Applications of AdS/CFT

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The AdS/CFT duality

- Large N gauge theory in 4d $\Leftrightarrow$ string theory on AdS$_5 \times$S^5
- 't Hooft coupling $\lambda = g_{YM}^2 N$
- $\lambda \gg 1 \Rightarrow$ QFT hard, **gravity easy**
- $\lambda \ll 1 \Rightarrow$ **QFT easy**, gravity hard

(Holographic) QCD

Strong coupling problems in QCD

- Understanding QCD $\rightarrow$ one of the main motivations for AdS/CFT

- QCD: $SU(3)$ gauge theory with fermions

- Coupling constant runs with energy scale

- Strongly coupled at low energies $\Rightarrow$ **CONFINEMENT**

UV $\Rightarrow$ quarks and gluons

IR $\Rightarrow$ gauge singlets (glueballs, mesons, baryons...)

What would we like to understand in QCD?

- Understand/prove confinement
- Calculate spectrum of asymptotic states (baryons, mesons etc.) from UV lagrangian
- Confinement/Deconfinement transition - equation of state of quark-gluon plasma
- Dynamical processes (scattering, quark-gluon plasma dynamics...)

How does AdS/CFT help?

- The duality between $\mathcal{N} = 4$ SYM and IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ is not the most useful setup to study QCD

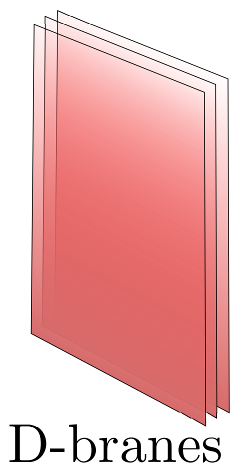
- The $\mathcal{N} = 4$ theory has too much supersymmetry and is conformal (i.e. has no scale like Λ_{QCD})$\Rightarrow$ no "particles"

- Need to consider some variation where the gauge theory is a **confining** gauge theory

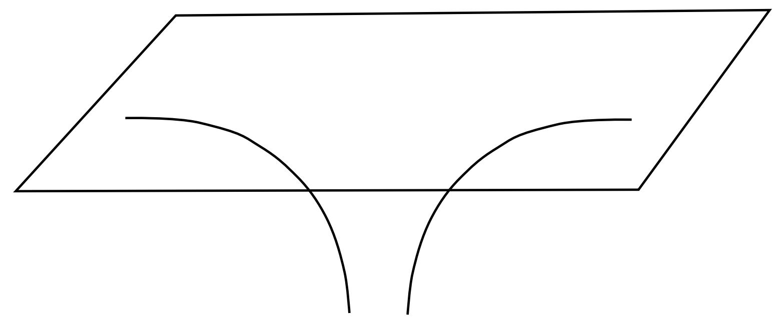
- The $\mathcal{N} = 4$ SYM has matter in adjoint of $SU(N)$. Need to include fields in fundamental representation (quarks).

Breaking conformal invariance

Stack of D4-branes in IIA string theory



$\Leftrightarrow$



5-dimensional $SU(N)$ supersymmetric gauge theory (not conformal!)

$$\Leftrightarrow$$

supergravity on

$$ds^2 \sim \frac{1}{z}\left(\frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2} + d\Omega_4^2\right)$$

Reducing supersymmetry

- The 5d SYM has too much SUSY (too many fields) $\Rightarrow$ need to reduce

- Compactify on circle with antiperiodic b.c. for fermions

- Fermions projected out, scalars get masses from loops

- Theory flows in the IR to 4d **pure** $SU(N)$ Yang-Mills

- This is a **CONFINING**, **NON-SUSY** gauge theory, qualitatively similar to QCD

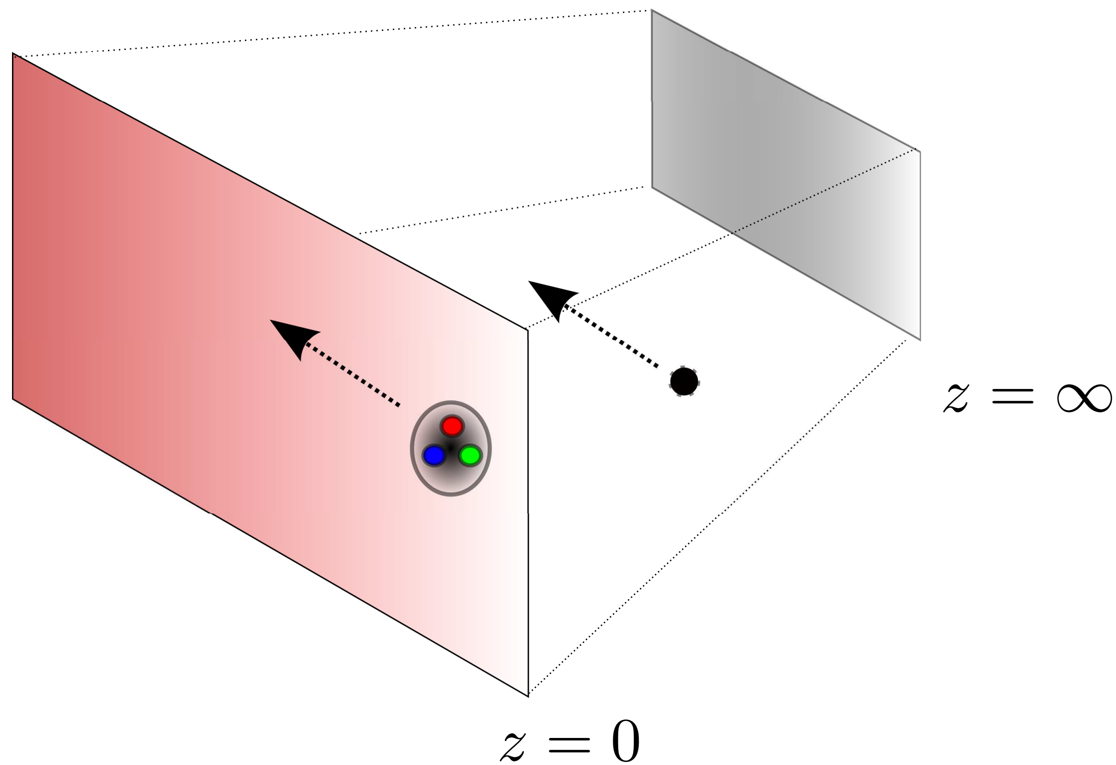
The gravitational description

• Performing the same circle reduction on the gravitational solution of the $D4$ branes we end up with the following solution of IIA supergravity

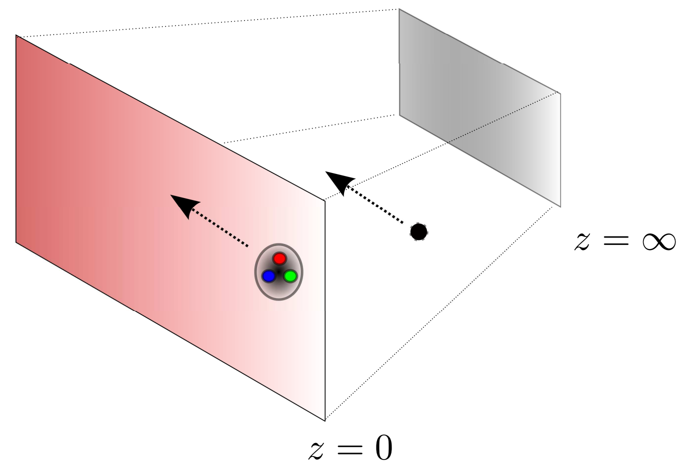
$$ds^2 \sim \frac{1}{z}\left(\frac{-dt^2 + d\vec{x}^2}{z^2} + \frac{dz^2}{z^2 f(z)} + \frac{f(z)}{z^2} d\phi^2 + d\Omega_4^2\right), \quad f(z) = 1 - \frac{z^6}{z_0^6}$$

• Circle $\phi \sim \phi + 2\pi$ whose size shrinks to zero at some $z = z_0$. The spacetime ends smoothly there.

• **This is (almost) the holographic dual of large N $SU(N)$ gauge theory in four dimensions!!!**


$z = \infty$
$z = 0$

• In AdS/CFT glueball/hadron masses $\Leftrightarrow$ energies of fields propagating in gravitational background.



• Consider scalar field Φ obeying the KG equation

$$\Box\Phi = m^2\Phi$$

• Look for solutions of the form

$$\Phi(x, z) = e^{ik_\mu x^\mu} f(z)$$

k_μ is the momentum of the glueball in the gauge theory. And

$$k^2 = \text{mass of glueball}$$

Computing the mass spectrum

• Plug ansatz into KG equation get a 2nd order ordinary differential equation for $f(z)$

$$z^5 \frac{d}{dz}\left(z\left(\frac{1}{z^6}-\frac{1}{z_0^6}\right)\frac{df(z)}{dz}\right)+M^2 f(z)=0$$

where $M^2 = k^2$.
We need to impose boundary conditions:

1. $f(z)$ should not blow-up at infinity
2. $f(z)$ should be smooth in interior $z = z_0$

• 2-boundary conditions for 2nd order ODE $\Rightarrow$ there is a solution **only for a discrete set of** M^2

Spectrum of allowed M^2

$\Leftrightarrow$

Masses of glueballs/hadrons in gauge theory!!!

Computing the mass spectrum

- Comparison (ratio of lightest glueballs)

$$\left(\frac{M_2}{M_1}\right)_{AdS/CFT} = 1.20$$

$$\left(\frac{M_2}{M_1}\right)_{lattice} = 1.36 \pm 0.32 \tag{1}$$

- This is impressive! We got a reasonable answer for the **non-perturbative** bound states of a **strongly coupled** gauge theory by solving a 2nd order classical ODE! (for lattice we would need lots of computer power).

- Many improvements, including flavor quarks, etc.

Confinement and Wilson loops

• What is a direct way to see confinement in AdS/CFT ?

• Consider a $q\bar{q}$ pair at distance L. The theory is confining if energy goes like $V(L) \sim kL$ for large L.

• Consider propagation of state for time T. We expect the following dependence

$$e^{-HT} \sim e^{-kLT} \sim e^{-k \cdot \mathrm{Area}}$$

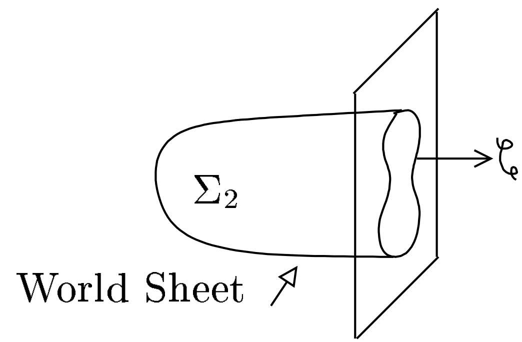
This amplitude can be related to the holonomy of the gauge field around the loop (called Wilson loop)

$$\mathcal{W} = \langle \mathrm{Tr} \mathcal{P} e^{i \oint_C A_\mu dx^\mu} \rangle$$

$$\text{Confinement} \qquad \Leftrightarrow \qquad \mathcal{W} \sim e^{-k \cdot \mathrm{Area}}$$

Wilson loops from AdS/CFT

- A quark in the gauge theory $\Leftrightarrow$ string in AdS ending on boundary



- Wilson loop $\sim$ string surface ending on boundary loop

Prescription: find **minimal area** surface ending on boundary loop. Calculate its area A^* and then

$$\mathcal{W} = exp\,(-A^*)$$

for confining gauge theories this leads to area law in boundary theory

Dynamical processes in QCD

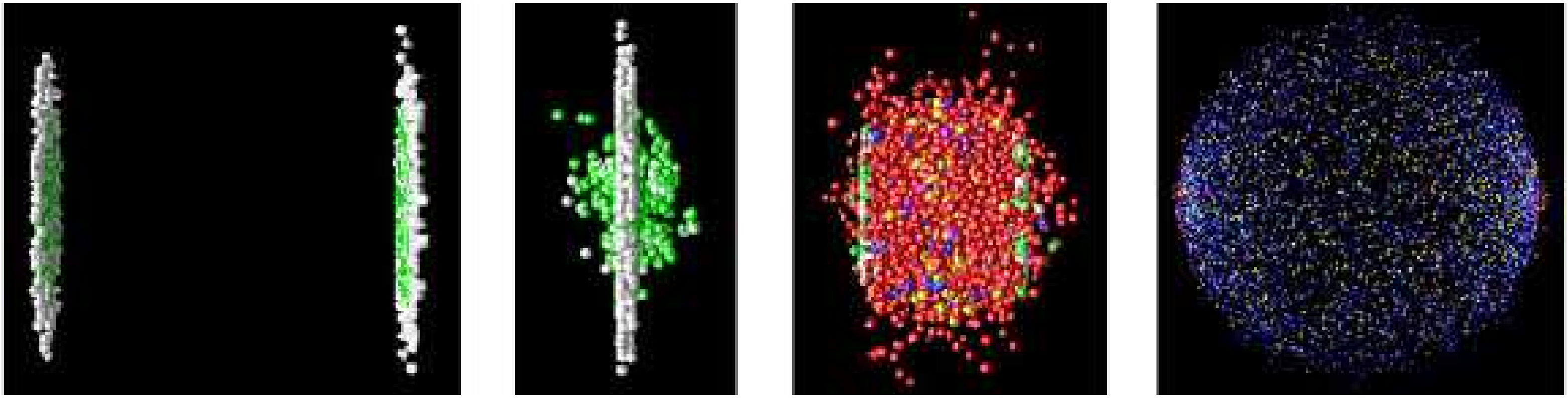
- So far $\Rightarrow$ "static" questions. Can also be addressed by lattice

- For dynamical processes (time-dependent) lattice/numerical computations are very difficult

- AdS/CFT can sometimes be used to study such questions

For example

Experiments at RHIC and LHC



Heavy ion collision $\Rightarrow$
Fireball of quark gluon plasma $\Rightarrow$
Decay via hadronization

Extremely complicated, numerics difficult

Many interesting questions about various stages of the process and about the physics of the (strongly coupled) fluid (QGP)

Quark-gluon Fireball $\Leftrightarrow$ Black Hole

Collision of particles$\Rightarrow$

Black Hole formation$\Rightarrow$

Hawking evaporation into particles

Many other computations

- Adding flavor quarks
- Quark energy loss
- Equation of state of plasma
- Phase diagram of QCD

. . .

Hydrodynamics and condensed matter

Black holes and Fluid dynamics

• Static, Uniform black hole $\Leftrightarrow$ uniform fluid (QGP) in gauge theory

• In gravity, fluctuations of black hole horizon $\Rightarrow$ solutions of Einstein equations

• In gauge theory, fluctuations of fluid are described by hydrodynamics (Navier Stokes)

$$\nabla \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} - \eta \nabla^2 \vec{v} = -\nabla P$$

EINSTEIN EQUATIONS

$$\Leftrightarrow$$

NAVIER-STOKES EQUATIONS

The viscosity bound

• Any fluid ⇒ "viscosity coefficient" η

• Measured by thermal 2-point function

$$\langle T_{\mu\nu}(x)\, T_{\rho\sigma}(0)\rangle$$

• Hard to calculate in strongly interacting theories (for example for QGP)

in AdS/CFT: $\qquad T_{\mu\nu} \qquad \Leftrightarrow \qquad g_{\mu\nu}\;$ (graviton)

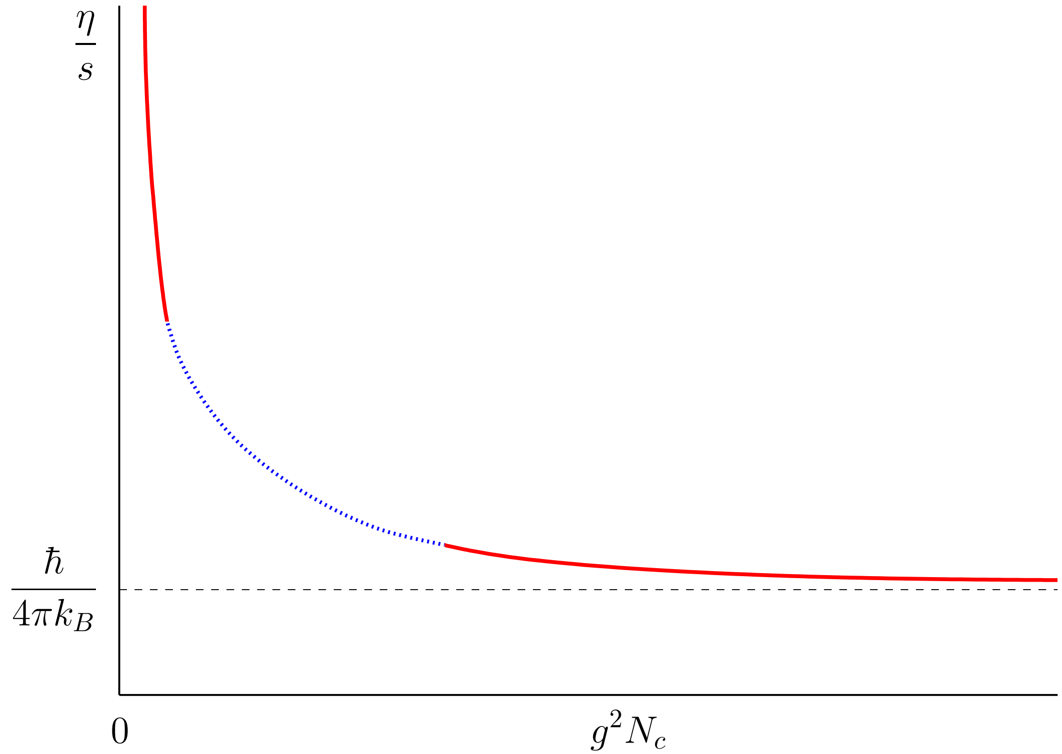
• Hence viscosity related to graviton scattering amplitude off a black hole horizon ⇒

EASY

gives

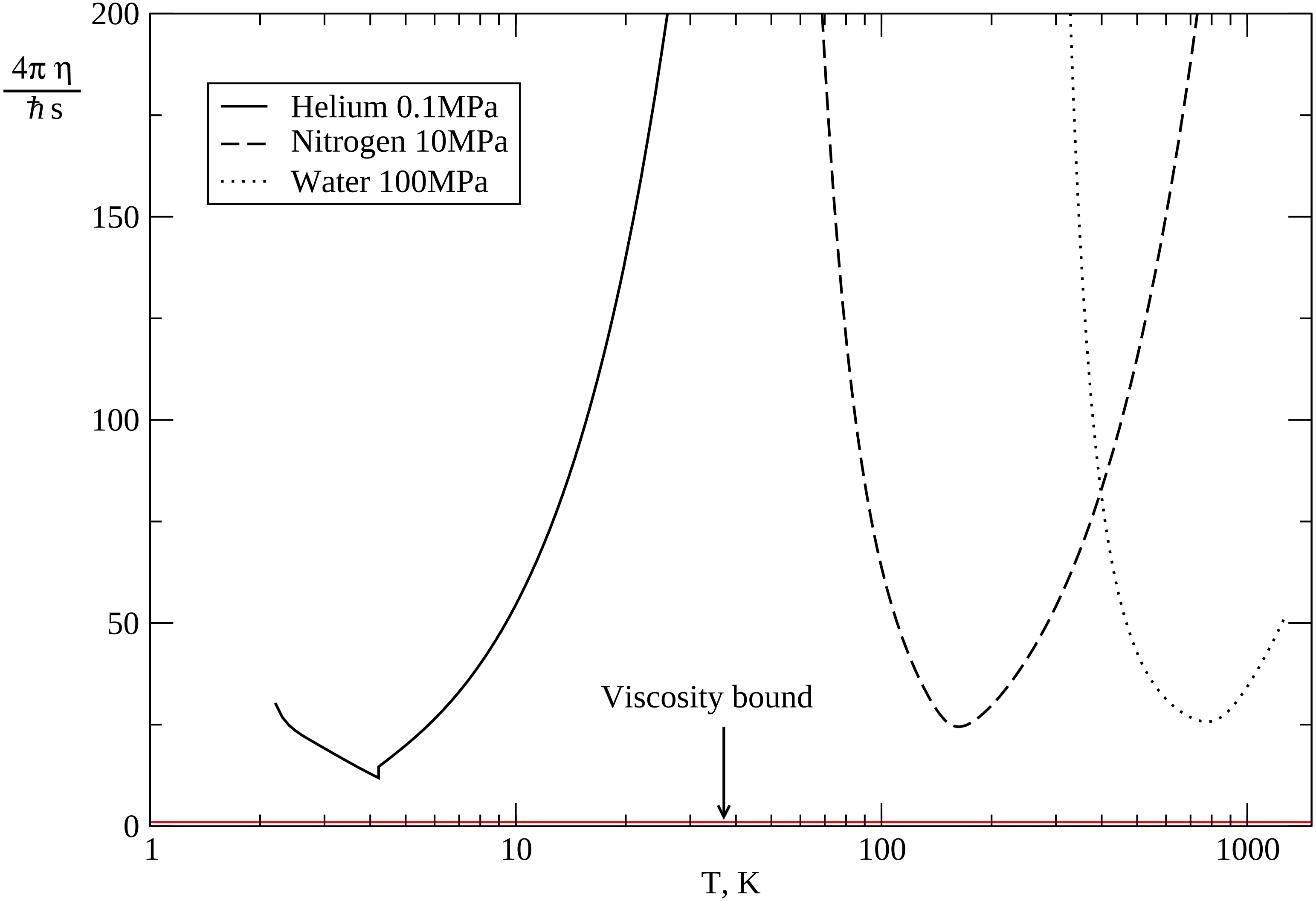
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

The viscosity bound

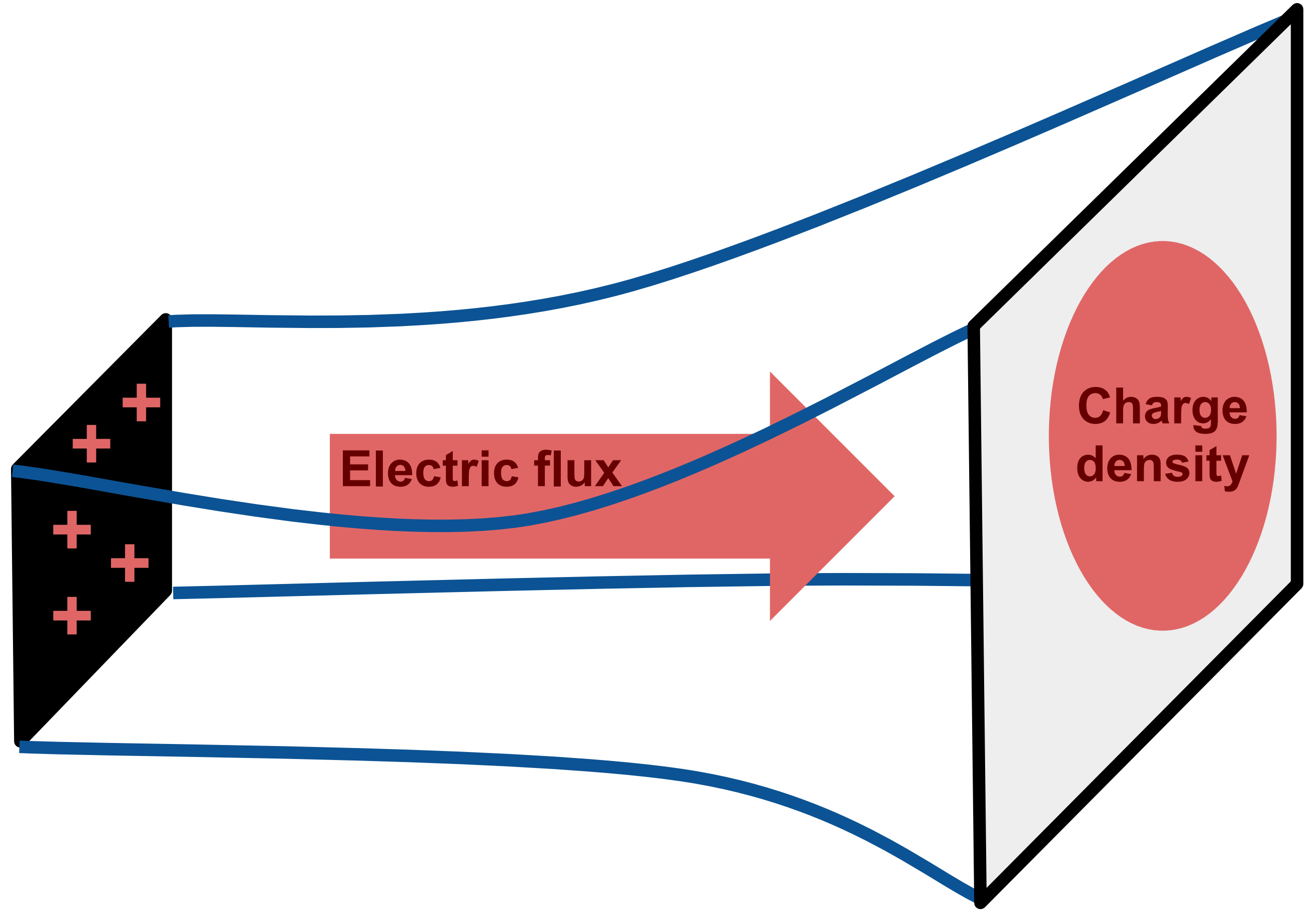


For all values of the coupling we find that

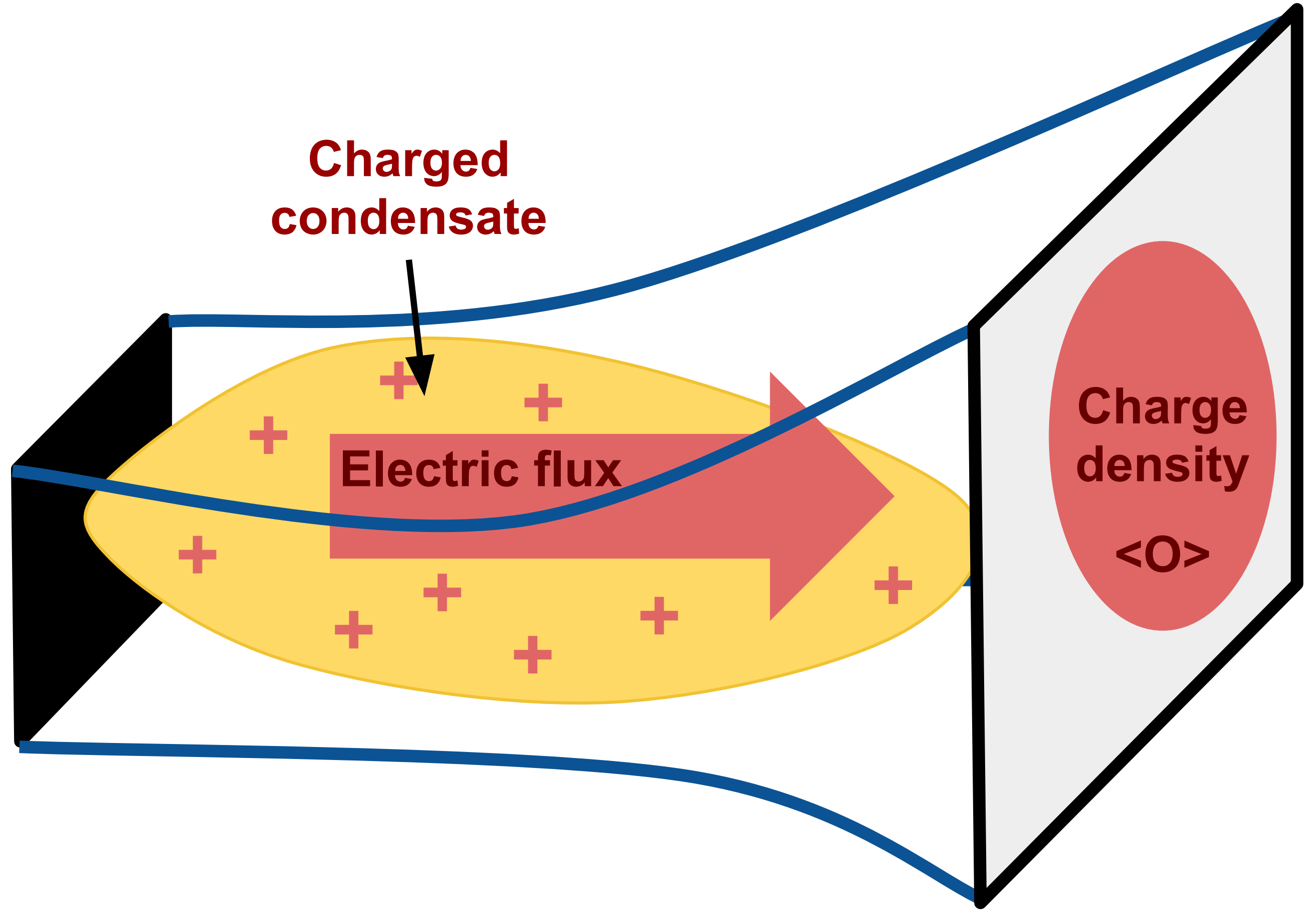
$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

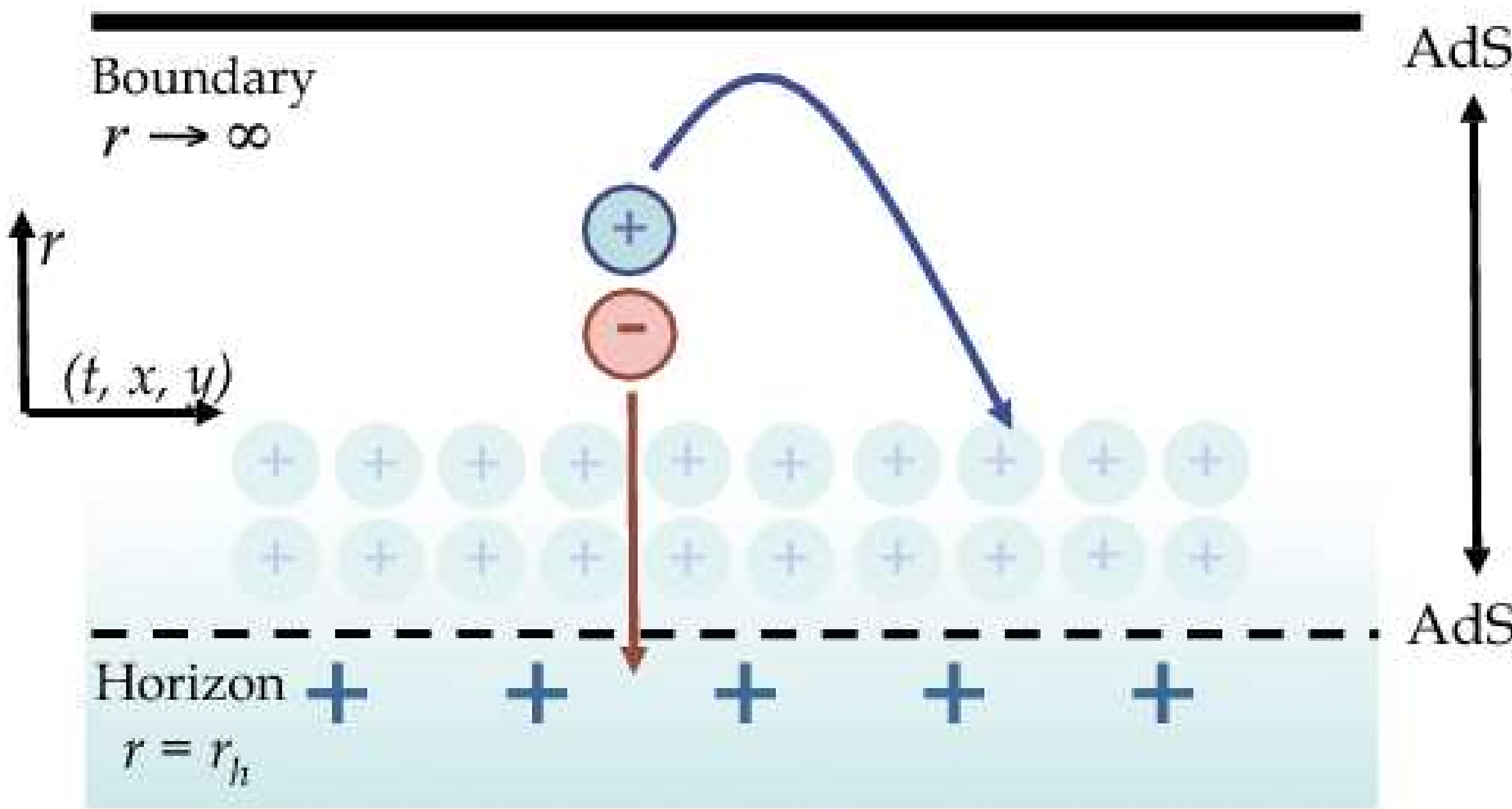


All known materials obey the bound


Electric flux
Charge density

Holographic superconductors





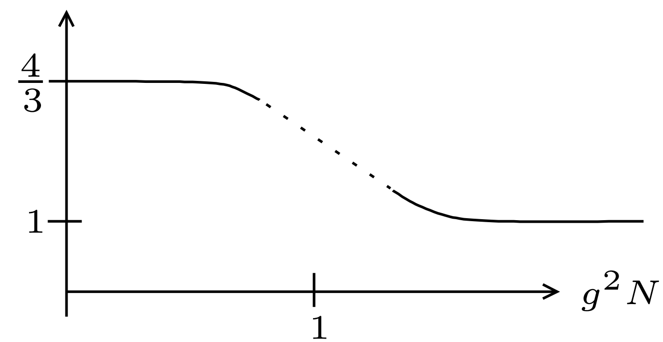
Quantum criticality, non-fermi liquids, strange metals

Gravity and Cosmology

- Black hole in AdS $\Leftrightarrow$ QGP at finite temperature

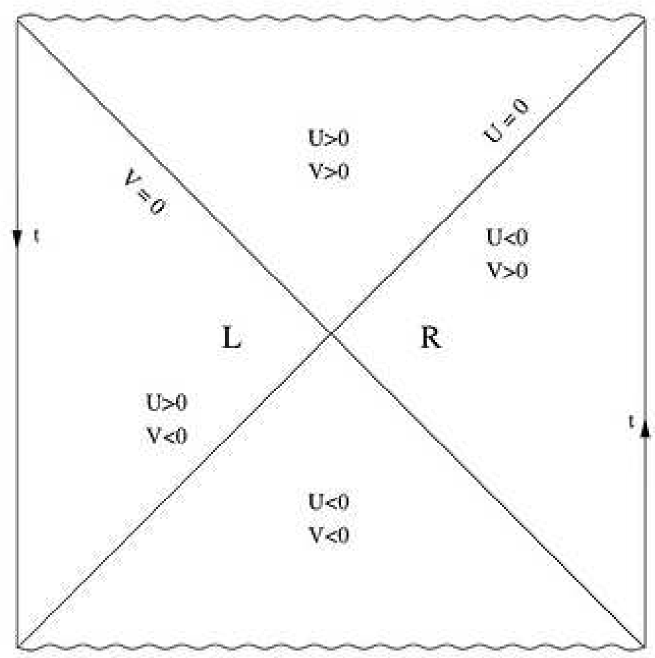
- Estimate black hole entropy from weakly coupled gauge theory

$$\frac{S}{\frac{\pi^2}{2} N^2 T^3 V}$$



- Correct entropy up to a factor of 3/4 !

- Numerical simulation of gauge theory $\Rightarrow$ exact agreement



- Information paradox: dual gauge theory is manifestly unitary
$\Rightarrow$ No information loss

- Resolution of singularity

Phenomenology and Cosmology

- AdS/CFT provides us with many examples of strongly coupled QFTs

- What if BSM physics is strongly coupled (for example technicolor etc.)?

- Randall-Sundrum models (warped extra dimensions)

- Inflation, power spectrum

- Holography for de-Sitter $(\Lambda > 0)$

Towards the future

- More work on applications
- Decode the AdS hologram
- Holography for flat space/de Sitter

The structure of space-time

- Is our universe a hologram?
- Observational predictions of holography?
- Is time emergent?

THANK YOU