ABJM Baryon Stability and Myers effect

Konstadinos Siampos

CPHT-Ecole Polytechnique

Corfu Strings 2011

- Based on work with:
  - Y. Lozano, M. Picos and K. Sfetsos, JHEP 07(2011)03, arXiv:1105.093[hep-th].

AdS/CFT and motivation

- The AdS/CFT correspondence is a tool to extract information for gauge theories at strong coupling from gravity. Prototype was the $AdS_5 \times S^5$ dual to $\mathcal{N}$=4 SYM [Maldacena 99]. Extensions: temperature, velocity, Coulomp branch, marginally deformed backgrounds...
- AdS_4/CFT_3: Type IIA string theory on $AdS_4 \times CP^3$ with an $\mathcal{N} = 6$ quiver CS matter theory with gauge group $U(N)_k \times U(N)_{-k}$ and marginal superpotential [ABJM Model]. The superpotential coupling proportional to k^{-2}, $N^{1/5} \ll k$ and allows for a weak coupling regime ($\lambda = N/k$). The Type IIA theory is then weakly curved when $k \ll N$.
- Bound states of quarks are dual to classical string/brane probe solutions. Discrepancies arise in many examples between field theory /experimental expectations and their gravitational description, baryons: colorless states.

Plan of the talk:

- Construction of baryons within the gravity/gauge theory duality.
- Macroscopical calculation of binding energy and charges.
- Stability analysis:
  - Based on general statements concerning the perturbative stability of such string solutions (transcendental equation), rather than (heavy) numerics.
  - Applications and resolutions of the discrepancies, non-sinlet solutions.
- Microscopical calculation of binding energy and charges.
- Conclusions and future directions.

Baryon potential within AdS/CFT

- Heavy baryon potential $E(L)$ is extracted from Wilson loop expectation values $\langle W(C)\rangle$.
- Within AdS/CFT, the interaction potential energy of the baryon is given by [prototype by Witten 1998]

$$e^{-\mathrm{i}ET} = \langle W(C)\rangle \simeq \exp\left(\mathrm{i}S[C]\right) \ ,$$

$S[C] = S_{NG} + S_{DBI} + S_{CS}$, **Note**: Quarks are external.

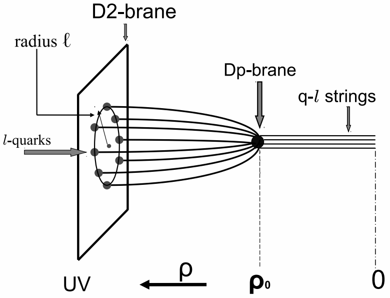

Figure: Baryon Configuration.

Dp-Brane energy

The DBI action of a Dp-brane in string units reads

$$S_p = -T_p \int d^{p+1}\xi \, e^{-\phi} \sqrt{|\det(P[g + 2\pi\mathcal{F}])|} \ , \quad T_p = \frac{1}{(2\pi)^p} \ ,$$

where g is the induced metric and $\mathcal{F} = F + \frac{1}{2\pi} B_2$ is the magnetic flux. The metric of a Dp-brane wrapping on $CP^{p/2}$ cycles (gauge choice is time and the angles of the $CP^{p/2}$ cycles) reads

$$ds^2_{\mathrm{ind}} = -\frac{16\rho^2}{L^2} d\tau^2 + L^2 ds^2_{CP^{\frac{p}{2}}} \ .$$

where $F = \mathcal{N} J$ and $B_2 = -2\pi J$, J is the Kähler form (equations of motion are satisfied) and $\mathcal{N} \in 2\mathbb{Z}$. The energy of the Dp-brane is [Lozano et. al. 2010].

$$E^{Dp}_{DBI} = -Q_p \frac{4\rho_0}{L} \ , \quad Q_p = \frac{T_p}{g_s} \, \mathrm{Vol}(CP^{\frac{p}{2}}) \, \left(L^4 + (2\pi)^2(\mathcal{N} - a_p)^2\right)^{\frac{p}{4}} \ ,$$

$a_{2,6} = 1$, $a_4 = 0$, due Freed-Witten anomaly of CP^2, not spin-manifold.

Note: $\mathcal{N}^2$ is comparable to $L^4 \gg 1$.

Dp-brane Charges

The CS action for a Dp-brane reads

$$S_{CS} = T_p \int d^{p+1}\xi \ \left[P\left(\sum_q C_q \ e^{B_2} \right) e^{2\pi F} \right]_{p+1} .$$

Both the D4 and D6-branes have CP^1 D2-branes dissolved. Therefore in the presence of a magnetic flux they capture the F_2 flux and develop a tadpole with charge $q = k \, \frac{\mathcal{N}^{\frac{p}{2}-1}}{2^{\frac{p}{2}-1}(\frac{p}{2}-1)!}$, [Lozano et al 2010].

There are three more couplings for $D6$:

- The $\int_{D6} F_2 \wedge B_2 \wedge B_2 \wedge A$ which cancels [Aharony et al 09] from higher curvature terms [Green et al 96, Cheung et al 97, Bachas et al 99].
- The $\int_{D6} F_2 \wedge F \wedge B_2 \wedge A$ which contributes to its k charge, $q_{D6} = N + k \, \frac{\mathcal{N}(\mathcal{N}-2)}{8}$, where the N units induced by the F_6 flux $S^{D6}_{CS} = 2\pi \ T_6 \int_{R\times\mathbb{P}^3} P[F_6] \wedge A = N \ T_{F1} \int dt A_t$.

Classical solution

Solving the e.o.m. and imposing the b.c. at the boundary and the baryon vertex (Figure) we find that the length and the energy of the distribution reads

$$
\begin{aligned}
\ell &= \frac{L^2\sqrt{x(1-x)}}{6\rho_0}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4};4x\,(1-x)\right)\;, \\
E_{bin} &= E_{Dp}+E_{lF1}+E_{(q-l)F1}= \\
&= l\;T_{F1}\,\rho_0\Big\{-{}_2F_1\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4};4x\,(1-x)\right)+2x-1\Big\}\;. \\
& x=l_{\min}/l\;,\, l\geqslant\frac{q}{2}(1+\sqrt{1-\beta^2})=l_{\min}\;,\sqrt{1-\beta^2}\equiv\frac{2Q_p}{L\,q\,T_{F_1}}\;.
\end{aligned}
$$

Thus, the binding energy reads
$E_{bin}=-f(x)\frac{(g_sN)^{2/5}}{\ell}\leqslant 0\;, f(x)\geqslant 0\;.$
[Conformal dependence (non-logarithmic), non-pertubative and concavity].

Stability analysis

Instabilities can emerge only from the longitudinal fluctuations of the l strings [Sfetsos, K.S. 2008]. Perturbing the embedding according to $r = r_{\rm cl} + \delta r(\rho)$ and expanding the Nambu-Goto action to quadratic order in the fluctuations, the zero mode solution vanishing in the UV reads

$$\delta r = A \int_{\rho}^{\infty} d\rho \frac{\rho^2}{(\rho^4 - \rho_1^4)^{3/2}} = \frac{A}{3\rho^3}\ {}_2F_1\Big(\frac{3}{2}, \frac{3}{4}; \frac{7}{4}; \frac{\rho_1^4}{\rho^4}\Big)\,.$$

imposing the boundary condition at the baryon vertex ρ_0 we find

$${}_2F_1\Big(\frac{3}{2}, \frac{3}{4}; \frac{7}{4}; 1-\gamma^2\Big) = \frac{3}{2\gamma(1+\gamma^2)}\,, \quad \gamma \equiv \sqrt{1 - \frac{\rho_1^4}{\rho_0^4}}\ .$$

The solution of the transcedental equation is $\gamma_c \simeq 0.538$, thus the bound of F-strings coming from the stability is more restrictive $l \geqslant \frac{q}{1+\gamma_c}(1 + \sqrt{1-\beta^2})$. As for the brane fluctuations they prove to be stable. Note that there are no boundary conditions for these fluctuations, the reason being that the $\mathbb{R} \times CP^{\frac{p}{2}}$ space has no boundary.

Microscopical energy

D0-brane charge on the Dp-branes wrapped on (fuzzy) $CP^{\frac{p}{2}}$ suggests a close analogy with the dielectric effect dielectric [Emparan 97, Myers 99]. The DBI action describing the dynamics of n coincident D0-branes expanded into fuzzy $CP^{p/2}$

$$S^{DBI}_{nD0} = -\frac{1}{g_s}\int d\tau \frac{4\rho}{L}\,\mathrm{STr}\sqrt{\det Q}\;,\; Q^i{}_j = \delta^i{}_j + \frac{i}{2\pi}[X^i, X^k]E_{kj}\;.$$

$E = g + B_2$. Thus, $Q^i{}_j = \delta^i{}_j + M^i{}_j$ with $M^i{}_j$ given by

$$M^i{}_j = -\frac{1}{\frac{p}{2}+1}\Lambda_{(m)} f_{ikl}X^l\left(\frac{pL^2}{8\pi}\delta^k{}_j - \sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{kjm}X^m\right),$$

We shall compute $\det(Q)$ by computing traces of powers of M for the fuzzy $CP^{p/2}$ space.

However, for $B_2 = 0$ in the limit

$$L \gg 1 \ , \quad m \gg 1 \longrightarrow r \simeq \frac{L^4}{m^2} = \text{finite} \ ,$$
$$\mathrm{Tr}(M^{2n}) \simeq p\,(-1)^n \left(\frac{r}{16\pi^2}\right)^n \mathbb{I} \,, \qquad \mathrm{Tr}(M^{2n+1}) \simeq 0 \ .$$

Thus the energy of n D0-branes expanding into a fuzzy $CP^{\frac{p}{2}}$ is then given to leading order in m by

$$E_{nD0} \simeq -\frac{n}{g_s}\left(1 + \frac{L^4}{16\pi^2 m^2}\right)^{\frac{p}{4}} \frac{4\rho_0}{L} \ , \ (L \gg 1 \ , L^p \ll n),$$

where $n = \dim(m, 0)$. For $m \sim \frac{\mathcal{N}}{2}$ the leading order in m coincides with the macroscopical result.

- For $B_2 \neq 0$ the discussion is more technical and would not be presented here. It turns out, that redefinition of m gets corrected: $\mathcal{N} = 2m + 2$, $p = 2, 4$ and $\mathcal{N} = 2m + 4$, $p = 6$.

Microscopical charges

We shall next show how fundamental strings that stretch from the Dp-brane to the boundary of AdS_4 strings arise in the microscopic setup. The relevant CS terms for n coincident D0-branes in the $AdS_4 \times CP^3$ are

$$
\begin{aligned}
S_{CS} \quad = \quad & \int d\tau \ \mathrm{STr}\Big\{ \Big[(i_X i_X) F_2 - \frac{1}{(2\pi)^2} (i_X i_X)^3 F_6 + \frac{i}{2\pi} (i_X i_X)^2 F_2 \wedge B_2 - \\
& - \frac{1}{2} \frac{1}{(2\pi)^2} (i_X i_X)^3 F_2 \wedge B_2 \wedge B_2 \Big] A_\tau \Big\} .
\end{aligned}
$$

Where we have expanded the background potentials on the non-Abelian scalars occurs through the Taylor expansion [Garousi et al 1998] and the pull-backs into the worldline are given in terms of gauge covariant derivatives, $D_\tau X^\mu = \partial_\tau X^\mu + i[A_\tau, X^\mu]$.

In the large m limit we find:

- $S_{CS_1} \simeq q \int d\tau A_\tau$, $q = \frac{2}{p}\, k\, \frac{\mathcal{N}^{\frac{p}{2}-1}}{2^{\frac{p}{2}-1}(\frac{p}{2}-1)!}$,
  the number of fundamental string charge in each CP^1, in agreement with the macroscopical result.
- $S_{CS_2} \simeq N \int d\tau A_\tau$, in agreement with the macroscopical result.
- $S_{CS_3} \simeq -k\, \frac{m^{\frac{p}{2}-2}}{(\frac{p}{2})!} \int d\tau A_\tau$, $S_{CS_4} \simeq \frac{3!}{8}\, k\, \frac{m^{\frac{p}{2}-3}}{(\frac{p}{2})!} \int d\tau A_\tau$.

To find the total k charge we add the subleading contributions in the large m expansion of S_{CS_1}. Next we use the corrected redefinition of m we recover precicely the units of F-charge for D2, D4 and D6 brane plus a $k/8$ contribution for the D6, coming from S_{CS_4}. Stability analysis goes along the same lines than in the macroscopical set-up; non-singlet classical stable solutions.

Dielectric higher-curvature terms

Generalizing the Chern–Simons action for multiple Dp-branes [Myers 1999] to include higher curvature terms we find for our background

$$S_{h.c.} = -\frac{i}{(2\pi)^2} \int_{\mathbb{R}} STr[(i_X i_X)^3 (F_2 \wedge \Omega_4)] A \ ,$$

where Ω_4 is given in term of the Pontryagin classes of the normal and the tangent bundle of the three CP^2 circles of the CP^3 manifold [Eguchi et al 1980, Bergman et al 2009].
Substituting F_2 and Ω_4 we find:

$$S_{h.c.} \simeq -\frac{\kappa}{8} \int_{\mathbb{R}} d\tau A_\tau \ .$$

Thus this higher curvature coupling cancels the S_{CS_4} contribution as in the macroscopical case.

Summary-future directions

- We have constructed macroscopically various configurations of magnetically charged particle-like branes in ABJM with reduced number of quarks. The stability analysis increases the classical lower bound for each value of the magnetic flux.
- We have given an alternative description in terms of D0-branes expanded into fuzzy $CP^{\frac{p}{2}}$ spaces that allows to explore the finite 't Hooft coupling region, $L^p \ll n$.
- We have constructed dielectric higher curvature couplings that to the best of our knowledge have not been considered before in the literature. This new coupling exactly cancels the $k/8$ contribution to the D6-brane tadpole.
- It would be interesting to extend to theories with reduced supersymmetry, like the Klebanov–Strassler backgrounds, where the internal geometry is the $T^{1,1}$ conifold. Non-singlet baryon vertex???

Review of the $AdS_4 \times CP^3$ background

In our conventions the $AdS_4 \times CP^3$ metric reads

$$ds^2 = L^2 \left(\frac{1}{4} ds^2_{AdS_4} + ds^2_{\mathbb{CP}^3} \right) ,$$

with L the radius of curvature in string units

$$L = \left(\frac{32\pi^2 N}{k} \right)^{1/4} , \qquad g_s = \frac{L}{k}$$

and where we have normalized the two factors such that $R_{\mu\nu} = -3g_{\mu\nu}$ and $8g_{\alpha\beta}$ for AdS_4 and CP^3, respectively. The explicit parameterization of AdS_4 we use is

$$ds^2_{AdS_4} = \frac{16\,\rho^2}{L^2} d\vec{x}^2 + L^2 \frac{d\rho^2}{\rho^2} , \quad d\vec{x}^2 = -d\tau^2 + dx_1^2 + dx_2^2 .$$

For the metric on CP^3 we use the parameterization in [Pope 1984,Warner 1985]

$$\begin{aligned} ds^2_{\mathbb{CP}^3} = \quad & d\mu^2 + \sin^2\mu \left[d\alpha^2 + \frac{1}{4}\sin^2\alpha \left(\cos^2\alpha \, (d\psi - \cos\theta \, d\phi)^2 + d\theta^2 \right.\right. \\ & \left.\left. + \sin^2\theta \, d\phi^2 \right) + \frac{1}{4}\cos^2\mu \left(d\chi + \sin^2\alpha \, (d\psi - \cos\theta \, d\phi) \right)^2 \right] , \end{aligned}$$

where

$$0 \leqslant \mu, \alpha \leqslant \frac{\pi}{2}, \quad 0 \leqslant \theta \leqslant \pi, \quad 0 \leqslant \phi \leqslant 2\pi, \quad 0 \leqslant \psi, \chi \leqslant 4\pi .$$

Inside CP^3 there is a CP^1 for $\mu = \alpha = \pi/2$ and fixed χ and ψ and also a CP^2 for fixed θ and ϕ.
In these coordinates the connection in $ds^2_{S^7} = (d\tau + \mathcal{A})^2 + ds^2_{\mathbb{CP}^3}$ reads

$$\mathcal{A} = \frac{1}{2}\sin^2\mu \left(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\, d\phi)\right).$$

The Kähler form

$$J = \frac{1}{2} d\mathcal{A},$$

is then normalized such that

$$\int_{CP^1} J = \pi, \qquad \int_{CP^2} J \wedge J = \pi^2, \qquad \int_{CP^3} J \wedge J \wedge J = \pi^3 .$$

Therefore,

$$\frac{1}{6} J \wedge J \wedge J = d\mathrm{Vol}(\mathbb{P}^3) \qquad \text{and} \qquad \mathrm{Vol}(\mathbb{CP}^3) = \frac{\pi^3}{6} .$$

The $AdS_4 \times CP^3$ background fluxes can then be written as

$$F_2 = \frac{2L}{g_s} J\,, \qquad F_4 = \frac{3L^3}{8g_s}\, d\mathrm{Vol}(AdS_4)\,, \qquad F_6 = -(\star_{10} F_4) = \frac{6\,L^5}{g_s}\, d\mathrm{Vol}(\mathbb{P}^3)$$

where $g_s = \dfrac{L}{k}$. The flux integrals satisfy

$$\int_{CP^3} F_6 = 32\,\pi^5\,N\,, \qquad \int_{CP^1} F_2 = 2\pi\,k\,.$$

The flat B_2-field that is needed to compensate for the Freed–Witten worldvolume flux in the D4-brane is given by [Aharony et al 2009]

$$B_2 = -2\pi J\;.$$

Fuzzy $CP^{\frac{p}{2}}$ manifold

$CP^{\frac{p}{2}}$ is the coset manifold $SU(\frac{p}{2}+1)/U(\frac{p}{2})$, and can be defined by the submanifold of $\mathbb{R}^{\frac{p^2}{4}+p}$ determined by the set of $p^2/4$ constraints

$$\sum_{i=1}^{\frac{p^2}{4}+p} x^i x^i = 1\,, \qquad \sum_{j,k=1}^{\frac{p^2}{4}+p} d^{ijk} x^j x^k = \frac{\frac{p}{2}-1}{\sqrt{\frac{p}{4}(\frac{p}{2}+1)}} x^i$$

where d^{ijk} are the components of the totally symmetric $SU(\frac{p}{2}+1)$-invariant tensor. The Fubini–Study metric of the $CP^{\frac{p}{2}}$ is given by

$$ds^2_{CP^{\frac{p}{2}}} = \frac{p}{4(\frac{p}{2}+1)} \sum_{i=1}^{\frac{p^2}{4}+p} (dx^i)^2\,.$$

A fuzzy version of $CP^{\frac{p}{2}}$ can then be obtained by imposing the conditions at the level of matrices. This is achieved with a set of coordinates X^i ($i = 1, \ldots, \frac{p^2}{4}+p$) in the irreducible totally symmetric representation of order m, $(m, 0)$, satisfying:

$$[X^i, X^j] = i\Lambda_{(m)} f_{ijk} X^k\,, \qquad \Lambda_{(m)} = \frac{1}{\sqrt{\frac{pm^2}{4(\frac{p}{2}+1)} + \frac{p}{4}m}}$$

with f_{ijk} the structure constants in the algebra of the generalized Gell-Mann matrices of $SU(\frac{p}{2}+1)$. The dimension of the $(m,0)$ representation is given by

$$\dim(m,0) = \frac{(m+\frac{p}{2})!}{m!(\frac{p}{2})!}\ .$$

The Kähler form of the fuzzy $CP^{\frac{p}{2}}$ is given by:

$$J_{ij} = \frac{1}{\frac{p}{2}+1}\sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{ijk} X^k\,.$$