Lectures on AdS/CFT

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September 9, 2011

Overview

- **Lecture 1:** Motivation and Background (confinement, large N , holographic bound, basic CFT, anti de-Sitter space)
- **Lectures 2+3:** Main statement of AdS/CFT, derivation, how to do computations, generalizations
- **Lecture 4:** Applications (QCD, hydrodynamics and condensed matter, black holes, cosmology)

Practicalities

- Subject is huge $\Rightarrow$ will move fast
- Intuitive explanations rather than technical derivations
- **PLEASE ASK QUESTIONS!!!** (during talks **and later**)
- If too formal/technical ask for more physical explanation
- References: If interested, ask me in private

Introduction

The AdS/CFT correspondence

The AdS/CFT correspondence is a duality (an exact equivalence) between two seemingly different theories

1. A four dimensional **quantum field theory** (a gauge theory-like QCD)

2. **Gravity** (string theory) in a higher dimensional spacetime

- Discovered in 1997 by J. Maldacena. Clarified by Gubser, Klebanov, Polyakov and independently by Witten.

- Very active field. Applications in other areas of physics (i.e. not string theory)

APPLICATIONS:

Use Gravity to learn about QFT

- QCD $\Rightarrow$ strong coupling phenomena (confinement, chiral symmetry breaking etc.)
- Fluid dynamics, condensed matter systems

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Use QFT to learn about Gravity

- Black Holes (singularities, entropy, Hawking radiation....)
- Cosmology (Big Bang, inflation, c.c. problem, ...)

- (so far the only) Non-perturbative definition of string theory/quantum gravity
- **SPACE AND TIME ARE EMERGENT CONCEPTS !!!**
- Is our world a hologram?

Origins

1. **Confinement in gauge theories, large N expansion**

2. **Black Holes and Holography**

Confinement in QCD

- QCD: $SU(3)$ gauge theory + fermions

- In the UV: quarks + gluons

- Coupling constant runs with energy scale (asymtpotic freedom)

- Theory becomes strongly coupled at low energies

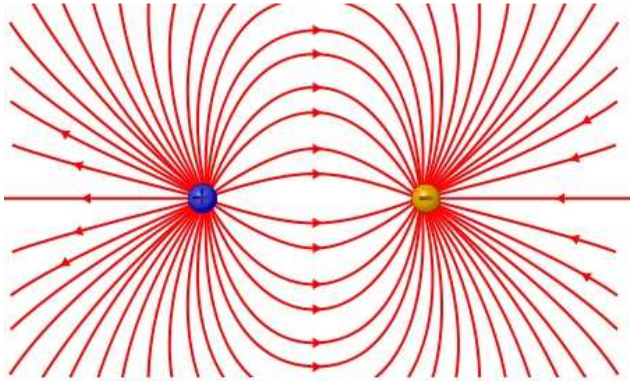
- Strong coupling $\Rightarrow$ "**color confinement**" : asymptotic states are $SU(3)$ singlets

- In the IR we see mesons, baryons, glueballs etc.

- Confinement difficult to understand analytically:

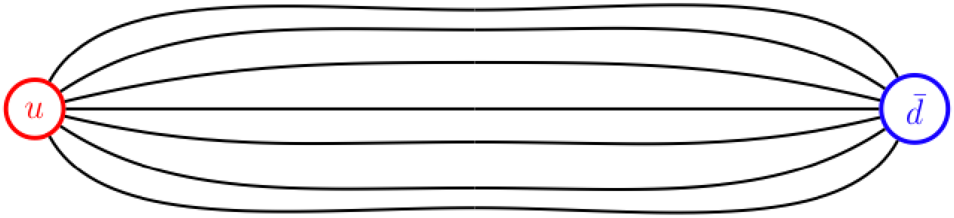
NO EXPANSION PARAMETER

- Consider two charges in $U(1)$ gauge theory



field lines spread out $\Rightarrow$ force $\sim 1/r^2$

- In non-abelian gauge theories



chromoelectric field independent of r $\Rightarrow$ energy in field linear with r $\Rightarrow$ confinement

Flux-tubes and string theory

- Fluxtube behaves like a string of constant tension

- Mesons can be understood as excitations of the fluxtube $\sim$ open strings (Regge trajectories)

- Glueballs $\sim$ closed strings

The large N expansion

- QCD has no obvious expansion parameter
- What if we replace $SU(3) \to SU(N)$?
- 't Hooft: theory simplifies in the large N limit
- In order to have good behavior we need to scale

$$N \to \infty$$

$$g_{YM} \to 0$$

keeping

$$\lambda \equiv g_{YM}^2 N$$

fixed. The parameter λ is called the "'t Hooft coupling"

Double-line diagrams at large N

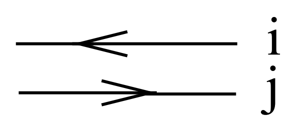
• Consider $U(N)$ gauge theory. The gauge field has the form

$$A_\mu = A_\mu^I T^I$$

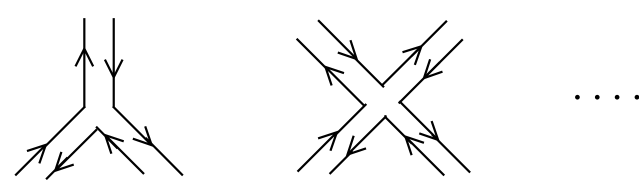
where T^I are the generators of the Lie algebra in the adjoint representation

• The adjoint representation can be understood as $\overline{\mathbf{N}} \otimes \mathbf{N}$. Hence we can trade the index $I \to (i, \bar{j})$

• The gluon propagator can then be represented as

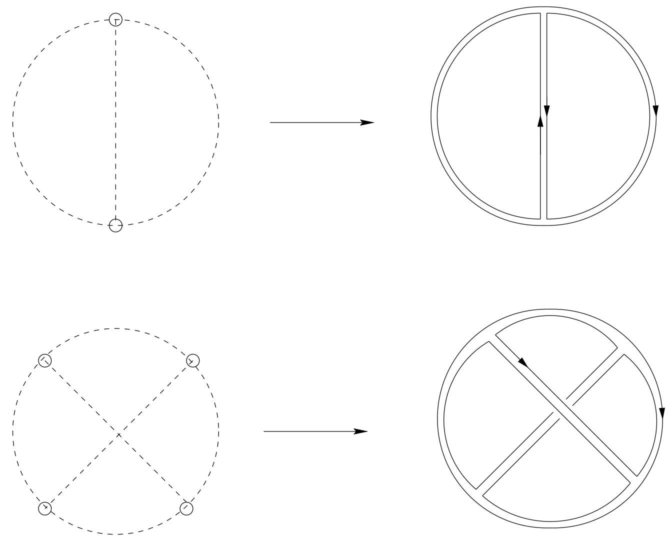


and vertices of the gauge theory are



Double-line diagrams at large N

• Feynman diagrams can be rewritten as double-line diagrams where the arrows have to be connected consistently



• Different diagrams contribute with different power of N in the large N limit.
• The double-line notation makes the counting of factors of N easier.

The lagrangian has the form

$$\mathcal{L} = -\frac{1}{4g_{YM}^2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

where $1/g_{YM}^2 = \frac{N}{\lambda}$.

• Every propagator carries a factor of $\frac{\lambda}{N}$. Every vertex carries a factor of $\frac{N}{\lambda}$. The summation over each closed line gives a factor of N

• If we have a diagram with V vertices, E propagators and F loops we find that it scales like

$$N^{V-E+F}\lambda^{E-V}$$

the quantity $V - E + F = \chi$ is the Euler character of a surface corresponding to the diagram

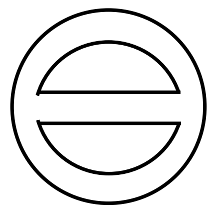
• For closed, oriented surfaces $\chi = 2 - 2g$ where g is the genus

• Power of N depends only on **topology** of the diagram. If g is the genus then the N-dependence is

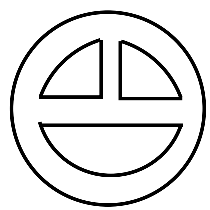
$$N^{2-2g}$$

EXAMPLES

genus 0:

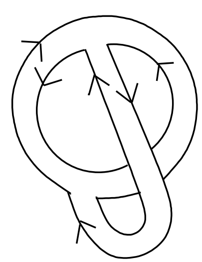


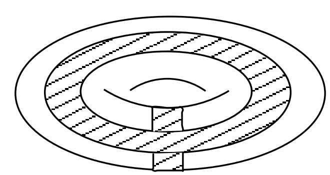
$$N^3(g^2) \sim N^2(g^2N)$$



$$N^4(g^2)^2 \sim N^2(g^2N)^2$$

genus 1:





Torus

$$N^0(g^2N)$$

Simplifications at large N

- Only planar (genus zero) diagrams contribute

- There is a systematic $1/N$ expansion

- Gauge singlets (mesons, glueballs etc.) become stable and free

- Large N limit is a "classical limit"

- While theory simplifies, still non-trivial dynamics $\Rightarrow$ we still have confinement

Large N expansion and string theory

• The genus expansion of large N gauge theories $\sim$ genus expansion of string theory, if we identify

$$\frac{1}{N} \sim g_s$$

• This suggests that a large N gauge theory is dual to a string theory

• At large λ the "holes" in double line diagrams close $\Rightarrow$ they become smooth surfaces (string worldsheet)

• String theory is inconsistent in four-dimensions, hence the dual string theory lives in higher dimensions

Black Hole entropy

- Schwarzchild black hole:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

- Event Horizon at $r = 2GM$, singularity at $r = 0$.
- Black Hole dynamics + Hawking radiation $\Rightarrow$

BLACK HOLES HAVE ENTROPY

$$S = \frac{A}{4G}$$

This has far-reaching implications for the nature of space-time.

The holographic bound

- Consider region of spacetime of size R. How many degrees of freedom do we need to describe it?

- $\#$ of degrees of freedom $\sim$ (maximal) entropy contained in region.

- In conventional local systems we entropy scales like **volume**

$$S \sim R^3$$

A Gedanken experiment

- Imagine adding matter to region. Entropy cannot decrease.

- If sufficient amount of matter $\Rightarrow$ collapse and black hole formation

- Entropy of final black hole goes like the **area** of the region!

$$S \sim R^2$$

in contrast our expectations for systems with local degrees of freedom

Quantum gravity is holographic

- Black hole entropy $\Rightarrow$ in theories with gravity $\#$ of degrees of freedom scales like the area, not volume

How is this possible?

- A natural mechanism to guarantee this would be to assume that somehow the degrees of freedom necessary to describe physics in a region M, live on the boundary of the region ∂M.

- These degrees of freedom on the boundary completely encode what happens in the interior.

- **Gravity is holographic.**

AdS/CFT: gravity and gauge theories

The two aforementioned ideas

1. That large N gauge theories can be described by string theories.
2. That quantum gravity is holographic.

have found a precise realization with the discovery of the AdS/CFT correspondence

Large N gauge theory in d dimensions

$$\Leftrightarrow$$

Quantum gravity (string theory) in $\geq d+1$dimensions

- Applications and fundamental physics
- Simplest case: gauge theory is conformal and gravity is in AdS
- $\mathcal{N}=4\ \mathrm{Yang-Mills} \quad \Leftrightarrow \quad \mathrm{IIB\ string\ theory\ on\ AdS_5 \times S^5}$

Conformal Field Theory

Scale invariance

• Most QFTs have scales (masses, couplings, etc.) $\Rightarrow$ non-trivial RG-flow

• Cutoff $\Rightarrow$ quantum violation of scale invariance

• Dynamically generated scales (like Λ_{QCD})

HOWEVER

• There are QFTs which have no scale and where $\beta = 0 \Rightarrow$ exact scale invariance.

• New symmetry generator: dilatation operator D

$$[D, P_\mu] = -iP_\mu, \qquad [D, M_{\mu\nu}] = 0$$

Conformal invariance

• In most cases scale invariant QFTs are invariant under a larger symmetry group, the conformal group

• In addition to $P_\mu, M_{\mu\nu}$ and D it contains new symmetry generators

$$\text{special conformal transformations} \qquad K_\mu$$

$$\begin{aligned} \text{Poincare} \qquad & x^\mu \to \Lambda^\mu_\nu\, x^\nu + a^\mu \\ \text{scale} \qquad & x^\mu \to \lambda\, x^\mu \\ \text{special conformal} \qquad & x^\mu \to \frac{x^\mu - b^\mu x^2}{1 - 2b\cdot x + b^2 x^2} \end{aligned} \tag{1}$$

• In d spacetime dimensions the conformal group is isomorphic to $SO(d,2)$.

No S-matrix in CFTs, correlation functions

- In theories with no mass gap $\Rightarrow$ no well defined asymptotic states

- $\Rightarrow$ No S-matrix. What are the physical observables?

- Correlation functions of "local operators" (gauge invariant ones)

$$\phi^k, \quad \phi\partial_\mu\phi, \quad F_{\mu\nu}F^{\mu\nu}, \qquad \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right), \quad \ldots$$

- In CFTs we would like to compute

$$\langle\mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)\rangle$$

where $\mathcal{O}_i$ are local operators like those mentioned above

- "Solving the CFT" $\Leftrightarrow$ computing such correlation functions

Classification of local operators

- Usual QFT $\Rightarrow$ classify states under Poincare group
- in CFT $\Rightarrow$ classify local operators under conformal group

$$[D, \mathcal{O}(0)] = -i\Delta\mathcal{O}(0)$$

Δ is the "conformal dimension of the operator".

From the algebra we have

$$[D, P_\mu] = -iP_\mu, \qquad [D, K_\mu] = iK_\mu$$

so P_μ raises the dimension of an operator while K_μ lowers it.

- Local operators annihilated by the K_μ's are called **conformal primaries**. The are characterized by Δ and their spin.

- All other local operators can be derived from primaries by acting with $P_\mu \sim -i\partial_\mu$. They are called **descendants**.

Correlation functions in CFTs

• Conformal invariance fixes form of 2-point function of conformal primaries

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$$

• and also the 3-point function

$$\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z)\rangle = \frac{C}{|x-y|^{\Delta}|y-z|^{\Delta}|x-z|^{\Delta}}$$

• 4- and higher-point correlation functions are constrained but NOT fixed by conformal invariance

The $\mathcal{N}=4$ SYM theory

• 4d QFT with maximum amount of supersymmetry (16 supercharges). The field content is

$$\begin{array}{lll} \text{gauge field} & A_\mu & \\ \text{fermions} & \lambda^i, & i=1,...,4 \\ \text{scalars} & \Phi^I, & I=1,...,6 \end{array} \tag{2}$$

all in the adjoint of the gauge group G.

• The Lagrangian of the theory has the schematic form

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi^I)^2 + \overline{\lambda}\not{D}\lambda + [\Phi^I,\Phi^J]^2 + \ldots\right)$$

• For given gauge group $G \Rightarrow$ Unique 4d QFT with $\mathcal{N}=4$ SUSY

Basic properties of the $\mathcal{N} = 4$ SYM

- The theory is conformal (β function is exactly zero).

- Theory is invariant under the superconformal group. Its bosonic subgroup is

$$SO(4,2) \times SO(6)$$

The $SO(6) = SU(4)$ is the R-symmetry of the theory.

- Exact $SL(2,Z)$ duality. Define complexified coupling $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$. Theory invariant under

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad (a,b,c,d) \in SL(2,Z)$$

Anti de-Sitter space

Geometry of AdS

• AdS_{d+1} is the maximally symmetric spacetime in $d+1$ dimensions

• It has constant negative curvature and is a solution of Einstein equations with negative cosmological constant

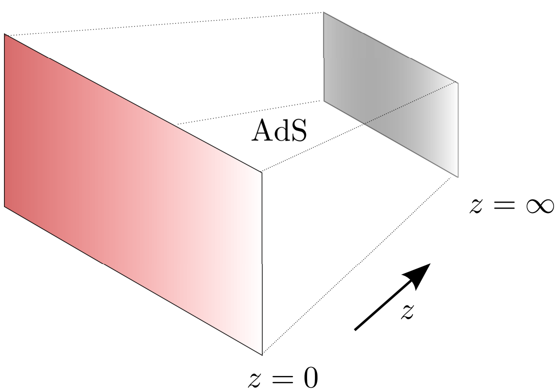
• The isometry group of AdS_{d+1} is $SO(d,2)$ (notice that it coincides with the conformal group in d dimensions!)

• There are various coordinate systems which can be used, each with its own advantages

Geometry of AdS

• One useful coordinate system is the so-called "Poincare patch", where the coordinates are $(z, t, \vec{x})$ and the metric has the form

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

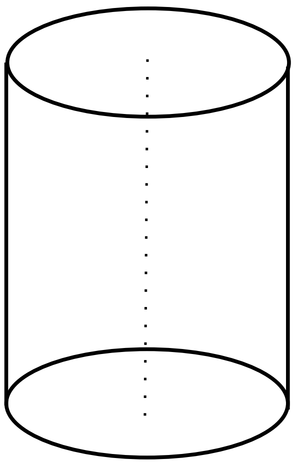


• We have Minkowski-space slices along $t, \vec{x}$ which are **warped** along the direction z

• Only the Poincare invariance along d directions and scaling is manifestly visible (not the full isometry group $SO(d, 2)$).

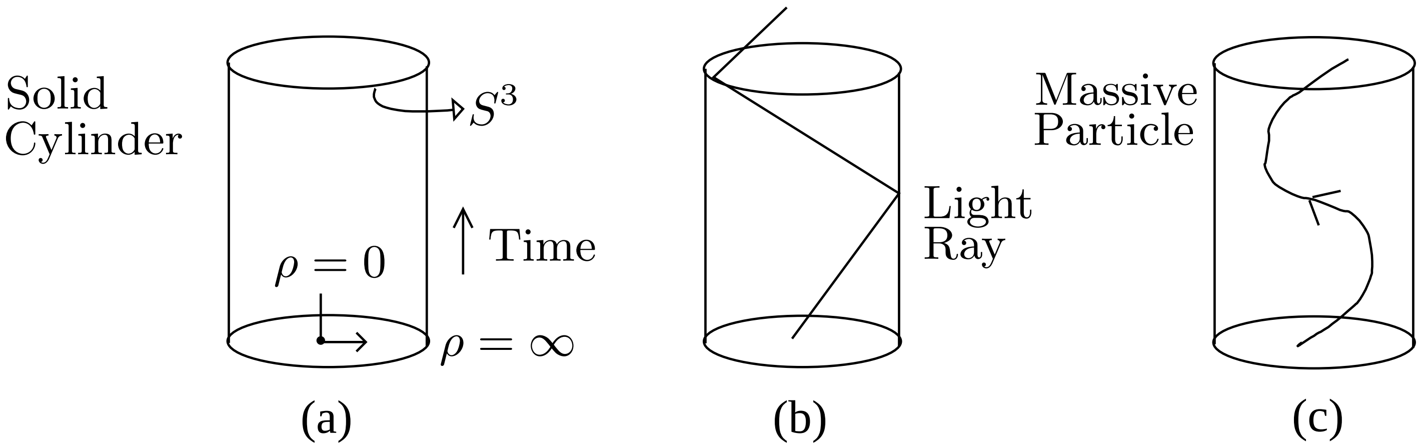
• Another useful coordinate system is the "global patch", with coordinates t, ρ, Ω_{d-1} and the metric

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho \, d\Omega^2_{d-1}$$



Now the $SO(d) \times \mathbf{R}$ isometry is manifest

- Attractive gravitational potential towards the "center" i.e. $\rho = 0$
- Penrose diagram of AdS $\Rightarrow$ "conformal boundary is $S^{d-1} \times R$
- Massless particles can reach the boundary $(\rho = \infty)$ in finite time, massive particles never reach the boundary



String theory on AdS

- (Super)string theory is consistent in 10 dimensions
- Consider the space AdS$_5 \times$S^5 (with F_5 flux)
- It is a consistent background for IIB string theory
- At low energies $\Rightarrow$ IIB supergravity on AdS$_5 \times$S^5
- Is equivalent to the 4d $\mathcal{N} = 4$ gauge theory

TO BE CONTINUED...