1 \; + \; c_2 \, M_{cut}^{4-\Delta_2} \, \mathcal{O}_2 + \ldots \right)$$

  with $c_1, c_2, \ldots \sim \mathcal{O}(1)$

- Vacuum energy term $\Delta = 0$ (most relevant operator) $\Rightarrow$ should be multiplied by M_{cut}^4

- In a natural theory vacuum energy should be of order M_{cut}^4

- Is there a symmetry/dynamical mechanism to suppress value of c.c.?

Spacetime is emergent

- Why don't quantum fluctuations gravitate?

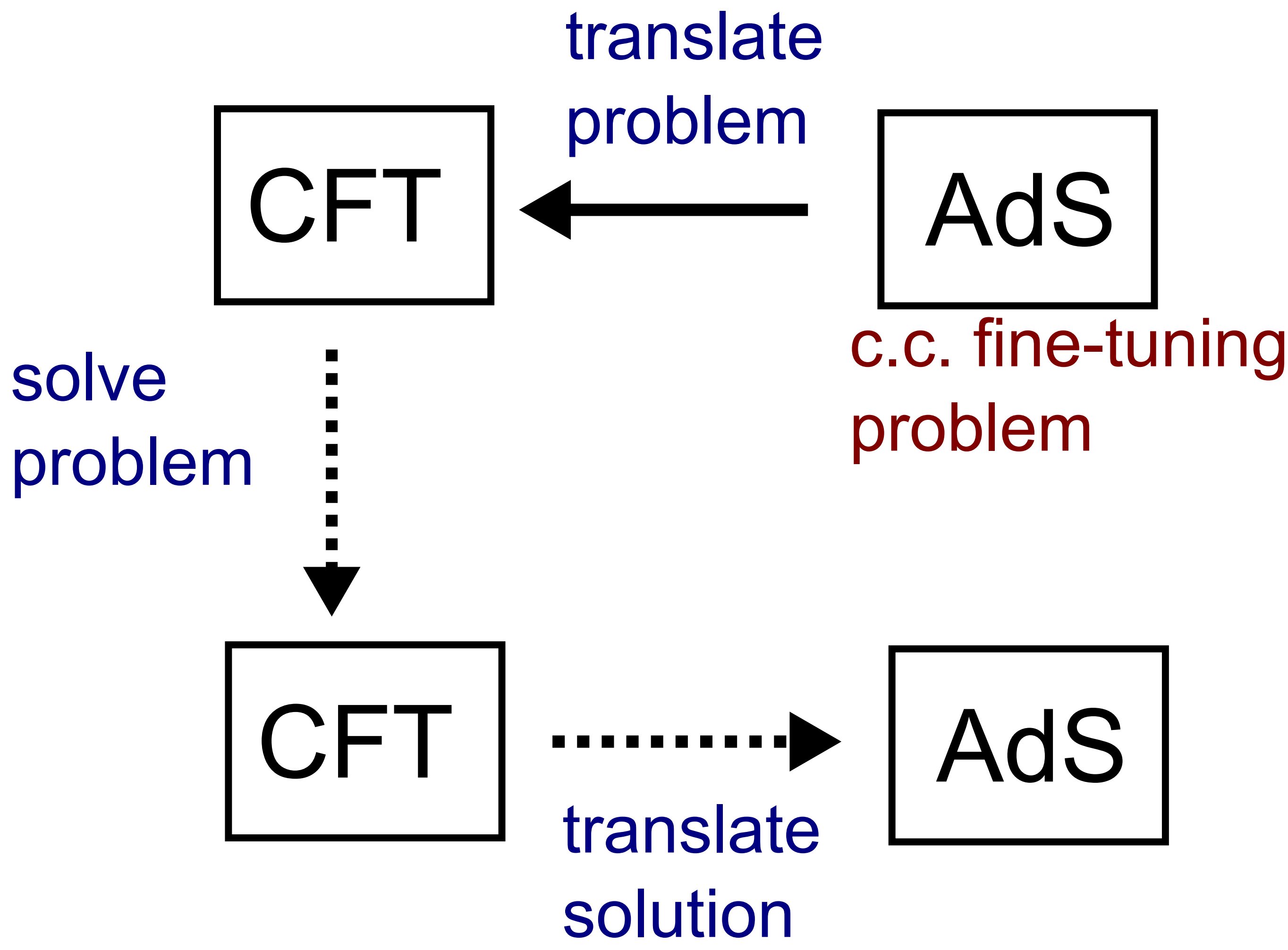
- Question independent of $\Lambda > 0$ or $\Lambda < 0$

- c.c. fine tuning also in AdS

- AdS/CFT: spacetime and gravity are EMERGENT from CFT

- Should be able to see (via the dual CFT) whether quantum fluctuations gravitate or not

Strategy


translate
problem
CFT
AdS
c.c. fine-tuning
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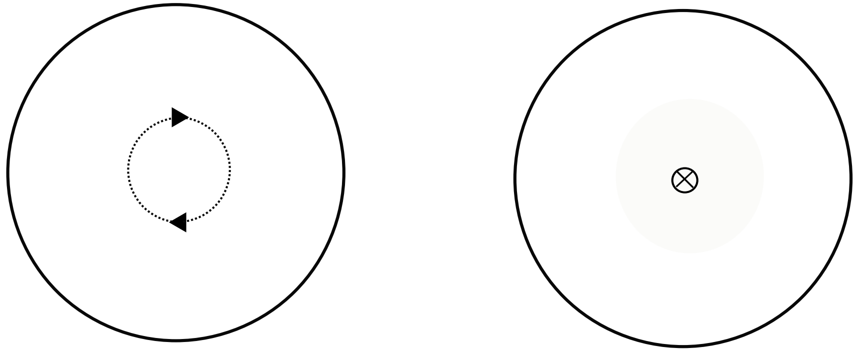
MAIN QUESTION

How do we translate the bulk cosmological constant fine-tuning problem into a CFT question?

How do we proceed?



- Need to understand meaning of "1-loop diagram" and "counterterm" ($\sim$ bare c.c.) in CFT language

- Difficult to map the bulk vacuum-to-vacuum amplitudes in CFT

- More familiar with computation of Witten diagrams (diagrams with external legs)

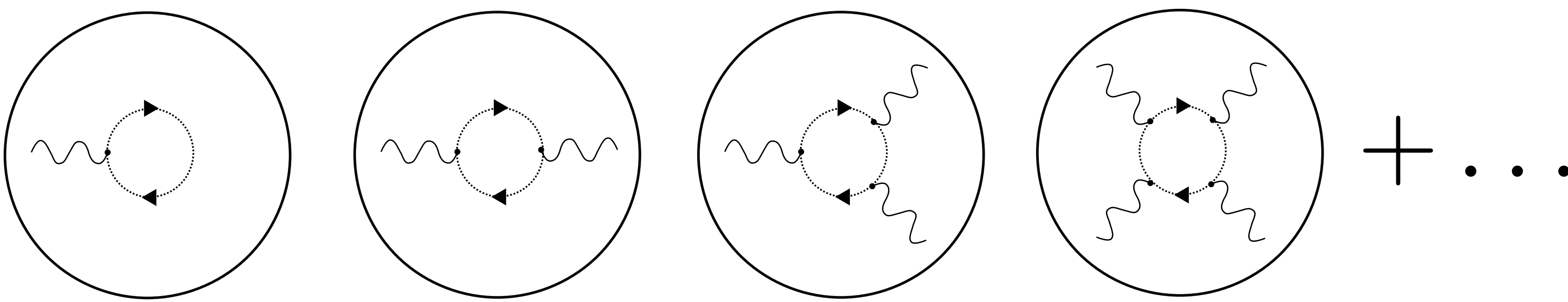
c.c. fine-tuning in correlation functions

- c.c. counterterm has the form

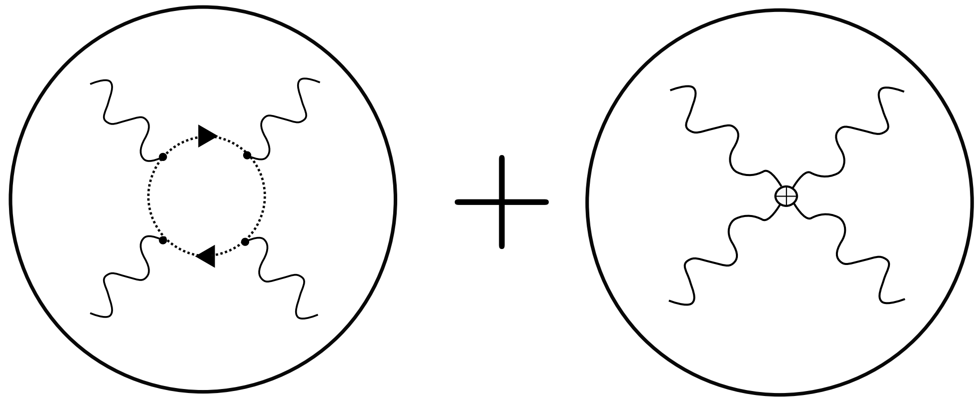
$$\int d^4x \sqrt{g}\, \Lambda_{bare}$$

- Expanding $\sqrt{g}$ generates graviton vertices with arbitrary number of external legs

- Necessary to (partly) cancel diagrams of the form



Example: graviton 4-point function



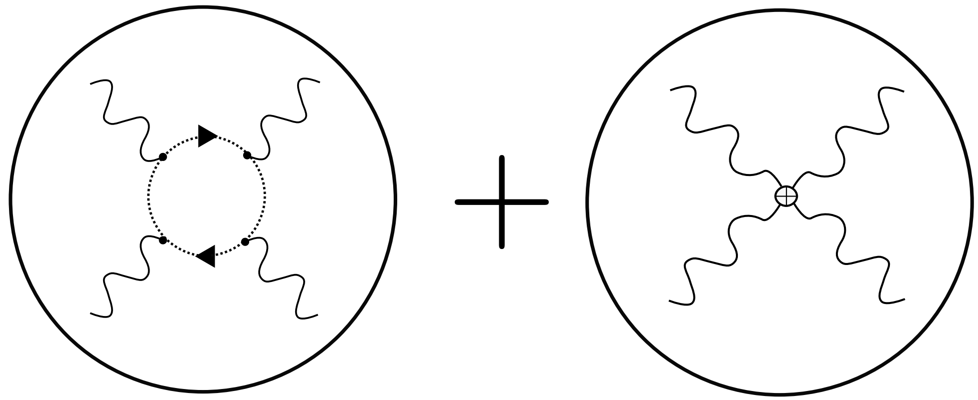
- If observed c.c. is small, then connected 4-point function should be small i.e. of the order

$$(M_P R)^{-2}$$

- If we take AdS as big as the universe, 4-point function should be of order

$$10^{-120}$$

Example: graviton 4-point function



- Estimate 1-loop diagram:

$$G_N^2 \int (d^4p)\, p^4 \left(\frac{1}{p}\right)^4 \sim M_P^{-4}\, M_{cut}^4 \sim \mathcal{O}(1)\,!$$

- two diagrams above are both $\mathcal{O}(1)$ and exactly cancel up to something of order $\mathcal{O}(10^{-120})$

Summary

- c.c. fine-tuning can be understood in terms of bulk correlation functions (Witten diagrams)

- via AdS/CFT $\Rightarrow$ c.c. fine-tuning should be visible in CFT correlation functions

Fine-tuning in CFT

- CFTs with holographic duals $\Rightarrow$ large N expansion
- Graviton scattering in AdS $\sim$ stress energy correlation functions in CFT
- Small (observerd) c.c. $\Leftrightarrow$

$$\langle \widetilde{T}(x_1) ... \widetilde{T}(x_n) \rangle_{con} \sim N^{2-n}$$

Fine-tuning in the CFT

???

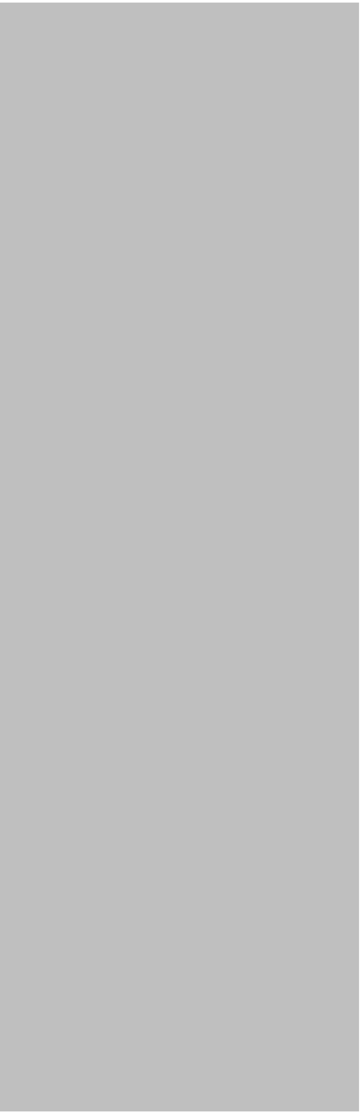
- **IS THE LARGE N EXPANSION NATURAL OR "FINE-TUNED"??**

- For example, 4-point function should be of order

$$\frac{1}{N^2}$$

is the $1/N$ suppression achieved in a **natural** way, or via cancellations between parametrically larger terms?

Splitting up of CFT correlators?



Fine-tuning in CFT

- Any CFT correlator ⇒ expanded in "conformal blocks"

$$\langle \widetilde{T}(x_1)\widetilde{T}(x_2)\widetilde{T}(x_3)\widetilde{T}(x_4)\rangle_{cont} = \sum_{\mathcal{A}} |C_{TT}^{A}|^2 \mathbf{G}_{\mathcal{A}}(x_1, x_2, x_3, x_4)$$

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OUR CONJECTURE

- Any CFT correlator $\Rightarrow$ expanded in "conformal blocks"

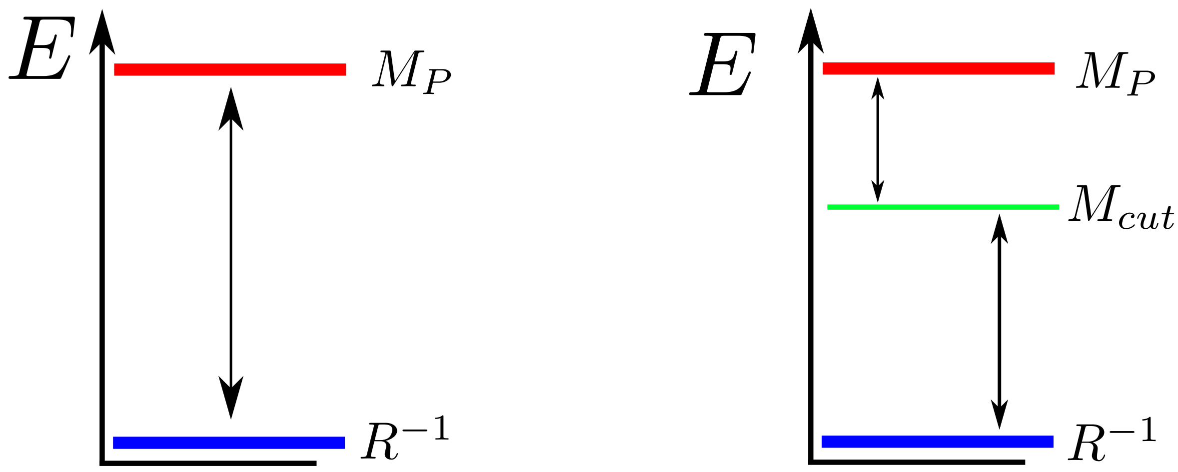
$$\langle \widetilde{T}(x_1)\widetilde{T}(x_2)\widetilde{T}(x_3)\widetilde{T}(x_4)\rangle_{cont} = \sum_{\mathcal{A}} |C_{TT}^{A}|^2 \mathbf{G}_{\mathcal{A}}(x_1, x_2, x_3, x_4)$$

OUR CONJECTURE

- Bulk c.c. fine tuning problem $\Leftrightarrow$ "un-naturalness" of large N expansion in CFT
- Final $1/N$ suppression of correlators is achieved via delicate cancellations between (parameterically larger) conformal blocks

(Notice: $1/N$ expansion and conformal block expansion are not the same thing!)

Comments



- Hierarchy vs fine-tuning
- Fine-tuned if

$$M_{cut}^4 M_P^{-2} R^2 \gg 1$$

- Need high cutoff in bulk theory

Large N gauge theories

- Large N gauge theories in 't Hooft limit $\Rightarrow$ weakly coupled string theories (Hagedorn growth)

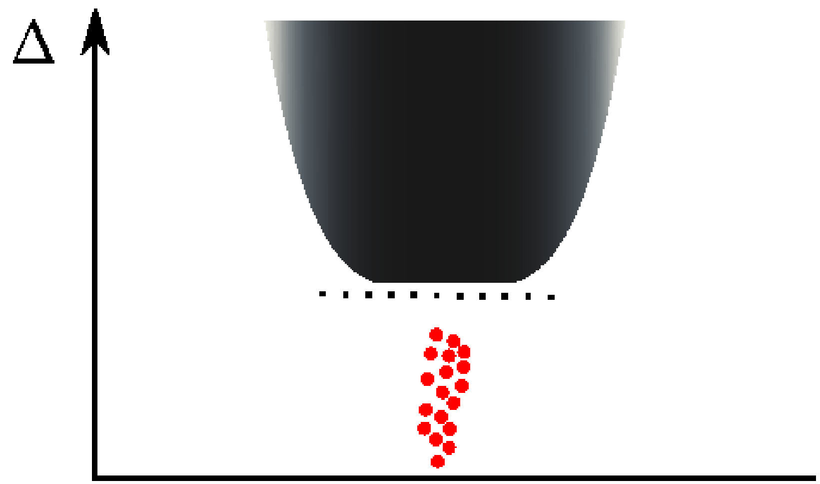
- String scale $M_s \sim f(\lambda)/R$ (does not scale with N), which implies

$$M_{cut} \sim M_s \ll M_P$$

- $\Rightarrow$ Cannot pose a **sharp** c.c. fine-tuning problem

- Same for higher spin gravity (Vasiliev)

CFTs with holographic duals



1. **Large central charge c**
2. **Few low-lying operators (c-independent)**
3. **Factorization of correlators**

   For semi-classical gravity

4. **Gap between spin 2 and higher**

Candidate CFTs with c.c. problem

Conditions

1. CFT must have a holographic dual (large c, few operators etc.)
2. CFT should not be supersymmetric
3. Conformal dimension Δ_{cut} where spin>2 states appear must satisfy

$$\Delta_{cut} \gg c^{1/d-1}$$

- 1 and 2 but not 3: large N gauge theories, $O(N)$/WZW/coset models, non-susy orbifolds of $\mathcal{N}=4$
- 1 and 3 but not 2: ABJM, $k=1, N \to \infty$
- 1 and 2 and 3?

Conformal block expansion

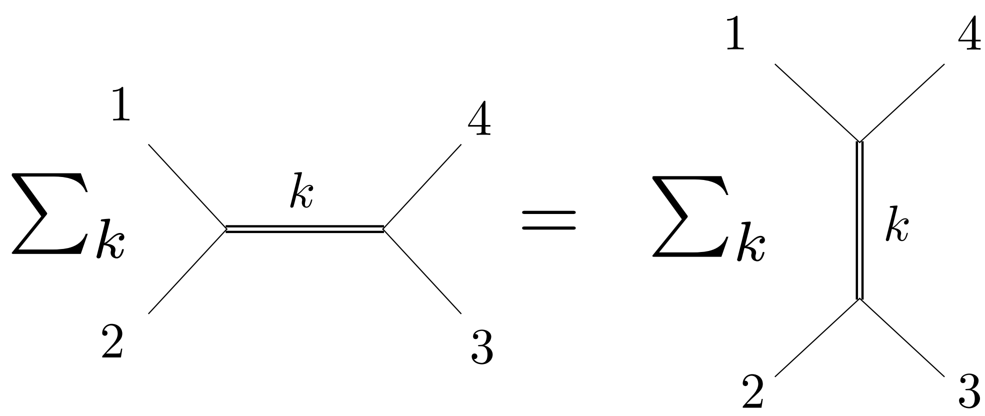
- Operator Product Expansion

$$\mathcal{O}_i(x)\mathcal{O}_j(y) \sim C_{ij}^k \mathcal{O}_k(y) + \ldots$$

- Any n-point function can be computed by OPEs

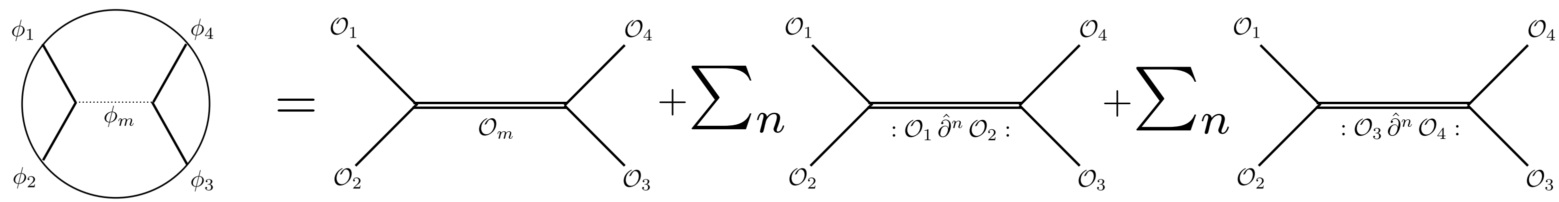
$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_m(x_3)\mathcal{O}_n(x_4)\rangle = \sum_k C_{12}^k C_{34}^k \mathbf{G}_k(x_1,x_2,x_3,x_4)$$

- Consistency condition: "conformal bootstrap"



Witten diagrams and conformal blocks

- Interactions in AdS $\Rightarrow$ Witten diagrams
- Witten diagram expansion $\sim$ conformal block expansion



- Witten diagram basis = easy solutions of conformal bootstrap

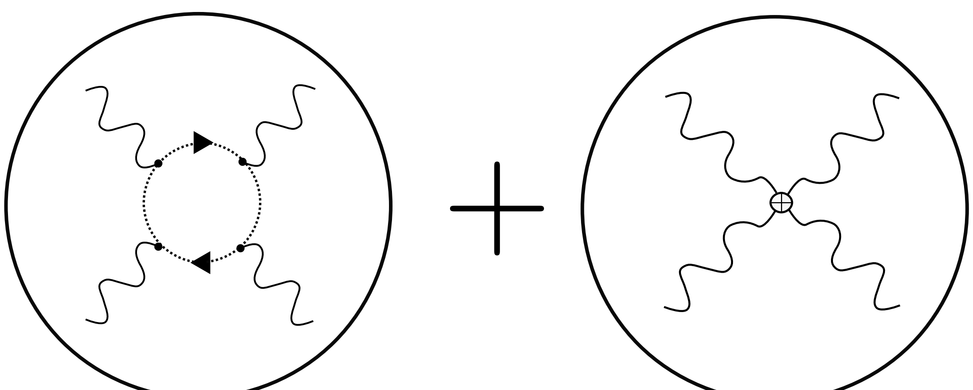
4-point function of $T_{\mu\nu}$

- It has a large N expansion

$$\langle \widetilde{T}(x_1)\widetilde{T}(x_2)\widetilde{T}(x_3)\widetilde{T}(x_4)\rangle =$$

$$= G_0(x_1, ..., x_4) + \frac{\mathbf{1}}{\mathbf{N^2}} G_1(x_1, ..., x_4) + \frac{\mathbf{1}}{\mathbf{N^4}} G_2(x_1, ..., x_4) + ...$$

- $G_2(x_1, ..., x_4)$ gets contributions (at least) from



- The term $\frac{\mathbf{1}}{\mathbf{N^4}} G_2(x_1, ..., x_4)$ is $\mathcal{O}(1/N^4)$. However if we split it into Witten diagrams $\Rightarrow$ each of them is $\mathcal{O}(1)$

- We have delicate cancellations between Witten diagrams

$\Rightarrow$

- In CFT language: expand $\frac{\mathbf{1}}{\mathbf{N^4}} G_2(x_1, ..., x_4)$ in conformal blocks. While $\frac{\mathbf{1}}{\mathbf{N^4}} G_2$ is order $\mathcal{O}(1/N^4)$, individual conformal blocks contributing to it are $\mathcal{O}(1)$ i.e. **parametrically larger**

- **(Apparently) fine-tuned cancellations between conformal blocks**

1-loop and "counterterm" diagrams

- 1-loop Witten diagram corresponds to conformal blocks of

$$:\Psi(\partial^2)^n\partial_1...\partial_l\Psi:$$

- Sum over n, l is $\sim$ momentum integral in bulk $\Rightarrow$ gives very large contribution

- Counterterm Witten diagram conformal block of

$$:T\partial...\partial T:$$

also comes with very large coefficient in order to cancel the divergence of the previous sum

CONCLUSION

- **COSMOLOGICAL CONSTANT FINE-TUNING IN ADS**

$\Rightarrow$

- **UN-NATURALNESS OF LARGE N EXPANSION IN CFT**
  (from conformal block point of view)

Can we study this in more detail?

Is there a specific CFT where this fine-tuning takes place?

- No example known yet

- No such CFT exists $\Leftrightarrow$ AdS gravity without (low energy) supersymmetry inconsistent

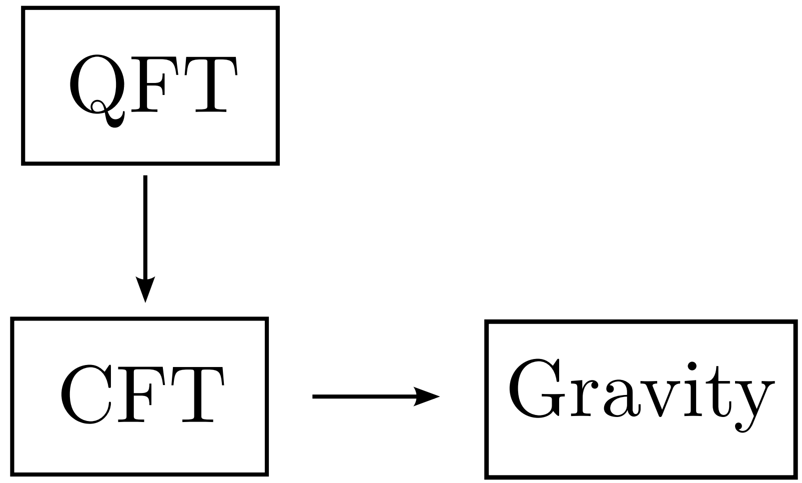
IS THERE A RESOLUTION OF THE FINE-TUNING FROM THE CFT?

- Assume large N expansion looks fine tuned from conformal block point of view

- Could there be another way to calculate correlators, where $1/N$ suppression looks natural?

- For example in large N gauge theories, conformal block expansion: expansion in term of exchanged gauge singlets (ie. **glueballs, mesons, etc.** in intermediate channels

- **BUT:** fundamentally correlators are computed by "double-line" diagrams in terms of **quarks and gluons**. In this description the $1/N$ expansion is manifest



QFT: description in terms of fundamental fields

CFT: description in terms of "gauge singlets"/CFT data (Δ_i, C_{ij}^k)

While mathematically equivalent, large N expansion may look natural in first but fine-tuned in second.

- If true $\Rightarrow$ Impossible to understand c.c. fine-tuning in terms of low energy fields (gravitons etc.)

- We need to understand what they are "made out of" (in the dual gauge theory)

- Underlying stabilizing mechanism $\Rightarrow$ Cancellations between terms of different order in perturbation theory (unlike supersymmetry)

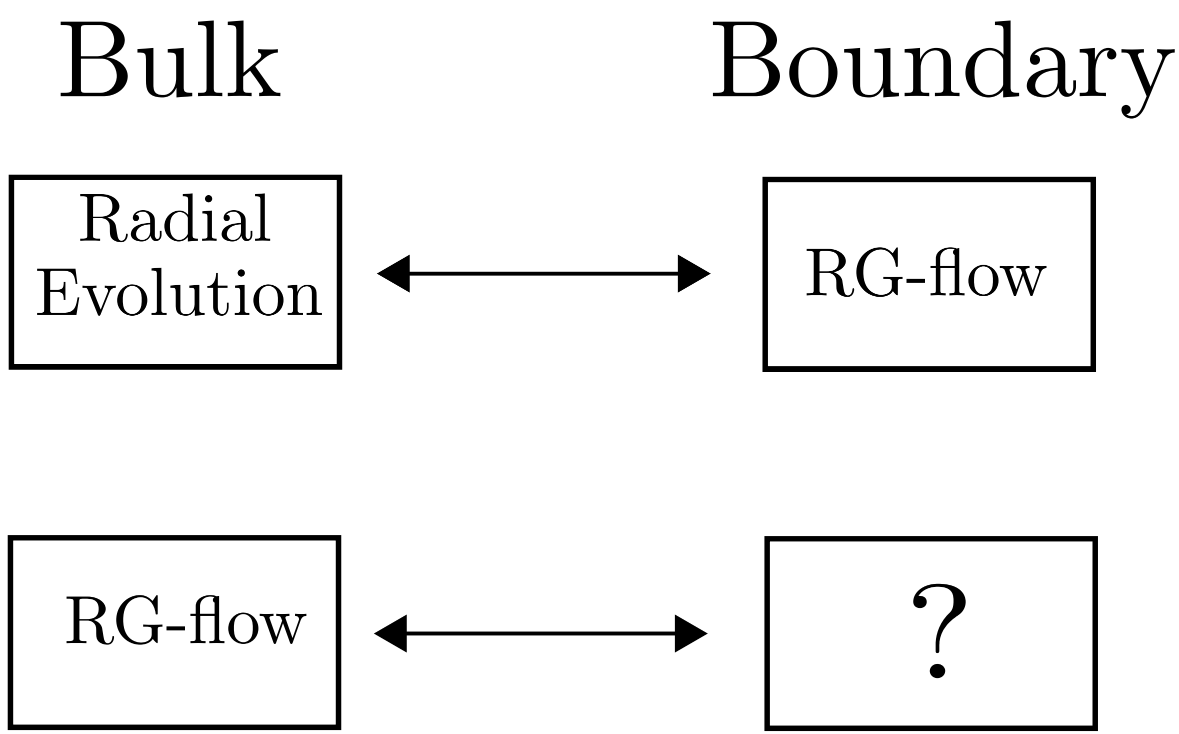
CONCLUSIONS

- To the extent that there is a consistent theory of AdS quantum gravity with the c.c. problem, we argued that in dual CFT the problem is manifest as an (apparent) fine-tuning of the large N expansion

- We argued that the CFT may admit an alternative description where this fine-tuning becomes naturally resolved

- WE NEED AN EXAMPLE! (does not have to be a full-fledged CFT)

THANK YOU


Bulk
Boundary
Radial Evolution
RG-flow
RG-flow
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