Thermal duality & non-singular superstring cosmology

Hervé Partouche
Ecole Polytechnique

Based on works in collaboration with :
C. Angelantonj, I. Florakis, C. Kounnas and N. Toumbas

arXiv: 0808.1357, 1008.5129, 1106.0946

Corfu Summer Institute, « 11^{th} Hellenic school and workshop on elementary particle physics and gravity », September 17, 2011

Introduction

- In General Relativity : Cosmological evolutions lead to a **initial Big-Bang singularity.**
- The **consistency of String Theory** is expected to **resolve them.**
- In this talk, we would like to discuss this question.

- **Consistency of a string vacuum** means there is no tachyon : the 1-loop vacuum amplitude Z is finite.

  - ✓ This IR statement can be rephrased from a UV point of view :

$$Z = \int_0^{+\infty} \frac{dl}{2l} \, V_d \int \frac{d^d k}{(2\pi)^d} \int_0^{+\infty} dM \, \big(\rho_B(M) - \rho_F(M)\big) \, e^{-(k^2+M^2)l/2}$$

  - ✓ One shows there must be a cancellation between the exponentially growing densities of bosons and fermions,

  - ✓ such that the **effective density of states is that of a 2-dimensional quantum field theory** [Kutasov, Seiberg].

  - ✓ **Effectively, there is a finite number of particles in the UV.**

- **For cosmological pusposes**, we should reconsider this question of consistency for CFT's on the worldsheet, which describe string backgrounds at finite T.

  - ✓ The 1-loop vacuum amplitude Z is computed in a target space where time is Euclidean and compactified on $S^1(R_0)$, ($\beta = 2\pi R_0$).

$$Z = \ln \mathrm{Tr}\, e^{-\beta H} =$$

$$V_{d-1} \int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} \int_0^{+\infty} dM \left[\underbrace{\rho_B(M)}_{\sim e^{\beta_H M}} \ln \left(\underbrace{\frac{1}{1 - e^{-\beta\sqrt{\mathbf{k}^2+M^2}}}}_{\sim e^{-\beta M}} \right) + \rho_F(M) \ln \left(1 + e^{-\beta\sqrt{\mathbf{k}^2+M^2}} \right) \right]$$

  - ✓ When $T > T_H$, Z diverges : **Hagedorn phase transition.**
  - ✓ Dealt with a condensation of tachyons.
  - ✓ However, the dynamical picture in a cosmological set up is unknown.

- **Another approach :** We are going to see that switching on Wilson lines along $S^1(R_0)$ yields a deformed canonical partition function

$$Z(R_0) = \ln \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right]$$

Total Right-moving Ramond charge of the multiparticle states

- It is not a canonical partition function, but has good properties :

✓ Converges for any R_0, due to alternative signs (no tachyon).

✓ T-duality symmetry

$$R_0 \to \frac{1}{2R_0} \quad \text{with fixed point} \quad R_c = \frac{1}{\sqrt{2}}$$

✓ $T = 1/\beta$ has a maximal value $T_c = \dfrac{1}{2\pi R_c}$.

- $T < T_c \quad \Longrightarrow \quad \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right] \quad \simeq \quad \mathrm{Tr}\, e^{-\beta H}$
- This will allow to interpret T as a conventional temperature.

- In fact, the alternative signs imply cancellations :
  - **We don't thermalize the infinite tower of states, but effectively only a finite part of it.**

- At zero T, consistent vacuum = **Finite number of particles.**
  At finite T, consistent = **Only them are thermalized.**
  - If not, we are « inconsistent » *i.e.* « unstable » *i.e.* need to condense tachyons to a trully stable background.

- Thanks to the consistency of these thermal backgrounds, we obtain **bouncing cosmologies** with :
  - **No Hagedorn divergence.**
  - **No singular Big Bang.**

Example of Type II model

- 1-loop vacuum amplitude in $S^1(R_0)\times T^{d-1}(V)\times T^{9-d}\times S^1(R_9)$

$$Z=\frac{V}{(2\pi)^{d-1}}\int_{\mathcal{F}}\frac{d^2\tau}{2\tau_2^{\frac{d+1}{2}}}\;\frac{1}{2}\sum_{a,b}(-)^{a+b+ab}\theta[{}^a_b]^4\;\frac{1}{2}\sum_{\bar a,\bar b}(-)^{\bar a+\bar b+\bar a\bar b}\bar\theta[{}^{\bar a}_{\bar b}]^4\;\frac{\Gamma_{(9-d,9-d)}}{(\eta\bar\eta)^{12}}$$

$$\frac{R_0}{\sqrt{\tau_2}}\sum_{n_0,\tilde m_0}e^{-\frac{\pi R_0^2}{\tau_2}|n_0\tau+\tilde m_0|^2}\qquad\frac{R_9}{\sqrt{\tau_2}}\sum_{n_9,\tilde m_9}e^{-\frac{\pi R_9^2}{\tau_2}|n_9\tau+\tilde m_9|^2}$$

$$(-)^{(a+\bar a)\tilde m_0+(b+\bar b)n_0}\qquad(-)^{\bar a\tilde m_9+\bar b n_9+\tilde m_9 n_9}$$

- ✓ Right-moving susy's are spontaneously broken at the scale $1/R_9$, *i.e.* the string scale because
- ✓ R_9 is stabilized at $R_c=1/\sqrt{2}$ where enhanced $SU(2)_R$.
- ✓ Temperature breaks Left-moving susy's.
- ✓ Alternatively : $(-)^{a\tilde m_0+bn_0+\tilde m_0 n_0}$

Properties

- Left movers have reversed GSO for $n_0 = 2l_0+1$:

$$\left(\Gamma_{m_0,2n_0} V_8 - \Gamma_{m_0+\frac{1}{2},2n_0} S_8\right) - \left(\Gamma_{m_0+\frac{1}{2},2l_0+1} O_8 - \Gamma_{m_0,2l_0+1} C_8\right)$$

- Right movers have reversed GSO for n_9 odd.

$$\Longrightarrow \quad \text{dangerous } O_8 \bar{O}_8, \text{ but } \; M^2_{O\bar{O}} = \left(\frac{1}{2R_0} - R_0\right)^2 \geq 0.$$

✓ NB : In the conventional model at finite T ,

$$M^2_{O\bar{O}} = R_0^2 - 2 \quad \Longrightarrow \quad R_H = \sqrt{2}$$

- In fact, the tachyon free model has a **T-duality symmetry :**

$$R_0 \to \frac{1}{2R_0} \quad \text{with} \quad S_8 \leftrightarrow C_8$$

✓ At the self-dual point $R_0 = R_c$: Enhanced $SU(2)_L$ + adj matter.

Physical interpretation

- **In the thermal model, for $R_0 > R_H$:**

  ✓ Unfold the fundamental domain

$$Z = \int_{\mathcal{F}} d^2\tau \sum_{n_0} (\cdots) = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int_0^{+\infty} d\tau_2 (\cdots) \quad \text{where} \quad n_0 = 0$$

  ✓ Implement level matching $\Rightarrow$ Z takes a « field theory form ».

  ✓ It can be rewritten in terms of a canonical partition function

$$e^Z = \mathrm{Tr}\, e^{-\beta H} \quad \text{where} \quad \beta = 2\pi R_0$$

- **In the tachyon-free model, for $R_0 > R_c$:**
  - Since $(-)^a = (-)^{(a+\bar{a})}\,(-)^{\bar{a}}$, the same procedure yields a **deformed canonical partition function :**

$$e^Z = \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right]$$

Total right-moving Ramond charge of the multiparticle states

where
$\beta = 2\pi R_0$ and spinors S_8 only, for $R_0 > R_c$
$\beta = 2\pi \frac{1}{2R_0}$ and spinors C_8 only, for $R_0 < R_c$ (by T-duality)

- Thus, $T = 1/\beta$ has a maximal value $T_c = \frac{\sqrt{2}}{2\pi}$.
- There are **2 phases**, in which the excitations along $S^1(R_0)$ are
  - **Pure KK modes for $R_0 > R_c$** . (Windings have been set to 0)
  - **Pure windings modes for $R_0 < R_c$** .

- Moreover, $$e^Z = \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right]$$

may differ from a canonical partition function for multiparticle states involving modes with Right moving Ramond charge 1.

- ✓ However, these modes have

$$M \geq \frac{1}{2R_9} = R_9 = \frac{1}{\sqrt{2}} = \pi\, T_c\,,$$

which is larger than the maximal T_c,

- ✓ Thus, the multiparticle states which contain them are Boltzmann suppressed when $T < T_c$ *i.e.* always :

$$e^Z = \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right] \simeq \mathrm{Tr}\, e^{-\beta H}$$

- ✓ **The tachyon free model is essentially a thermal model, and T is a temperature.**

- Then, **why is there no Hagedorn divergence ?**

In fact, the convergence of $e^Z = \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right]$ when $T \approx M_{string}$ is due to the alternative signs :

- For a gas of a single Bosonic (or Fermionic) degree of freedom, with Right-moving Ramond charge $\bar{a}$,

$$\ln \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right] = \mp \sum_{\mathbf{k}} \ln\left(1 \mp (-)^{\bar{a}} e^{-\beta\sqrt{\mathbf{k}+M^2}}\right)$$

- (Boson, $\bar{a}$) and (Fermion, $1-\bar{a}$) have opposite contributions in Z, when they are degenerate.

- This reduces the effective number of degrees of freedom which contribute in the thermal system.

- **Particular case :** The pairing of degenerate space-time (Boson, $\bar{a}$) with (Fermion, $1-\bar{a}$) **can even be an exact symmetry of the massive spectrum.**

  - Sush a symmetry can only exist in $d=2$: When the Right-moving sector on the worldsheet admits a **Massive Spectrum Degeneracy Symmetry** (MSDS) [Kounnas].

  - In a model with this symmetry : Exact cancellation of all the massive modes contributions.

$$e^{Z} = \mathrm{Tr}\left[e^{-\beta H}(-)^{\bar{a}}\right] \equiv \mathrm{Tr}\, e^{-\beta H}$$

For a **finite number** of massless bosonic and fermionic species

E.g. « **Hybrid model** » : Type II on $S^1(R_0) \times S^1 \times \mathscr{M}_8$

$$Z = \frac{V}{2\pi} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^{3/2}} \, \frac{1}{2} \sum_{a,b} (-)^{a+b+ab} \theta[^a_b]^4 \, \frac{\Gamma_{E_8}}{\eta^{12}} \, (\bar{V}_{24} - \bar{S}_{24})$$

$$\frac{R_0}{\sqrt{\tau_2}} \sum_{n_0, \tilde{m}_0} e^{-\frac{\pi R_0^2}{\tau_2} |n_0 \tau + \tilde{m}_0|^2} \; (-)^{a\tilde{m}_0 + bn_0 + \tilde{m}_0 n_0}$$

- ✓ $\bar{V}_{24} - \bar{S}_{24} = 24$ ← MSDS=Almost susy ! : Only 24 unpaired massless modes in $\mathrm{NS}_{\mathrm{Right}}$

- ✓ $$\frac{Z}{V} = 24 \begin{cases} \dfrac{1}{R_0} & \text{for } R_0 > R_c \\ 2R_0 & \text{for } R_0 < R_c \end{cases}$$

- ✓ $\dfrac{F}{V} \equiv -\dfrac{Z}{\beta V} = -\dfrac{n\sigma_2}{\beta^2}$ ← Free energy density for radiation

- In general, the cancellation between the massive $(\text{Bosons}, \bar{a})$ and $(\text{Fermions}, 1-\bar{a})$ is approximate :

  - ✓ Occurs thanks to the **asymptotic supersymmetry generated by the Right-moving sector,**

  - ✓ which is guaranted since we have **broken the Right-moving supersymmetry** **spontaneously.**

  - ✓

$$\frac{F}{V} \simeq -\frac{n\sigma_d}{\beta^d}$$

What happens at $R_0 = R_c$ where enhanced $SU(2)_L$ + adj matter ?

✓ The additional massless states induce a non-analyticity in Z.

$$\text{E.g. Hybrid :} \quad Z = 24\left(\frac{1}{2R_0} + R_0\right) - 24\left|\frac{1}{2R_0} - R_0\right|$$

mass of 24 complex scalars with $m_0 = n_0 = \pm 1$

✓ **At $R_0 = R_c$**, their vertex operators have $p_L = \pm 1$, $p_R = 0$, which is what is needed to map (pure KK along $S^1(R_0)$, S_8)↔(pure winding along $S^1(R_0)$, C_8)

$$\left(\psi^0 e^{-iX_L^0}\right)(z)\, e^{-\phi/2} S_\alpha e^{\frac{i}{2}(X_L^0 + X_R^0)} \bar{V}_{24}(w) \sim \frac{1}{z-w}\, e^{-\phi/2}\, \gamma^0_{\alpha\dot\beta} C_{\dot\beta} e^{\frac{i}{2}(-X_L^0 + X_R^0)} \bar{V}_{24}(w) + \text{reg.}$$

✓ **They trigger the Pure Winding → Pure Momentum phase transition.**

Wilson lines interpretation

- We started with a factorized torus $\underset{(-)^a}{\underset{\updownarrow}{S^1(R_0)}} \times \underset{(-)^{\bar{a}}}{\underset{\updownarrow}{S^1(R_9)}}$

$$\Longleftrightarrow \quad \frac{\sqrt{G}}{\tau_2} \sum_{\vec{n},\vec{\tilde{m}}} e^{-\frac{\pi}{\tau_2}(n\tau+\tilde{m})_I (G+B)_{IJ} (n\bar{\tau}+\tilde{m})_J} (-)^{(a+\bar{a})\tilde{m}_0+(b+\bar{b})n_0} (-)^{\bar{a}\tilde{m}_9+\bar{b}n_9+\tilde{m}_9 n_9}$$

$$G_{IJ} = \begin{pmatrix} R_0^2 + (A_0)^2 R_9^2 & A_0 R_9^2 \\ A_0 R_9^2 & R_9^2 \end{pmatrix}, \quad B_{IJ} = \begin{pmatrix} 0 & A_0' R_9^2 \\ -A_0' R_9^2 & 0 \end{pmatrix}$$

where $A_0 = 1,\ A_0' = 1/2$ (they vanish in thermal models)

✓ They are constant gauge potentials $A_\mu = (A_0, \vec{0}), A'_\mu = (A'_0, \vec{0})$ along $S^1(R_0)$, for the $U(1)$'s obtained by dimensional reduction of G_{IJ} and B_{IJ} along $S^1(R_9)$.

- From this point of view, we deform the Euclidean path integral $\mathrm{Tr}\, e^{-\beta H} \equiv \int \mathcal{D}\varphi\, e^{-S[\varphi]}$ by switching on Wilson lines.
- Since $S^1(R_0)$ is compact, they cannot be gauged away: They are vacuum parameters.
- This yields in general

$$\mathrm{Tr}\, e^{-\beta H - 2i\pi(A_0 Q + A_0' Q')}$$

- In our case, the $U(1)$ charges are

$$Q = \tilde{m}_9 - \frac{\bar{a} + n_9}{2} \quad \text{and} \quad Q' = n_9$$

and we recover $\quad e^{-2i\pi(A_0 Q + A_0' Q')} \equiv (-)^{\bar{a}}$

Cosmology

- We consider the low energy effective action for the massless modes, by integrating out the infinite tower of massive ones.

- We first consider it in Euclidean time *i.e.* on $S^1(R_0) \times T^{d-1}(V)$ in the large V limit.

- **In each phase:** The action exact in α',

  - at the 1-loop level (*i.e.* supposing weak coupling),
  - up to 2-derivatives,

$$\int dx^0 d^{d-1}\mathbf{x}\; \underset{\uparrow}{\beta} a^{d-1} \left[e^{-2\phi} \left(\frac{R}{2} + 2(\partial\phi)^2 \right) + \frac{Z}{\beta V} \right]$$

$$\boxed{2\pi R_0 \text{ or } 2\pi \frac{1}{2R_0}}$$

- At a fixed x_c^0 such that $R_0 = R_c$, there are additional massless scalars φ^I with $n_0 = m_0 = \pm 1$.

  - ✓ They don't even exist at other x^0, since in each phase there are only pure KK or pure winding modes along $S^1(R_0)$.

  - ✓ **They depend on the $(d-1)$ spatial coordinates only**

  - ✓ and parametrize a moduli space

$$\varphi^I \ : \ \mathbb{R}^{d-1} \longrightarrow \mathcal{M} = \frac{SO(2, 12-d)}{SO(2) \times SO(12-d)}$$

✓ Their tree level action is thus

$$\int d^{d-1}\mathbf{x}\, a(x_c^0)^{d-1}\, e^{-2\phi}\left(-G_{I\bar{J}}\,\partial_i\varphi^I\partial^i\bar{\varphi}^{\bar{J}}\right)$$

✓ We look for maps $\varphi^I : \mathbb{R}^{d-1} \longrightarrow \mathcal{M}$, which satisfy the equations of motions for φ^I.

✓ Simple solutions exist for $d = 2,\, 3$. Generic solutions exist for $2 \leq d \leq 6$ at the boundary of $\mathcal{M}$:

$$G_{I\bar{J}}\,\partial_i\varphi^I\partial^i\bar{\varphi}^{\bar{J}} = \kappa^2$$

- In total, there is a **bulk action + brane-like objet,**

$$\int dx^0 d^{d-1}\mathbf{x}\ \beta a^{d-1}\left[e^{-2\phi}\left(\frac{R}{2}+2(\partial\phi)^2\right)+\frac{Z}{\beta V}\right]-\kappa^2\int dx^0 d^{d-1}\mathbf{x}\ a^{d-1}\,e^{-2\phi}\,\delta(x^0-x_c^0)$$

- The analytic continuation $x^0 \to i\ x^0$ is formally identical :

  ✓ κ^2 yields a **negative contribution to the pressure and vanishing energy density,**

$$P_B = -\kappa^2\,\delta(x^0 - x_c^0)\ , \qquad \rho_B = 0$$

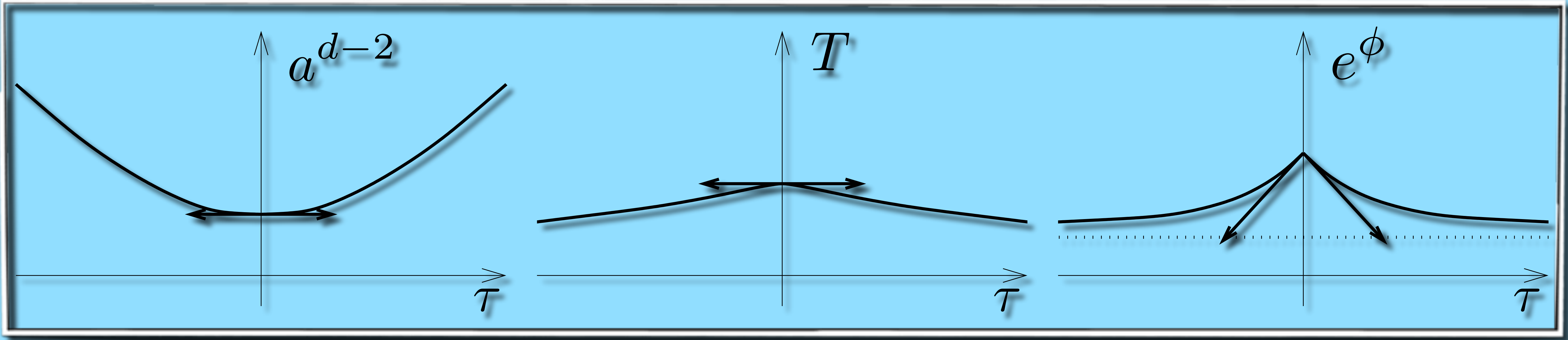
  ✓ This is an **unusual state equation.** It arises **from the phase transition and not from exotic matter.**

- If we approximate $\dfrac{Z}{\beta V} \simeq \dfrac{n\sigma_d}{\beta^d}$ to solve explictly the equations of motion, we find in string frame and conformal time :

$$\ln \frac{a}{a_c} = \ln \frac{T_c}{T} = \frac{1}{d-2} \left[\eta_+ \ln\left(1 + \frac{\omega|\tau|}{\eta_+}\right) - \eta_- \ln\left(1 + \frac{\omega|\tau|}{\eta_-}\right) \right]$$

$$\phi = \phi_c + \frac{\sqrt{d-1}}{2} \left[\ln\left(1 + \frac{\omega|\tau|}{\eta_+}\right) - \ln\left(1 + \frac{\omega|\tau|}{\eta_-}\right) \right]$$

where $\omega = \dfrac{d-2}{\sqrt{2}} \sqrt{n\sigma_d}\, a_c\, T_c^{d/2}\, e^{\phi_c}$, $\quad \eta_\pm = \sqrt{d-1} \pm 1$



- Cosmology which bounces at the phase transition.
- Radiation dominated at late/early times.
- Perturbative if e^{ϕ_c} is chosen small.
- $\partial_\tau \leq \mathcal{O}(e^{\phi_c})$: All the cosmology is a 1-loop effect !

  $\Rightarrow$ Ricci curvature is small.

  Higher derivative terms are negligible.

NB: A similar solution with spatial curvature $k{=}-1$ also exists. It is curvature dominated.

Summary

- String models describe systems, where only the effective number of states « à la Kutasov-Seiberg » are thermalized.

- These models have two thermal phases, where the thermal excitations are either pure KK modes or pure winding modes along $S^1(R_0)$.

- Additional massless states with both windings and momenta trigger the phase transition at the maximal temperature T_c.

- They induce a negative contribution to the pressure,

- which implies a bounce in the dilaton and Ricci curvature in the cosmological evolution, for $d \leq 6$.