New superconformal models in six dimensions

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[HS, E Sezgin, R Wimmer, arXiv:1108.4060] [B de Wit, H Nicolai, HS, arXiv:0801.1294]



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motivation: superconformal models in six dimensions

actions for the dynamics of multiple M5 branes

- single brane: (2,0) chiral tensor multiplet: $\{B_{\mu\nu}, \chi^i, \phi^{ij}\} (dB)^- = 0$
- multiple branes: non-abelian deformation
- o no-go theorems

lessons from M2 branes

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[Bagger, Lambert, Gustavsson '0'7] [Aharony, Bergman, Jafferis, Maldacena, '08]

- non-dynamical fields may be crucial (BLG)
- full supersymmetry may not be manifest (ABJM)

study (1,0) non-abelian superconformal models in six dimensions



- o non-abelian tensor hierarchy
- parameters and constraints
- superconformal field equations
 - 0 (1,0) supersymmetry0 action (?)
- examples
 - gauge groups
 - o vacua and spectra
- conclusions / outlook





- **ield content** $\{A^r_{\mu}, B^I_{\mu\nu}, C_{\mu\nu\rho\,r}\}$
- covariant field strengths (Yang-Mills)

$$\mathcal{F}^r_{\mu\nu} \equiv 2\partial_{[\mu}A^r_{\nu]} - \mathbf{f_{st}}^r A^s_{\mu}A^t_{\nu}$$

non-abelian gauge transformations

$$\delta A^r_{\mu} = D_{\mu} \Lambda^r$$

with structure constants $f_{rs}{}^t$ and gauge generators X_r



b field content
$$\{A^r_{\mu}, B^I_{\mu\nu}, C_{\mu\nu\rho r}\}$$

[B de Wit, HS, '05] [B de Wit, H Nicolai, HS, '08] [J. Hartong, T. Ortin, '09]

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covariant field strengths

$$\mathcal{F}_{\mu\nu}^{r} \equiv 2\partial_{[\mu}A_{\nu]}^{r} - f_{st}^{r}A_{\mu}^{s}A_{\nu}^{t} + h_{I}^{r}B_{\mu\nu}^{I}$$

$$\mathcal{H}_{\mu\nu\rho}^{I} \equiv 3D_{[\mu}B_{\nu\rho]}^{I} + 6d_{rs}^{I}A_{[\mu}^{r}\partial_{\nu}A_{\rho]}^{s} - 2f_{pq}^{s}d_{rs}^{I}A_{[\mu}^{r}A_{\nu}^{p}A_{\rho]}^{q} + g^{Ir}C_{\mu\nu\rho r}$$

non-abelian gauge transformations

$$\begin{split} \delta A^{r}_{\mu} &= D_{\mu}\Lambda^{r} - \boldsymbol{h}^{r}_{I}\Lambda^{I}_{\mu} \\ \Delta B^{I}_{\mu\nu} &= 2D_{[\mu}\Lambda^{I}_{\nu]} - 2\,\boldsymbol{d}^{I}_{rs}\Lambda^{r}\mathcal{F}^{s}_{\mu\nu} - \boldsymbol{g}^{Ir}\Lambda_{\mu\nu\,r} \\ \Delta C_{\mu\nu\rho\,r} &= 3D_{[\mu}\Lambda_{\nu\rho]\,r} + 3\,\boldsymbol{b}_{Irs}\,\mathcal{F}^{s}_{[\mu\nu}\,\Lambda^{I}_{\rho]} + \boldsymbol{b}_{Irs}\,\mathcal{H}^{I}_{\mu\nu\rho}\,\Lambda^{s} + \dots \\ \Delta B^{I}_{\mu\nu} &\equiv \delta B^{I}_{\mu\nu} - 2d^{I}_{rs}\,A^{r}_{[\mu}\,\delta A^{s}_{\nu]} \\ \Delta C_{\mu\nu\rho\,r} &\equiv \delta C_{\mu\nu\rho\,r} - 3\,\boldsymbol{b}_{Irs}\,B^{I}_{[\mu\nu}\,\delta A^{s}_{\rho]} - 2\,\boldsymbol{b}_{Irs}\,d^{I}_{pq}\,A^{s}_{[\mu}\,A^{p}_{\nu}\,\delta A^{q}_{\rho]} \end{split}$$

in terms of constant tensors $d_{rs}^{I}, b_{Irs}, f_{rs}^{t}, g^{Ir}, h_{I}^{r}$ and gauge generators X_{r}

ield content
$$\{A^r_{\mu}, B^I_{\mu\nu}, C_{\mu\nu\rho r}\}$$

covariant field strengths

[B de Wit, HS, '05] [B de Wit, H Nicolai, HS, '08] [J. Hartong, T. Ortin, '09]

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$$\mathcal{F}_{\mu\nu}^{r} \equiv 2\partial_{[\mu}A_{\nu]}^{r} - f_{st}^{r}A_{\mu}^{s}A_{\nu}^{t} + h_{I}^{r}B_{\mu\nu}^{I}$$
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in terms of constant tensors $d_{rs}^{I}, b_{Irs}, f_{rs}^{t}, g^{Ir}, h_{I}^{r}$ and gauge generators X_{r}

$$\mathcal{F}_{\mu\nu}^{r} \equiv 2\partial_{[\mu}A_{\nu]}^{r} - f_{st}^{r}A_{\mu}^{s}A_{\nu}^{t} + h_{I}^{r}B_{\mu\nu}^{I}$$

$$\mathcal{H}_{\mu\nu\rho}^{I} \equiv 3D_{[\mu}B_{\nu\rho]}^{I} + 6 d_{rs}^{I}A_{[\mu}^{r}\partial_{\nu}A_{\rho]}^{s} - 2f_{pq}^{s}d_{rs}^{I}A_{[\mu}^{r}A_{\nu}^{p}A_{\rho]}^{q} + g^{Ir}C_{\mu\nu\rho r}$$

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consistency requires a number of constraints on the parameters :

gauge group generators	$(X_r)_s{}^t = \cdot$	$-f_{rs}{}^t + d_{rs}^I h_I^t$
$D_{\mu} = \partial_{\mu} - A^r_{\mu} X_r$	$(X_r)_I{}^J = 2$	$h_I^s d_{rs}^J - g^{Js} b_{Isr}$

close into the algebra $[X_r, X_s] = (X_r)_s^{t} X_t$

- charged tensor fields require Stückelberg-type coupling
- generalized Bianchi identities $D\mathcal{F}^r = h_I^r \mathcal{H}^I$, etc.
- continues to 4-forms, 5-forms, ...

$$\mathcal{F}_{\mu\nu}^{r} \equiv 2\partial_{[\mu}A_{\nu]}^{r} - f_{st}^{r}A_{\mu}^{s}A_{\nu}^{t} + h_{I}^{r}B_{\mu\nu}^{I}$$

$$\mathcal{H}_{\mu\nu\rho}^{I} \equiv 3D_{[\mu}B_{\nu\rho]}^{I} + 6 d_{rs}^{I}A_{[\mu}^{r}\partial_{\nu}A_{\rho]}^{s} - 2f_{pq}^{s}d_{rs}^{I}A_{[\mu}^{r}A_{\nu}^{p}A_{\rho]}^{q} + g^{Ir}C_{\mu\nu\rho r}$$

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consistency requires a number of constraints on the parameters :

$$2 \left(d_{r(u}^{J} d_{v)s}^{I} - d_{rs}^{I} d_{uv}^{J} \right) h_{J}^{s} = 2 f_{r(u}{}^{s} d_{v)s}^{I} - b_{Jsr} d_{uv}^{J} g^{Is} ,$$

$$\left(d_{rs}^{J} b_{Iut} + d_{rt}^{J} b_{Isu} + 2 d_{ru}^{K} b_{Kst} \delta_{I}^{J} \right) h_{J}^{u} = f_{rs}{}^{u} b_{Iut} + f_{rt}{}^{u} b_{Isu} + g^{Ju} b_{Iur} b_{Jst}$$

$$f_{[pq}{}^{u} f_{r]u}{}^{s} - \frac{1}{3} h_{I}^{s} d_{u}^{I} f_{qr]}{}^{u} = 0$$

$$h_{I}^{r} g^{Is} = 0$$

$$f_{rs}{}^{t} h_{I}^{r} - d_{rs}^{J} h_{J}^{t} h_{I}^{r} = 0$$

$$g^{Js} h_{K}^{r} b_{Isr} - 2 h_{I}^{s} h_{K}^{r} d_{rs}^{J} = 0$$

$$- f_{rt}{}^{s} g^{It} + d_{rt}^{J} h_{J}^{s} g^{It} - g^{It} g^{Js} b_{Jtr} = 0$$

for the constant tensors $d_{rs}^{I}, b_{Irs}, f_{rs}^{t}, g^{Ir}, h_{I}^{r}$

$$\mathcal{F}_{\mu\nu}^{r} \equiv 2\partial_{[\mu}A_{\nu]}^{r} - f_{st}^{r}A_{\mu}^{s}A_{\nu}^{t} + h_{I}^{r}B_{\mu\nu}^{I}$$

$$\mathcal{H}_{\mu\nu\rho}^{I} \equiv 3D_{[\mu}B_{\nu\rho]}^{I} + 6 d_{rs}^{I}A_{[\mu}^{r}\partial_{\nu}A_{\rho]}^{s} - 2f_{pq}^{s}d_{rs}^{I}A_{[\mu}^{r}A_{\nu}^{p}A_{\rho]}^{q} + g^{Ir}C_{\mu\nu\rho r}$$

consistency requires a number of constraints on the parameters :

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too many constraints ? are there solutions ?

$$\mathcal{F}_{\mu\nu}^{r} \equiv 2\partial_{[\mu}A_{\nu]}^{r} - f_{st}^{r}A_{\mu}^{s}A_{\nu}^{t} + h_{I}^{r}B_{\mu\nu}^{I}$$

$$\mathcal{H}_{\mu\nu\rho}^{I} \equiv 3D_{[\mu}B_{\nu\rho]}^{I} + 6 d_{rs}^{I}A_{[\mu}^{r}\partial_{\nu}A_{\rho]}^{s} - 2f_{pq}^{s}d_{rs}^{I}A_{[\mu}^{r}A_{\nu}^{p}A_{\rho]}^{q} + g^{Ir}C_{\mu\nu\rho r}$$

some examples (which satisfy all constraints)

o abelian limit

 $f_{rs}{}^t = 0, \ q^{Ir} = 0, \ h_I^r = 0$ free theory with nontrivial Chern-Simons term

• Yang-Mills with neutral tensor fields

 $q^{Ir} = 0, \ h_I^r = 0, \ d_{rs}^I = d^I \eta_{rs}$ no Stückelberg-type couplings

[E. Bergshoeff, E. Sezgin, E. Sokatchev, '96]

Yang-Mills with adjoint tensor fields

$$h_s^r = 0, \ g^{rs} = \eta^{rs}, \ b_{t\,rs} = f_{rst}$$

coupling of three-forms, charged tensors



superconformal symmetry



(1,0) supermultiplets

vector
$$\{A^r_{\mu}, \lambda^r_i, Y^r_{ij}\}$$
tensor $\{B^I_{\mu\nu}, \chi^I_i, \phi^I\}$ three-form $\{C_{\mu\nu\rho\,r}\}$ off-shellon-shell??



(1,0) supermultiplets

vector $\{A_{\mu}^{r}, \lambda_{i}^{r}, Y_{ij}^{r}\}$ tensor $\{B_{\mu\nu}^{I}, \chi_{i}^{I}, \phi^{I}\}$ three-form $\{C_{\mu\nu\rho\,r}\}$ • closure of the supersymmetry algebra on the tensor multiplet implies $\mathcal{H}_{\mu\nu\rho}^{I-} = -d_{rs}^{I}\bar{\lambda}^{r}\gamma_{\mu\nu\rho}\lambda^{s}$ $\gamma^{\sigma}D_{\sigma}\chi^{iI} = \frac{1}{2}d_{rs}^{I}\mathcal{F}_{\sigma\tau}^{r}\gamma^{\sigma\tau}\lambda^{is} + 2d_{rs}^{I}Y^{ij\,r}\lambda_{j}^{s} + (d_{rs}^{I}h_{J}^{s} - 2b_{Jsr}g^{Is})\phi^{J}\lambda^{ir}$ $D^{\mu}D_{\mu}\phi^{I} = -\frac{1}{2}d_{rs}^{I}(\mathcal{F}_{\mu\nu}^{r}\mathcal{F}^{\mu\nu\,s} - 4Y_{ij}^{r}Y^{ij\,s} + 8\bar{\lambda}^{r}\gamma^{\mu}D_{\mu}\lambda^{s})$ $-2(b_{Jsr}g^{Is} - 8d_{rs}^{I}h_{J}^{s})\bar{\lambda}^{r}\chi^{J} - 3d_{rs}^{I}h_{J}^{r}h_{K}^{s}\phi^{J}\phi^{K}$

tensor multiplet is onshell: Yukawa couplings, (cubic) scalar potential



(1,0) supermultiplets

vector $\{A^r_{\mu}, \lambda^r_i, Y^r_{ij}\}$ tensor $\{B^I_{\mu\nu}, \chi^I_i, \phi^I\}$ three-form $\{C_{\mu\nu\rho\,r}\}$

• closure of the supersymmetry algebra on the tensor multiplet implies

$$\mathcal{H}_{\mu\nu\rho}^{I-} = -d_{rs}^{I}\bar{\lambda}^{r}\gamma_{\mu\nu\rho}\lambda^{s}$$

$$\gamma^{\sigma}D_{\sigma}\chi^{iI} = \frac{1}{2}d_{rs}^{I}\mathcal{F}_{\sigma\tau}^{r}\gamma^{\sigma\tau}\lambda^{is} + 2d_{rs}^{I}Y^{ijr}\lambda_{j}^{s} + \left(d_{rs}^{I}h_{J}^{s} - 2b_{Jsr}g^{Is}\right)\phi^{J}\lambda^{is}$$

$$D^{\mu}D_{\mu}\phi^{I} = -\frac{1}{2}d_{rs}^{I}\left(\mathcal{F}_{\mu\nu}^{r}\mathcal{F}^{\mu\nu s} - 4Y_{ij}^{r}Y^{ijs} + 8\bar{\lambda}^{r}\gamma^{\mu}D_{\mu}\lambda^{s}\right)$$

$$-2\left(b_{Jsr}g^{Is} - 8d_{rs}^{I}h_{J}^{s}\right)\bar{\lambda}^{r}\chi^{J} - 3d_{rs}^{I}h_{J}^{r}h_{K}^{s}\phi^{J}\phi^{K}$$

tensor multiplet is onshell: Yukawa couplings, (cubic) scalar potential

• supersymmetry of these equations implies

$$b_{Irs} \left(Y_{ij}^{s} \phi^{I} - 2\bar{\lambda}_{(i}^{s} \chi_{j)}^{I} \right) = 0$$

$$b_{Irs} \left(\mathcal{F}_{\mu\nu}^{s} \phi^{I} - 2\bar{\lambda}^{s} \gamma_{\mu\nu} \chi^{I} \right) = \frac{1}{4!} \varepsilon_{\mu\nu\lambda\rho\sigma\tau} \mathcal{H}_{r}^{(4)\,\lambda\rho\sigma\tau}$$

vector multiplet also onshell: three-forms are dual to vectors

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can these equations be lifted to an action ?

yes! provided that there is a constant metric η_{IJ} and the parameters are related as $h_I^r = \eta_{IJ}g^{Jr}$, $2 d_{rs}^I = \eta^{IJ}b_{Jrs}$

$$\mathcal{L} = -\frac{1}{8}D^{\mu}\phi_{I} D_{\mu}\phi^{I} - \frac{1}{2}\bar{\chi}_{I}\gamma^{\mu}D_{\mu}\chi^{I} + \frac{1}{16}b_{Irs}\phi^{I}\left(\mathcal{F}_{\mu\nu}^{r}\mathcal{F}^{\mu\nu\,s} - 4Y_{ij}^{r}Y^{ij\,s} + 8\bar{\lambda}^{r}\gamma^{\mu}D_{\mu}\lambda^{s}\right) - \frac{1}{96}\mathcal{H}_{\mu\nu\rho}^{I}\mathcal{H}_{I}^{\mu\nu\rho} - \frac{1}{48}b_{Irs}\mathcal{H}_{\mu\nu\rho}^{I}\bar{\lambda}^{r}\gamma^{\mu\nu\rho}\lambda^{s} - \frac{1}{4}b_{Irs}\mathcal{F}_{\mu\nu}^{r}\bar{\lambda}^{s}\gamma^{\mu\nu}\chi^{I} + b_{Irs}Y_{ij}^{r}\bar{\lambda}^{i\,s}\chi^{j\,I} + \frac{1}{2}\left(b_{Jsr}g_{I}^{s} - 4b_{Isr}g_{J}^{s}\right)\phi^{I}\bar{\lambda}^{r}\chi^{J} + \frac{1}{8}b_{Irs}g_{J}^{r}g_{K}^{s}\phi^{I}\phi^{J}\phi^{K} - \frac{1}{48}\mathcal{L}_{top} - \frac{1}{24}b_{Irs}b_{uv}^{I}\bar{\lambda}^{r}\gamma^{\mu}\lambda^{u}\bar{\lambda}^{s}\gamma_{\mu}\lambda^{v},$$

Yukawa couplings, cubic scalar potential, topological term

$$\int_{\partial M_7} \mathcal{L}_{\text{top}} \propto \int_{M_7} \left(b_{Irs} \, \mathcal{F}^r \wedge \mathcal{F}^s \wedge \mathcal{H}^I - \mathcal{H}^I \wedge D \mathcal{H}_I \right)$$

duality has to be imposed by hand ("democratic") - or HT/PST style no dimensionfull constants, classically superconformal

indefinite metrics (ghosts):
$$g^{Ir}\eta_{IJ}g^{Js}~\equiv~0~K_{rs}~\equiv~b_{I\,rs}\,\phi^{I}$$

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b models with G adjoint tensor multiplets $\{B_{\mu\nu}^r\}$

• vectors in G $\{A^r_\mu\}$

$$h_s^r = 0, \ g^{rs} = \eta^{rs}, \ b_{t\,rs} = f_{rst}$$

YM field strength, Stückelberg-type coupling of tensors to three-forms, potential vanishes, no action

O vectors in G x G
$$\left\{A_{\mu}^{r}, A_{\mu}^{r'}\right\}$$
 [Chu, arXiv:1108.513]
 $f_{rs}{}^{t} = f_{rs}{}^{t}, \quad f_{rs'}{}^{t'} = -f_{s'r}{}^{t'} = \frac{1}{2}f_{rs'}{}^{t'}$
 $d_{rs'}^{t} = d_{s'r}^{t} = -\frac{1}{2}f_{rs'}{}^{t}, \quad h_{s'}^{r'} = \delta_{s'}^{r'}$
 $g^{rs} = 0 = g^{rs'}$

no three-forms, Stückelberg-type coupling of vectors to tensors, potential vanishes, no action



models with an action

16 vectors, 10 self-dual tensors, based on magic gamma identities of SO(10)

• SO(5) gauge group $SO(5) \subset GL(5) \subset SO(5,5)$

$$b_{rs}^{I} \equiv \gamma_{rs}^{I}, \quad f_{rs}^{t} \equiv -4 \gamma_{rs}^{IJK} \gamma_{IJp}^{t} g_{K}^{p} \qquad g_{K}^{p} \propto 1$$

tensors: two copies of fundamental representation indefinite metric, potential vanishes

• T₈ gauge group $T_8 \subset (SO(8) \times SO(1,1)) \ltimes T_8 \subset SO(9,1)$ $g^{Ir} \equiv \zeta^r \zeta^s \zeta^t \gamma^I_{st}$

nilpotent gauge group indefinite metric, cubic potential $\mathcal{L} = -\frac{1}{8}D^{\mu}\phi^{i}D_{\mu}\phi^{i} - \frac{1}{8}\partial^{\mu}\phi^{+}D_{\mu}\phi^{-} + g^{3}(\phi^{+})^{3}$



maximally supersymmetric vacuum

 $0 \stackrel{!}{\equiv} \delta \lambda^{ir} = \frac{1}{8} \gamma^{\mu\nu} \mathcal{F}^{r}_{\mu\nu} \epsilon^{i} - \frac{1}{2} Y^{ijr} \epsilon_{j} + \frac{1}{4} h^{r}_{I} \phi^{I} \epsilon^{i}$

constant scalar fields with $\phi^I_0 \, h^r_I ~\equiv~ 0$

fluctuations around the vacuum

fall into multiplets

(V) :
$$\Box A_{\mu} = 0$$
, $\partial \lambda = 0$,

$$(\mathbf{T}) \quad : \quad \Box \varphi = 0 \,, \quad \partial \hspace{-0.15cm} \hspace{0.15cm} \chi = 0 \,, \quad (dB)^- = 0 \,,$$

free vector / tensor multiplets

(TV) :
$$\Box \varphi = 0$$
, $\kappa dA = {}^*\!dC$, $(dB)^- = -gC^-$, $\partial \lambda = 0$, $\partial \chi = -2g\kappa\lambda$,

 $(VT) : \Box \varphi = 0, \quad \Box A_{\mu} = 0, \quad (dB)^{-} = 0, \quad \partial \chi = 0, \quad \partial \lambda = 2g \, \chi \, .$

non-decomposable vector-tensor multiplets indefinite metric, higher-derivative theory?



conclusions / outlook

non-abelian (1,0) superconformal models in six dimensions

- coupling to three-form gauge potentials
- \triangleright equations of motion \longrightarrow action

consistency constraints and solutions

- b models with G adjoint tensor fields (no action)
- b compact gauge group SO(5) with action (distinguished?)
- vacua and spectra
- understand their structure / quantization
 - b ghosts: gauge fixing, imposing further constraints ...
 - b cubic potential
- classification: solutions to the consistency constraints
 - > Jacobi identities, fundamental identities,
- extension to (2,0) theories, relation to D=5 SYM
 - include hypermultiplets, non-propagating vector multiplets