Combining F-Term Hybrid Inflation With a Peccei-Quinn Phase Transition

C. Pallis

Department of Physics
University of Cyprus

Based on:

- G. Lazarides and C. Pallis, *Phys. Rev. D.* **82**, *063535 (2010)* [arXiv:1007.1558].

Outline

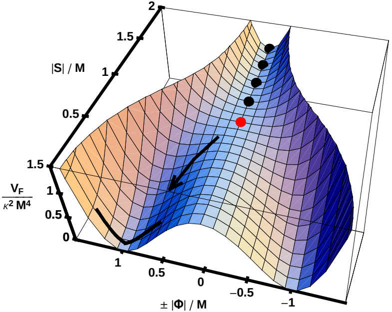
The Relevant Superpotential Terms

The FHI can be Realized Adopting the Following Superpotential[1]:

$$W_{\rm FHI} = \kappa S\left(\bar\Phi\Phi - M^2\right),\ \text{Where } \kappa \text{ and } M \text{ are Real Parameters With } M \sim M_{\rm GUT} = 2.86\cdot 10^{16}\ {\rm GeV}$$

- $W_{\rm FHI}$ is Renormalizable and Consistent With the $U(1)_R$ Symmetry: $S \;\rightarrow\; e^{i\alpha}\, S,\ \bar\Phi\Phi \;\rightarrow\; \bar\Phi\Phi,\ W \rightarrow\; e^{i\alpha}\, W$.
- S: a Left Handed Superfield, Singlet Under a GUT Gauge Group, e.g., $G_{\rm LR} = {\rm SU(3)_c}\times{\rm SU(2)_L}\times{\rm SU(2)_R}\times{\rm U(1)_{B-L}}$.
- $\bar\Phi$ and Φ: Pair of Left Handed Superfields Which are ${\rm SU(2)_R}$ Doublets With $B-L=-1,\ 1$ Respectively – no Cosmic Strings Are Produced Since $SU(2)_R\times U(1)_{B-L}\rightarrow U(1)_Y$.

The SUSY Potential



- The SUSY Potential is $V_{\rm SUSY} = V_{\rm F} + V_{\rm D}$, Where
  - $V_{\rm D} = 0$, with $|\bar\Phi| = |\Phi|$.
  - $V_{\rm F} = \kappa^2 M^4\left((\mathbf{\Phi}^2-1)^2 + 2\mathbf{S}^2\mathbf{\Phi}^2\right)$, with $\mathbf{\Phi} = |\Phi|/M = |\bar\Phi|/M$ and $\mathbf{S} = |S|/M$.
- $W_{\rm FHI}$ Gives Rise to FHI Since There is a F-Flat Direction with $\mathbf{\Phi}=\mathbf{0}$ and $V_{\rm F}\simeq\kappa^2M^4 = {\rm cst}$ Which is a Local Minimum of $V_{\rm F}$ for $\mathbf{S} > 1 = \mathbf{S}_c$.
- $W_{\rm FHI}$ Also Leads to the Spontaneous Breaking of $G_{\rm LR}$, Since the SUSY Vacuum is

$$\langle S\rangle = 0\ \text{ and }\ |\langle\bar\Phi\rangle| = |\langle\Phi\rangle| = v_G = M$$

[1] G.R. Dvali, Q. Shafi, and R.K. Schaefer (1994); G. Lazarides, R.K. Schaefer and Q. Shafi (1997).

The Peccei-Quinn (PQ) Solution to the Strong CP Problem

Due to the Structure of the Vacuum of $\mathrm{SU}(3)_{\rm c}$ the Langrangian of QCD Includes a CP-Violating Term, Involving the Strong Coupling Constant, g_3, the Gluon Field-Strength Tensor, $\mathcal{G}$, and its Dual, $\widetilde{\mathcal{G}}$. I.e.,

$$\mathcal{L}_{\rm QCD} \ni \frac{g_3^2}{32\pi^2}\bar\theta\, \mathcal{G}^{\rm a}_{\mu\nu}\, \widetilde{\mathcal{G}}_{{\rm a}\mu\nu} \;\; \text{With}\;\; \bar\theta \lesssim 5\cdot 10^{-10} \;\; (!?) \;\; \text{Since}\;\; d_{\rm n} \simeq 5\cdot 10^{-16}\bar\theta < 10^{-26}{\rm e\ cm}$$

With $d_{\rm n}$ the Neutron Electric Dipole Moment. A Possible Solution to This *Strong CP Problem* is to Introduce a Global Color Anomalous $U(1)_{\rm PQ}$ Symmetry Which Induces New Terms in $\mathcal{L}_{\rm QCD}$ As Follows:

$$\mathcal{L}_{\rm QCD} \ni \frac{g_3^2}{32\pi^2}\left(\bar\theta + c_a\frac{a}{f_a}\right)\mathcal{G}^{\rm a}_{\mu\nu}\widetilde{\mathcal{G}}_{{\rm a}\mu\nu} \;\; \text{With}\;\; 10^9 \lesssim f_a/{\rm GeV} \lesssim 10^{12} \;\; \text{and} \;\; <a> = -\bar\theta f_a/c_a.$$

Where c_a is a Model-Dependent Parameter, $a(x)$ is the Goldstone Boson - Axion - Associated With the Spontaneous Breaking of $U(1)_{\rm PQ}$ and f_a is the Energy Scale Where $U(1)_{\rm PQ}$ is Broken Spontaneously.

Therefore, Minimizing the Potential With Respect to a Sets The Total CP-Violating Term To Zero.

The Relevant Superpotential Terms

Obviously, the Spontaneous Breaking of $U(1)_{\rm PQ}$ Within SUSY Can be be Realized Through the VEVs Which Acquire a Pair of PQ-Charged Fields Adopting[2] Only Renormalizable Superpotential Terms Similar To Those Which Lead to FHI. I.e., If We Impose $P \to P,\ Q \to e^{i\alpha}\,Q, \bar Q \to e^{-i\alpha}\,Q$, Under $U(1)_{\rm PQ}$ We Obtain $W_{\rm PQ} \to W_{\rm PQ}$ Where

$$W_{\rm PQ} = \kappa_a P\left(\bar Q Q - f_a^2/4\right), \;\; \text{Which Leads To VEVs}\;\; \phi_Q = f_a \;\; \text{With}\;\; 2Q = 2\bar Q = \phi_Q.$$

Can we Obtain a Concrete Model Combining Both Ingredients (FHI and PQPT)?

[2] *J.E. Kim (1984); T. Goto and M. Yamaguchi (1992)*

The Relevant Superpotential

The Key Point Of our Construction is That P Can Be Regarded as the Linear Combination of the $G_{\rm LR}$ Singlets With $({\rm PQ},{\rm R})=(0,4)$ That Does Not Couple to $\bar\Phi\Phi$. Therefore, The Superpotential can Be Written As

$$
\begin{aligned}
W &= \kappa S\left(\bar\Phi\Phi - M^2\right) \quad \text{FHI}\\
&+ \kappa_a P(\bar Q Q - f_a^2/4) \quad \text{PQPT}\\
&+ \lambda S\,\bar Q Q \quad \text{Unavoidable Coupling}\\
&+ \underbrace{\lambda_\mu \frac{\bar Q^2 h^2}{m_{\rm P}}}_{\text{to Generate } \mu = \lambda_\mu f_a^2/m_{\rm P} \sim 1~{\rm TeV}} \left(m_{\rm P} \simeq 2.44\cdot 10^{18}~{\rm GeV}\right)\\
&+ \underbrace{y_{\nu ij}\frac{\bar\Phi l^c_i \bar\Phi l^c_j}{m_{\rm P}}}_{\text{to Generate Masses for RHNs}}\\
&+ \underbrace{y_{lij} l_i h l^c_j + y_{qij} q_i h q^c_j}_{\text{W of MSSM with } \mu=0}\\
&+ \underbrace{\lambda_{D{\rm a}}\bar Q \bar D_{\rm a} D_{\rm a} + \lambda_{H{\rm a}}\bar Q H_{\rm a}^2}_{\text{to Avoid Domain Walls}}
\end{aligned}
$$

Super-fields	Representations under $G_{\rm LR}$	Global Symmetries R	PQ	B	D
Matter Fields					
l_i	$(\mathbf{1},\mathbf{2},\mathbf{1},-\mathbf{1})$	0	-2	0	0
l^c_i	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	2	0	0	0
q_i	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1/3})$	1	-1	$1/3$	0
q^c_i	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{2},-\mathbf{1/3})$	1	-1	$-1/3$	0
Higgs Fields					
S	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{0})$	4	0	0	0
$\bar\Phi$	$(\mathbf{1},\mathbf{1},\mathbf{2},-\mathbf{1})$	0	0	0	0
Φ	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	0	0	0	0
P	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{0})$	4	0	0	0
$\bar Q$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{0})$	0	-2	0	0
Q	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{0})$	0	2	0	0
h	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{0})$	2	2	0	0
Extra Matter Fields					
$\bar D_{\rm a}$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{2/3})$	2	1	0	-1
$D_{\rm a}$	$(\mathbf{3},\mathbf{1},\mathbf{1},-\mathbf{2/3})$	2	1	0	1
$H_{\rm a}$	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{0})$	2	1	0	0

Evading the Domain-Wall Problem

We Introduce[3] n Pairs Of Left-Handed Superfields $\bar{D}_{\rm a}$ and $D_{\rm a}$ $({\rm a}=1,...,n)$ Which Are ${\rm SU}(3)_{\rm c}$ $\bar{\mathbf{3}}$ And $\mathbf{3}$ Respectively.
To Restore Gauge Unification, We Include An Equal Number of ${\rm SU}(2)_{\rm L}\times{\rm SU}(2)_{\rm R}$ $(\mathbf{2},\mathbf{2})$ Superfields $H_{\rm a}$.
We Give Intermediate Scale Masses To $\bar{D}_{\rm a}-D_{\rm a}$ and $H_{\rm a}$ Through Superpotential Terms:

$$m_{D_{\rm a}}=m_{\bar{D}_{\rm a}}\simeq\lambda_{D_{\rm a}}f_a \text{ and } m_{H_{\rm a}}\simeq\lambda_{H_{\rm a}}f_a.$$

Soft SUSY Breaking and Instanton Effects Explicitly Break ${\rm U}(1)_R\times{\rm U}(1)_{\rm PQ}$ to a Discrete Subgroup.
The Explicitly Unbroken Subgroup of ${\rm U}(1)_R\times{\rm U}(1)_{\rm PQ}$ is $Z_4\times Z_{2(n-6)}$ and Can Be Found, For Every n, as Follows:

$$\left.\begin{matrix}e^{irR(W)}=1\\ e^{ir\sum_i R(i)+p\sum_i PQ(i)}=1\end{matrix}\right\}\Rightarrow\begin{cases}4r=0\ (\mathrm{mod}\ 2\pi)\\ -12r+2(n-6)p=0\ (\mathrm{mod}\ 2\pi)\end{cases}\text{ where }\begin{cases}e^{irR}\in{\rm U}(1)_R\\ e^{ipPQ}\in{\rm U}(1)_{\rm PQ},\end{cases}$$

Where r $[p]$ is a ${\rm U}(1)_R$ $[{\rm U}(1)_{\rm PQ}]$ And the Sum Over i Is Applied Over All ${\rm SU}(3)_{\rm c}$ $\mathbf{3}$ And $\bar{\mathbf{3}}$ Of The Model.
It is Then Important To Ensure That This Subgroup is not Spontaneously Broken by $\langle\bar{Q}\rangle$ and $\langle Q\rangle$, i.e.,

$$e^{2ip_s}\langle Q\rangle=\langle Q\rangle \text{ and } e^{-2ip_s}\langle\bar{Q}\rangle=\langle\bar{Q}\rangle\ \Rightarrow\ 2p_s=0\ (\mathrm{mod}\ 2\pi)\,,$$

Since Otherwise Cosmologically Disastrous Domain Walls Will Be Produced at PQPT. This Implies $\boxed{n=5 \text{ or } n=7.}$
The One Loop Evolution of the Three Gauge Coupling Constants g_i, With $i=1,\ 2$ and 3, Within MSSM Obey the Relations[4]:

$$\frac{d\alpha_i}{dt}=-\frac{b_i}{2\pi}\alpha_i^2 \text{ With } \alpha_i(t)=\frac{g_i^2(t)}{4\pi} \text{ and } t=\ln(\Lambda/\ {\rm GeV})\ \Rightarrow\ \frac{1}{\alpha_3}=\frac{(1+B_\alpha)}{\alpha_2}-\frac{B_\alpha}{\alpha_1} \text{ Where } B_\alpha=\frac{b_3-b_2}{b_2-b_1}$$

If we Assign $B-L=2/3$ and $-2/3$ to $\bar{D}_{\rm a}$ and $D_{\rm a}$ Respectively and Find The Contribution of $\bar{D}_{\rm a}, D_{\rm a}$ and $H_{\rm a}$ to $b_1,\ b_2$, and b_3, We Can Prove That B_α Remains Unaltered.

[3] *G. Lazarides and Q. Shafi (1982); H. Georgi and M.B. Wise (1982); G. Lazarides and Q. Shafi (2000).*
[4] *See e.g. M. Peskin, arXiv:0801.1928.*

The Relevant Kälher Potential

We Expand the Kähler Potential up to Fourth Order in the Relevant Fields as Follows:

$$K = |S|^2 + |P|^2 + a(SP^* + S^*P) + b\frac{|S|^4}{4m_{\rm P}^2} + c\frac{|P|^4}{4m_{\rm P}^2} + d\frac{|S|^2|P|^2}{m_{\rm P}^2} + \frac{e|S|^2 + f|P|^2}{2m_{\rm P}^2}(SP^* + S^*P) + \frac{g}{4m_{\rm P}^2}\left[(SP^*)^2 + (S^*P)^2\right]$$

Where All The Coefficients a, b, c, d, e, f and g Are Taken, For Simplicity, Real.

The SUSY Potential

The SUSY Potential, $V_{\rm F}$, Comes from the SUGRA Scalar Potential

$$V_{\rm SUGRA} = e^{K/m_{\rm P}^2}\left(K^{MN^*}F_M F^*_{N^*} - 3\frac{|W|^2}{m_{\rm P}^2}\right) \text{ Where } M, N = \{S, P, \Phi, \bar\Phi, Q, \bar Q\}.$$

In The Limit Where $m_{\rm P} \to \infty$, $V_{\rm SUGRA}$ tends to its SUSY Limit, $V_{\rm F}$, Which Turns out to be:

$$\begin{aligned} V_{\rm F} \simeq\ & \left|\kappa\left(\bar\Phi\Phi - M^2\right) + \lambda\bar Q Q\right|^2/(1-a^2) + \kappa_a^2\left|\bar Q Q - f_a^2/4\right|^2/(1-a^2) + \kappa^2|S|^2\left(|\bar\Phi|^2 + |\Phi|^2\right) \\ + \ & |\lambda S + \kappa_a P|^2\left(|\bar Q|^2 + |Q|^2\right) - \left[a\kappa_a\left(\bar Q^* Q^* - f_a^2/4\right)[\kappa\left(\bar\Phi\Phi - M^2\right) + \lambda\bar Q Q]/(1-a^2) + \text{Complex Conjugate}\right] \end{aligned}$$

The D–term Contribution Vanishes Along The Direction $|\bar\Phi| = |\Phi|$.
For $M \gg f_a$, we Find That The SUSY Vacuum Lies at

$$\langle S\rangle = 0,\ |\langle\bar\Phi\rangle| = |\langle\Phi\rangle| \simeq M,\ \langle P\rangle = 0, \text{ and } |\langle\phi_Q\rangle| = f_a, \quad \text{Where } \phi_Q = 2Q = 2\bar Q.$$

Therefore, $G_{\rm LR}\times U(1)_{\rm PQ}$ is Spontaneously Broken Down to $G_{\rm SM}\times Z_2$.
The Same $V_{\rm F}$ Gives Also Rise to a Stage of FHI and a PQPT, as We Show in the Following.

Flat Directions of the SUSY Potential

$V_{\rm F}$ Possesses TWO D– and F–Flat Directions at

(1) $\bar\Phi=\Phi=0$ and $\bar Q=Q=0$
(2) $S=0,\ \bar\Phi=\Phi=M,$ and $\bar Q=Q=0$

with a Constant Potential Energy Density $\begin{cases} V_{\rm HI0}\simeq\kappa^2M^4/(1-a^2) \\ V_{\rm PQ0}=\kappa_a^2f_a^4/16 \end{cases}$

Stability of the Flat Direction (1)

The Direction (1) Can Be Used As Inflationary Path Since It Corresponds To A Classically Flat Valley Of Minima For

$$\textbf{(a)}\quad |S|>M/\sqrt{1-a^2}\quad \text{and}\quad \textbf{(b)}\quad |\sigma_a|>\sqrt{\kappa(\lambda-a\kappa_a)/(1-a^2)}M,\quad \text{Where}\quad \sigma_a=\lambda S+\kappa_aP$$

.

The Mass Spectrum Along The Inflationary Trajectory:

Superfields of Origin	Fields	Mass Squared
	Bosons	
$\bar\Phi,\Phi$	4 Complex Scalars	$\kappa^2\left(\lvert S\rvert^2\pm\frac{M^2}{(1-a^2)}\right)$
$\bar Q, Q$	2 Complex Scalars	$\lvert\sigma_a\rvert^2\pm\frac{\kappa(\lambda-a\kappa_a)M^2}{(1-a^2)}$
	Fermions	
$\bar\Phi,\Phi$	2 Dirac Spinors	$\kappa^2\lvert S\rvert^2$
$\bar Q, Q$	1 Dirac Spinors	$\lvert\sigma_a\rvert^2$

Since $V_{\rm PQ0}\ll V_{\rm HI0}$, $V_{\rm PQ0}$ can Dominate Over Radiation after the end of FHI Leading to a *Peccei-Quinn Phase Transition* (PQPT)

The Cosmological Scenario Above Can Be Attained If (a) is Violated Before (b).

The Inflationary Potential

The Inflationary Potential can Be Written As $\boxed{V_{\rm HI} = V_{\rm HI0} + V_{\rm HIs} + V_{\rm HIc}}$, Where

- $V_{\rm HI0}$: The Dominant Contribution to $V_{\rm HI}$ Along the F-Flat Direction, $V_{\rm HI0} = \kappa^2 M^4/(1-a^2)$.
- $V_{\rm HIs}$: SUGRA Corrections to $V_{\rm HI}$ Which can be Found by Expanding $V_{\rm SUGRA}$ on the Inflationary Trajectory

$$\begin{aligned} V_{\rm HIs} &\simeq \frac{V_{\rm HI0}}{(1-a^2)m_{\rm P}^2}\Big[A_1|S|^2 + A_{12}\left(S^*P + PS^*\right) + A_2|P|^2\Big] + \frac{V_{\rm HI0}}{4(1-a^2)^2 m_{\rm P}^4}\Big[B_1|S|^4 + B_2|P|^4 \\ &+ B_3|S|^2|P|^2 + \left(B_4|S|^2 + B_5|P|^2\right)\left(S^*P + PS^*\right) + B_6\left((S^*P)^2 + (P^*S)^2\right)\Big] \\ &= \frac{V_{\rm HI0}}{2m_{\rm P}^2}\left(m_+^2\left(s^2+q^2\right) + m_-^2\sigma^2\right) + \cdots \quad \text{Where} \end{aligned}$$

  - $\boxed{K_{SP} = |S|^2 + |P|^2 + a(SP^* + S^*P) = \left(\sigma^2 + s^2 + q^2\right)/2}$ The Quadratic Part of K
  - $m_\pm^2 = \left(D_1 \pm \sqrt{D_2}\right)/(1-a^2)^2$ with $D_1 = A_1 - 2aA_{12} + A_2$ and $D_2 = D_1^2 + 4(1-a^2)(A_{12}^2 - A_1A_2)$.
  - $A_1 = 2ae - a^4 - a^2(d-1) - b,\ A_2 = 1 - d - a(a + ac - 2f),$
    $A_{12} = a(1 + d - a(a+f) + g) - e$ and $B_{1-6} = f(a, ..., g)$
    Note That for $a \neq 0$ and $b = c = d = e = f = g = 0$ We Obtain $m_-^2 = 0$. However We Need $m_-^2 \leq 0$ in Order to Obtain Observationally Acceptable Values for the Spectral Index, $n_{\rm s}$ – See Below.
- $V_{\rm HIc}$: One-Loop Radiative Corrections to $V_{\rm HI}$ due to SUSY Breaking Along the Inflationary Trajectory

$$V_{\rm HIc} \simeq \frac{\kappa^2 V_{\rm HI0}}{16\pi^2(1-a^2)}\left(2\ln\frac{\kappa^2 x M^2}{(1-a^2)\Lambda^2} + 3\right) + \frac{(\lambda - a\kappa_a)^2 V_{\rm HI0}}{32\pi^2(1-a^2)}\left(2\ln\frac{\kappa(\lambda - a\kappa_a)x_a M^2}{(1-a^2)\Lambda^2} + 3\right)$$

with $x = |S|^2(1-a^2)/M^2$ and $x_a = |\sigma_a|^2(1-a^2)/\kappa(\lambda - a\kappa_a)M^2$.

The Inflationary Dynamics

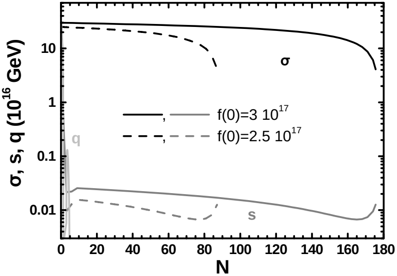
The *Equations Of Motion* (e.o.m.) of the Various Fields are

$$\ddot{f} + 3H\dot{f} + V_{\mathrm{HI},f} = 0 \;\Rightarrow\; H^2 f'' + 3H^2 f' + V_{\mathrm{HI},f} = 0 \text{ with } f = \sigma,\ s, \text{ and } q \text{ and } ' = d/dN \text{ with } N = \ln\left(R/R_{\mathrm{HIi}}\right)$$

Here $R(t)$ is the Scale Factor of the Universe and R_{HIi} its Value at the Onset of FHI.
We Impose The Following Initial Conditions (at $N = 0$): $f(0) = (1.5 - 4.5)\cdot 10^{17}$ GeV and $f'(0) = 0$ with $f = \sigma,\ s,$ or q.
If $f(0)$ is Large Enough, s Reaches An Attractor And Our Results Are Independent Of The Precise $f(0)$. E.g.,

- $q \simeq 0$
- $s \simeq \dfrac{2\kappa^2 C_1 + (\lambda - a\kappa_a)^2 C_2}{8(1-a^2)\pi^2 m_+^2 \sigma} m_{\mathrm{P}}^2,$
- $\sigma = \dfrac{m_{\mathrm{P}}(1-a^2)}{\sqrt{2C_3}}\left[-m_-^2 - D\tan\left(DN - \arctan\dfrac{1}{D}\left(m_-^2 + \dfrac{2C_3\sigma_{\mathrm{HIi}}^2}{m_{\mathrm{P}}^2(1-a^2)^2}\right)\right)\right]^{1/2}$

Where $D = \sqrt{C_3(2\kappa^2 + (\lambda - a\kappa_a)^2)/2\pi^2(1-a^2)^3 - m_-^4}$
With $C_{1,2,3} = f(A's, B's)$.



- The Field s Turns Out To Be Just Mildly, And Not Drastically[5] Reduced w.r.t. σ.

[5] I. Izawa, M. Kawasaki, and T. Yanagida (1997); M. Kawasaki, N. Sugiyama, and T. Yanagida (1998)

The Stage of PQPT

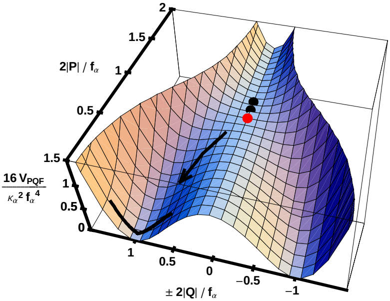
For $\lambda \simeq (0.05-0.1)$, we get $T_{1\mathrm{rh}} > V_{\mathrm{PQ0}}^{1/4}$. Therefore, After FHI we Obtain *Matter Domination* (MD) for $T \geq T_{1\mathrm{rh}}$ and *Radiation Domination* (RD) for $V_{\mathrm{PQ0}}^{1/4} \lesssim T \lesssim T_{1\mathrm{rh}}$. The Value of s at the Beginning of PQPT is:

$$s_{\mathrm{PQi}} \simeq (\rho_{1\mathrm{rh}}/V_{\mathrm{HI0}})^{1/4}\, s_{\mathrm{HIf}} \quad \text{with} \quad \rho_{1\mathrm{rh}} = \pi^2 g_{1\mathrm{rh}*} T_{1\mathrm{rh}}^4/30.$$

For $T \lesssim V_{\mathrm{HI0}}^{1/4}$, W is Dominated by the Term $W_{\mathrm{PQ}} = \kappa_a P(\bar{Q}Q - f_a^2/4)$ and the Relevant F-Term Scalar Potential is

$$V_{\mathrm{PQF}} = \kappa_a^2 \left|\bar{Q}Q - f_a^2/4\right|^2 + \kappa_a^2 |P|^2 \left(|\bar{Q}|^2 + |Q|^2\right) \Rightarrow V_{\mathrm{PQF}} = V_{\mathrm{PQ0}} = \kappa_a^2 f_a^4/16 \quad \text{for} \quad \bar{Q} = Q = 0.$$

Assuming Gravity Mediated Soft SUSY Breaking, the Potential for $\bar{Q} = Q = 0$ has the form:



$$V_{\mathrm{PQ}} \simeq V_{\mathrm{PQ0}} + m_P^2 |P|^2 + V_{\mathrm{T}} + V_{\mathrm{PQc}} \quad \text{for} \quad |P| \geq f_a/2, \quad \text{Where}$$

- $V_{\mathrm{T}} = -\sqrt{2V_{\mathrm{PQ0}}}\, |\mathrm{a}_P||P|$ is a Tadpole Contribution[a] With $\mathrm{a}_P \sim 1$ TeV.
- $V_{\mathrm{PQc}} = \kappa_a^2 V_{\mathrm{PQ0}} \left(\ln \kappa_a^2 |P|^2/\Lambda^2 + 3/2\right)/16\pi^2$ Are Loop Corrections Arising due to the SUSY Breaking.

• V_{PQ} Does Not Give Rise To Another FHI, Since V_{PQc} Spoils the η-Criterion;
• When $|P| < f_a/2$, an Instability Occurs Along the $|P|$-axis Triggering Thereby a PQPT;
• If $|P_{\mathrm{PQi}}| > f_a/2$ We Obtain An Out-Of-Equilibrium Decay Of the PQ System, i.e., A Secondary Reheating.

[a] V.N. Şenoğuz and Q. Shafi (2005).

Reheating Processes and Gravitino ($\widetilde{G}$) Abundance

The Energy Density, ρ_1 [ρ_2], of the Oscillatory System Which Reheats The Universe At The Temperature $T_{1\rm rh}$ [$T_{2\rm rh}$], the Energy Density Of Produced Radiation, $\rho_{\rm R}$, and the Number Density of $\widetilde{G}$, $n_{\widetilde{G}}$, Satisfy The Equations:

$$\left.\begin{aligned}\dot\rho_1+3H\rho_1+\Gamma_1\rho_1&=0,\\ \dot\rho_2+3H\rho_2+\Gamma_2\rho_2&=0\\ \dot\rho_{\rm R}+4H\rho_{\rm R}-\Gamma_1\rho_1-\Gamma_2\rho_2&=0\\ \dot n_{\widetilde{G}}+3Hn_{\widetilde{G}}-C_{\widetilde{G}}\left(n^{\rm eq}\right)^2&=0.\end{aligned}\right\}\Rightarrow\left\{\begin{aligned}Hf_1'&=-\Gamma_1f_1\\ Hf_2'&=-\Gamma_2f_2\\ Hf_{\rm R}'&=(\Gamma_1f_1+\Gamma_2f_2)R\\ Hf_{\widetilde{G}}'&=C_{\widetilde{G}}\left(n^{\rm eq}\right)^2R^3\end{aligned}\right.\ \text{where}\ \left\{\begin{aligned}f_1&=\rho_1R^3\\ f_2&=\rho_2R^3\\ f_{\rm R}&=\rho_{\rm R}R^4\\ f_{\widetilde{G}}&=n_{\widetilde{G}}R^3\end{aligned}\right.\ \text{and}\ \begin{aligned}H&=\left(\rho_1+\rho_2+\rho_{\rm R}\right)^{1/2}/\sqrt{3}m_{\rm P}\\ C_{\widetilde{G}}&=\frac{3\pi}{16\zeta(3)m_{\rm P}^2}\sum_{i=1}^3c_ig_i^2\ln\left(\frac{k_i}{g_i}\right)\\ n^{\rm eq}&=\zeta(3)T^3/\pi^2\end{aligned}$$

where g_i (with $i=1,2,3$) are the Gauge Coupling Constants of MSSM, $(k_i)=(1.634,1.312,1.271)$ and $(c_i)=(33/5,27,72)$. We use the Following Initial Conditions: $\rho_1(0)=V_{\rm HI0},\ \rho_{\rm R}(0)=n_{\widetilde{G}}(0)=0,$ and $\rho_2(\bar N_{\rm PQ})=V_{\rm PQ0}$, Where $\bar N_{\rm PQ}$ is The Value of $\bar N$ Corresponding To The Temperature $T_{\rm PQ}$ Which Is Defined As The Solution Of The Equation $\rho_{\rm R}\left(T_{\rm PQ}\right)=V_{\rm PQ0}$.

Reheating after FHI

The Corresponding Reheat Temperature is

$$T_{1\rm rh}=\left(\frac{72}{5\pi^2g_{1\rm rh*}}\right)^{1/4}\sqrt{\Gamma_1m_{\rm P}}\ \text{ with }\ \Gamma_1=\frac{1}{16\pi}\lambda^2m_{\rm inf}$$

(since $\lambda S\bar QQ\in W$) where $g_{1\rm rh*}\simeq438.75$ [$g_{1\rm rh*}\simeq513.75$] for $n=5$ [$n=7$]. The $\widetilde{G}$ Abundance is

$$Y_{1\widetilde{G}}=\frac{n_{\widetilde{G}}}{\mathsf{s}}(T_{1\rm rh})\simeq1.9\cdot10^{-12}\left(\frac{T_{1\rm rh}}{10^{10}\ {\rm GeV}}\right).$$

Reheating after PQPT

The Corresponding Reheat Temperature is

$$T_{2\rm rh}=\left(\frac{72}{5\pi^2g_{2\rm rh*}}\right)^{1/4}\sqrt{\Gamma_2m_{\rm P}}\ \text{ with }\ \Gamma_2=\frac{1}{2\pi}\lambda_\mu^2\left(\frac{f_a}{2m_{\rm P}}\right)^2m_{\rm PQ}$$

(since $\lambda_\mu\bar Q^2h^2/m_{\rm P}\in W$) where $g_{2\rm rh*}=228.75$. The $\widetilde{G}$ Abundance is

$$Y_{2\widetilde{G}}=\frac{n_{\widetilde{G}}}{\mathsf{s}}(T_{2\rm rh})\simeq\left(\frac{\pi^2}{30}g_{1\rm rh*}\right)^{1/4}\frac{T_{2\rm rh}}{V_{\rm PQ0}^{1/4}}Y_{1\widetilde{G}}.$$

We Observe that $Y_{2\widetilde{G}}$ is Suppressed Relative to $Y_{1\widetilde{G}}$ by the Ratio $T_{2\rm rh}/V_{\rm PQ0}^{1/4}\ll1$.

Cosmological Requirements

We Impose on our Cosmological Scenario the Following Constraints:

(i) The Violation of the Instability Conditions Occurs According to the Desired Order.

(ii) The Number of e-foldings $N_{\rm HI*}$ That the Scale $k_* = 0.002/{\rm Mpc}$ Suffered During FHI has to be Sufficient to Resolve the Horizon and Flatness Problems of *Standard Big Bang* (SBB) Cosmology:

$$N_{\rm HI*} = N_{\rm HI} - N_* \simeq 23 + \frac{2}{3}\ln\frac{V_{\rm HI0}^{1/4}}{1\ {\rm GeV}} - \frac{1}{3}\ln\frac{V_{\rm PQ0}^{1/4}}{1\ {\rm GeV}} + \frac{1}{3}\ln\frac{T_{\rm 1rh}T_{\rm 2rh}}{1\ {\rm GeV}^2} \simeq 50 \ \text{ for } T_{\rm rh1} = 10^{10}\ {\rm GeV} \ \text{ and } T_{\rm rh2} \simeq 10^5\ {\rm GeV}$$

Where N_* $[N_{\rm HI}]$ is the Value of N from the Onset of FHI until k_* Crossed Outside the Horizon of FHI [the End of FHI]. This is Found from the Condition:

$$\max\{\epsilon(\sigma(N_{\rm HI})), |\eta(\sigma(N_{\rm HI}))|\} = 1, \quad \text{with} \quad \epsilon \simeq \frac{m_{\rm P}^2}{2}\left(\frac{V_{{\rm HI},\sigma}}{V_{\rm HI}}\right)^2 \quad \text{and} \quad \eta \simeq m_{\rm P}^2\,\frac{V_{{\rm HI},\sigma\sigma}}{V_{\rm HI}}.$$

(iii) The Power Spectrum, $P_{\mathcal{R}*}$, of the Curvature Perturbation at $k = k_*$ is to be Confronted With the WMAP7 Data:

$$P_{\mathcal{R}*}^{1/2} = V_{\rm HI}^{3/2}/2\sqrt{3}\,\pi m_{\rm P}^3|V_{{\rm HI},\sigma}|\Big|_{N=N_*} \simeq 4.93\cdot 10^{-5}.$$

(iv) The Spectral Index, $n_{\rm s}$, is to be Consistent With the fitting of the WMAP7 Results by the ΛCDM Model ($\alpha_{\rm s} \sim 0$).

$$n_{\rm s} = 1 - 6\epsilon(N_*) \ + \ 2\eta(N_*) = 0.963 \pm 0.028 \ \Rightarrow \ 0.935 \lesssim n_{\rm s} \lesssim 0.991, \ \text{ at 95\% c.l.}$$

(v) In Order For The PQPT to Take Place After A Short Temporary Domination of $V_{\rm PQ0}$, we Require $\left|P_{\rm PQi}\right| > f_a/2$, Where the value of $|P|$ at the Beginning of PQPT.

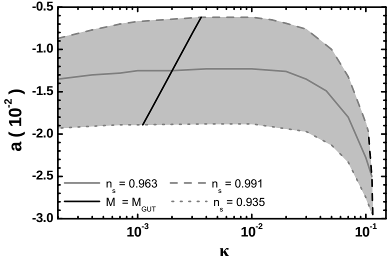
(vi) Assuming Unstable $\widetilde{G}$, We Impose an Upper Bound[6] on $Y_{\widetilde{G}}$ In Order to Avoid Problems With the SBB Nucleosynthesis:

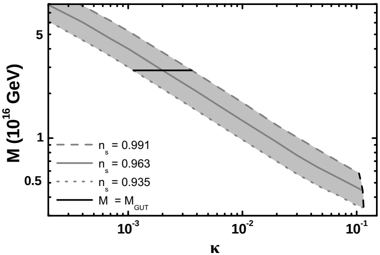
$$Y_{\widetilde{G}} \lesssim \begin{cases}10^{-14}\\ 10^{-13}\end{cases} \text{ for } \widetilde{G} \text{ Mass } m_{\widetilde{G}} \simeq \begin{cases}0.69\ {\rm TeV}\\ 10.6\ {\rm TeV}.\end{cases}$$

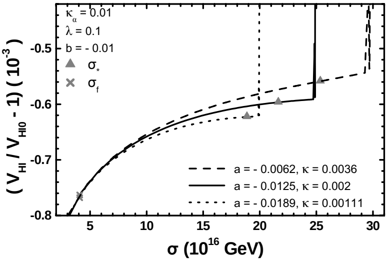
[6] *M. Kawasaki, K. Kohri, and T. Moroi (2005); J.R. Ellis, K.A. Olive, and E. Vangioni (2005).*

Imposing the Inflationary Requirements

We Can Delineate The Allowed (Lightly Gray Shaded) Regions in the $\kappa - a$ and $\kappa - M$ as Follows:





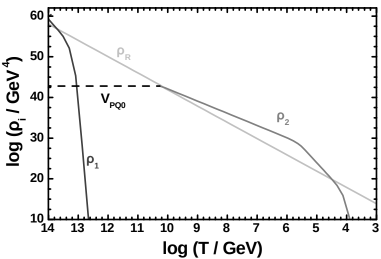


- Our Cosmological Set-Up Depends On The Following Parameters: $\kappa,\ \kappa_a,\ \lambda,\ M,\ f_a,\ \lambda_\mu,\ n,\ a,\ b,\ c,\ d,\ e,\ f,$ and g.
- We Fix $n = 5$, $\kappa_a = -b = 0.01, \lambda = c = d = e = f = g = 0.1$, $f_a = 10^{12}$ GeV and $\lambda_\mu = 0.01 \Rightarrow \mu \sim 1$ TeV.
- We Can Obtain $M = M_{\rm GUT}$ for $\kappa = 0.002$ and $a = -0.0125$.
- The Reduction of $n_{\rm s}$ Can be Attained Without Disturbing the Monotonicity of $V_{\rm HI}$. For Large σ's, $V_{\rm HI}$ Develops an Oscillatory Behavior but for Lower σ's $V_{\rm HI}$ Remains Monotonic.

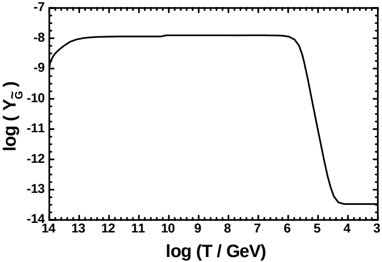
Post-Inflationary Cosmological Evolution

For $\kappa = 0.002$, $M = M_{\rm GUT}$ and $n_{\rm s} = 0.963$ We Obtain $T_{\rm 1rh} = 4.7 \cdot 10^{13}$ GeV, $[T_{\rm 2rh} = 3 \cdot 10^4$ GeV$]$ Resulting to $Y_{1\widetilde{G}} \simeq 9 \cdot 10^{-9}$ $[Y_{2\widetilde{G}} \simeq 1.9 \cdot 10^{-14}]$ Which is Consistent With the $\widetilde{G}$ Constraint for $m_{\widetilde{G}} < 10$ TeV.

- The Evolution of The Energy Densities



- The Evolution of The $\widetilde{G}$ Abundance



Conclusions

Combining FHI With a PQPT Based on Renormalizable Superpotential terms, We can Obtain:

- Observationally Viable FHI at the SUSY GUT Scale With Natural Values, $\pm(0.01 - 0.1)$, for the Model Parameters;
- A Simultaneous Resolution of the Strong CP and μ Problems of MSSM;
- A Second Stage Of Reheating After PQPT, Which Leads to Observationally Safe Values of the $\widetilde{G}$ Abundance.

An Important Open Issue of Our Scenario is this Of Baryogenesis Which Is Currently Under Consideration.