# Green-Schwarz Mechanism in Heterotic (2,0) Gauged Linear Sigma Models: Torsion and NS5 Branes

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Corfu, 2011

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Fortsch.Phys. 59 (2011) 454-493 [arXiv:1012.3350]

JHEP 1108 (2011) 083 [arXiv:1107.0320]

# Heterotic String Phenomenology

Overview:

- Approaches to String Phenomenology
- Model Building on Orbifolds
- Model Building on their Resolutions

# Objective

One of the aims of String Phenomenology is to find the Standard Model of Particle Physics from String constructions:

The $E_8 \times E_8$ Heterotic Strings incorporates features of GUT theories and can lead to the Supersymmetric Standard Models (MSSMs).

Two approaches are often considered to achieve this goal:

- *singular* Orbifold constructions Dixon,Harvey,Vafa,Witten'85
- smooth Calabi–Yau compactifications with gauge bundles Candelas,Horowitz,Strominger,Witten'85

have lead to MSSM models Bouchard,Donagi'05

# Heterotic Strings on Orbifolds

Orbifolds: are flat spaces except for the *singular* fixed points.

$T^2/\mathbb{Z}_3$ $T^2/\mathbb{Z}_2$

Heterotic strings on orbifolds are free CFTs: Ibanez,Mas,Nilles,Quevedo'88

On $T^6/\mathbb{Z}_{6-\text{II}}$ give rise to a large pool ($\approx 200$) of possible MSSMs.

Buchmuller,Hamaguchi,Lebedev,Ratz'04, Lebedev,Nilles,Raby,Ramos-Sanchez,Ratz, Vaudrevange,Wingerter'07

# Model Building on Resolutions

We have constructed MSSM-like models on resolutions of $T^6/\mathbb{Z}_{6\text{-II}}$ using methods of toric geometry: Lüst,Reffert,Scheidegger,Stieberger'06

- In full resolution the hyper charge is always broken, because in blow-up it is broken by a flux. SGN,Held,Rühle,Trapletti'09

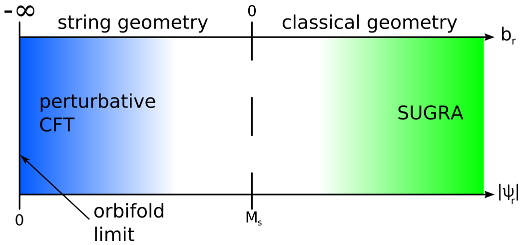
To avoid this we have constructed $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold models: Blaszczyk,SGN,Ratz,Rühle,Trapletti,Vaudrevange'10

- The GUT symmetry is broken by a freely acting Wilson line.
- Such models can in principle be completely blown up.

# Orbifold / Resolution Model Matching

Matching of orbifold and resolution models is hampered at least by the fact that we are comparing different regions in the moduli space:

Aspinwall,Greene,Morrison'93




**We need an approach that is able to interpolate between both regimes.**

**Gauge Linear Sigma Model (GLSM) is such a framework...**

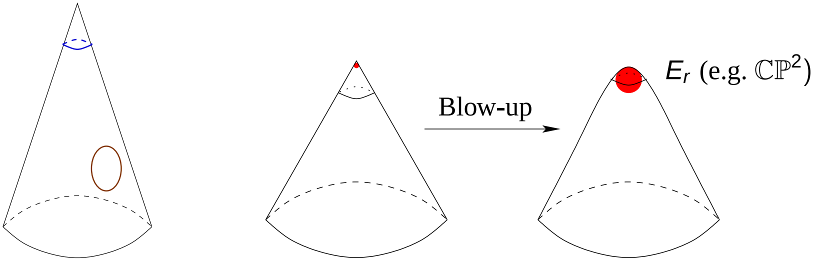
# Heterotic Resolution Gauged Linear Sigma Models

Overview:

- Twisted States on Orbifolds
- Characterization of a GLSM

# Basic Picture of a Resolution

An orbifold theory contains so-called twisted states $|T_r\rangle$.




A non-vanish twisted state Vacuum Expectation Values (VEV) $\langle T_r \rangle \neq 0$

$$\Downarrow$$

The orbifold singularity gets resolved, i.e. blown-up:

- An exceptional cycle $E_r$ becomes large $\text{Vol}(E_r) \sim b_r \gg 0$.

# Non–Compact Heterotic Orbifolds

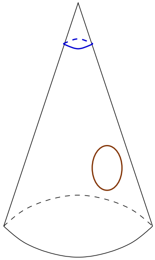
The input data for a heterotic $\mathbb{C}^3/\mathbb{Z}_N$ orbifold model is quite limited:

- the geometrical orbifold twist $v = (v^1, v^2, v^3)$:

$$\theta(z^1, z^2, z^3) = \left(e^{2\pi i v^1} z^1, e^{2\pi i v^2} z^2, e^{2\pi i v^3} z^3\right)$$

- the gauge shift emdedding $V = (V^1, \ldots, V^{16})$:

$$\theta(\lambda^1, \ldots, \lambda^{16}) = \left(e^{2\pi i V^1} \lambda^1, \ldots, e^{2\pi i V^{16}} \lambda^{16}\right)$$



The entries of $v^a = n^a/N$ and $V^\alpha = N^\alpha/N$ are quantized in units of $1/N$, i.e. $n^a, N^\alpha \in \mathbb{Z}$.

# Twisted States on Orbifolds

A non–oscillator $r$–twisted states $|T_r\rangle = |p_r, P_r\rangle$is determined by
Dixon,Harvey,Vafa,Witten'85, Ibanez,Nilles,Quevedo'87

1. a right-moving shifted momentum:

$$p_r = p + rv\,, \quad p \in SO(8) \text{ vector weight lattice;}$$

2. a left-moving shifted momentum:

$$P_r = P + rV\,, \quad P \in SO(32) \text{ root lattice.}$$

They fulfill the level matching condition:

$$\boldsymbol{P_r^2 = 1 + p_r^2}$$

# GLSM Multiplets: Chiral & chiral-Fermi Superfields

The heterotic string on complex manifolds has at least (2,0) worldsheet supersymmetry:

| superfield type | superfield symbol | dim | charge | bos. DOF on | bos. DOF off | ferm. DOF on | ferm. DOF off |
|---|---|---|---|---|---|---|---|
| chiral | $\Psi^a$ | 0 | $q_I^a$ | $z^a$ | - | $\psi^a$ | - |
| chiral-Fermi | $\Lambda^\alpha$ | 1/2 | $Q_I^\alpha$ | - | $h^\alpha$ | $\lambda^\alpha$ | - |

- $z^a$, $a = 0, \ldots, 3$ are the complex target space coordinates,
- $\psi^a$, $a = 0, \ldots, 3$ are their right–moving superpartners,
- $\lambda^\alpha$, $\alpha = 1, \ldots, 16$ are the left–moving fermions that generate the target space gauge degrees of freedom (DOF)

# Bosonic Gaugings & Fayet-Iliopoulos Terms

| superfield type | superfield symbol | dim | charge | bos. DOF on | bos. DOF off | ferm. DOF on | ferm. DOF off |
|---|---|---|---|---|---|---|---|
| gauge | $(V, A)^I$ | (0,1) | 0 | $A^I_\sigma, A^I_{\bar\sigma}$ | $\widetilde{D}^I$ | $\phi^I$ | - |

The action of the gauge multiplets contains Witten'93,Distler,Kachru'93

$$S_{\text{FI}} = \int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \, \rho_I(\Psi)\, F_I + \text{h.c.}$$

is expressed in terms of

- the gauge superfield strengths $F_I = -\frac{1}{2}\overline{D}_+\big(A - i\bar\partial V\big)_I$
- and Fayet-Ilopoulos parameters $\rho_I$

# Bosonic Gaugings & Fayet-Iliopoulos Terms

When we eliminate the auxiliary fields $\widetilde{D}_I$ we end up with the worldsheet potential

$$V_{FI} = \sum_I \frac{e_I^2}{2} \Big( \sum_a q_I^a \, |z^a|^2 - b_I \Big)^2 \,,$$

where the lowest components $\rho_I| = b_I + i\beta_I$ are the Kähler parameters $b_I$ and axions $\beta_I$.

In order to preserve (2,0) worldsheet supersymmetry, this potential needs to vanish. Witten'93

# Some Consistency Requirements on a GLSM

No pure or mixed gauge anomalies:

$$\sum_\alpha Q_I^\alpha Q_J^\alpha = \sum_a q_I^a q_J^a \qquad \rightsquigarrow \qquad c_2(\mathbb{V}) = c_2(TX)$$

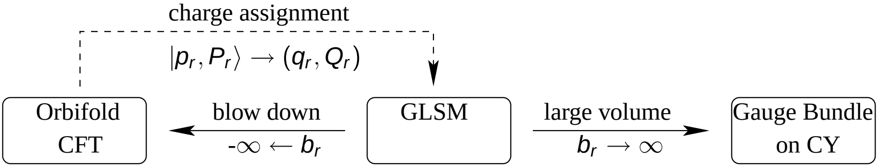
No divergent FI-terms on the worldsheet:

(Otherwise the FI–parameters flow to $\pm\infty$ in the IR)

$$\sum_a q_I^a = 0 \qquad \rightsquigarrow \qquad c_1(TX) = 0$$

Distler'93

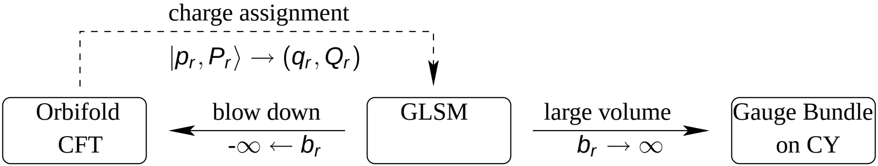
# From a Twisted State VEV to a GLSM




This charge assignment has to be such that:

- the number of DOF is as in the free theory
- the sum of charges zero

# From a Twisted State VEV to a GLSM




This charge assignment has to be such that:

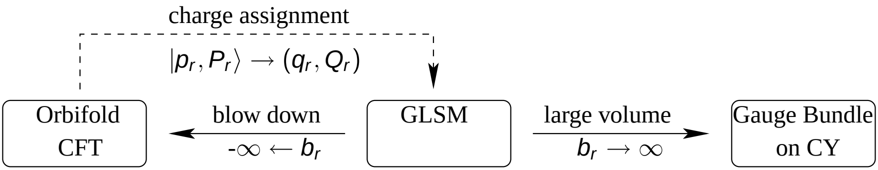
- the number of DOF is as in the free theory
- the sum of charges zero

The left-moving momenta define the charges of $\Psi^a$: SGN'10

$$(q_r)^a = (p_r)^a = \frac{n_r^a}{N} \qquad \sum_a n_r^a = N$$

And we introduce a new chiral superfield $\Psi^{\text{-}r}$ with charge $(q_r)^{\text{-}r} = -1$.

# Non-Oscillatory Blow-Up Modes




We set the charges $Q_r^\alpha$ of the chiral-Fermi superfields $\Lambda^\alpha$: SGN'10

$$Q_r^\alpha = P_r^\alpha$$

The level matching condition ensures pure anomaly cancellation:

$$Q_r^2 = P_r^2 = 1 + p_r^2 = q_r^2$$

for twisted states without oscillator excitations ($\widetilde{N}_r = 0$).

# Anomalies on Resolutions of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$

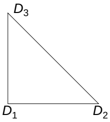
Overview:

- Phases of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold Resolutions
- Bianchi Identities on Different Triangulations
- Target Interpretation of GLSM Anomalies

# Phases of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold Resolutions

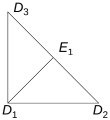
**orbifold phase:**
no exceptional divisors

$$b_1 \leq 0 \qquad b_2 \leq 0 \qquad b_3 \leq 0$$



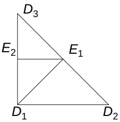

**partial resolution:**
one exceptional divisor

$$b_1 \geq 0 \qquad b_2 \leq 0 \qquad b_3 \leq 0$$

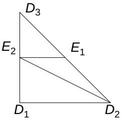



**partial resolution:**
two exceptional divisors

$$b_1 \geq b_2 \geq 0 \qquad b_3 \leq 0$$

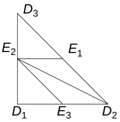



$$b_2 \geq b_1 \geq 0 \qquad b_3 \leq 0$$

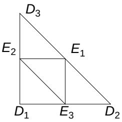



**full resolution:** three exceptional divisors

$$b_1,\ b_3 \geq 0 \qquad b_2 \geq b_1 + b_3$$




$$b_1 + b_2 \geq b_3 \geq 0 \qquad b_1 + b_3 \geq b_2 \geq 0 \qquad b_2 + b_3 \geq b_1 \geq 0$$



# Bianchi Identities on Different Triangulations

The Bianchi identities on triangulation "$E_1$" read: SGN,Ha,Trapletti'08

$$Q_2^2 + Q_3^2 = 3\,, \qquad Q_2^2 - 2\,Q_1 \cdot Q_3 = 1\,, \qquad Q_3^2 - 2\,Q_1 \cdot Q_2 = 1$$

and on triangulation "$S$":

$$Q_1^2 + 2\,Q_2 \cdot Q_3 = 2\,, \qquad Q_2^2 + 2\,Q_1 \cdot Q_3 = 2\,, \qquad Q_3^2 + 2\,Q_1 \cdot Q_2 = 2$$

The pure and mixed GLSM anomaly cancellations require: SGN'10

$$Q_1^2 = Q_2^2 = Q_3^2 = \frac{3}{2}\,, \qquad Q_1 \cdot Q_2 = Q_2 \cdot Q_3 = Q_3 \cdot Q_1 = \frac{1}{4}$$

Hence the GLSM anomaly conditions ensure that the Bianchi identities in any triangulation are fulfilled.

# GLSM Phases and Bianchi Identities

The GLSM anomaly cancellation conditions always seem to ensures that the integrated Bianchi identities are fulfilled:

- When going from one smooth phase to the next, we might go through a flop where supergravity cannot be trusted. But in both smooth phases the corresponding integrated Bianchi identities need to be satisfied.
- The whole process should be describable in the GLSM, and therefore the GLSM anomaly conditions should imply the Bianchi identities in any smooth geometry that can ever arise.

**What to do when the GLSM anomalies don't cancel?**

# Green–Schwarz Mechanism on the Worldsheet

Overview:

- Field dependent Fayet–Iliopoulos Terms
- Possible Interpretation: Torsion and NS5–Branes
- Consequences for the geometry

# Non–invariant Fayet–Iliopoulos terms

Remember, that the Fayet–Iliopoulos term on the worldsheet read:

$$W_{\text{FI}} = \frac{1}{2\pi}\,\rho_J(\Psi)\,F^J\ .$$

If under gauge transformations the FI parameters transform as

$$\rho_J \to \rho_J + T_{IJ}\,\Theta^I\ ,$$

we can generalize the anomaly conditions to

Blaszczyk,SGN,Rühle'11, Quigley,Sethi'11

Pure anomalies: $$T_{II} = \tfrac{1}{2}\Big(Q_I\cdot Q_I - q_I\cdot q_I\Big)\ ,\quad I = J\ ,$$

Mixed anomalies: $$T_{IJ} = (1-c_{IJ})\,(Q_I\cdot Q_J - q_I\cdot q_J)\ ,\quad I < J\ ,$$

$$T_{JI} = c_{IJ}\,(Q_I\cdot Q_J - q_I\cdot q_J)\ ,\quad I > J\ .$$

This like a Green–Schwarz mechanism on the worldsheet.

# Field dependent FI–terms

Field dependent FI–terms can be interpreted as torsion:
Adams,Ernebjerg,Lapan'06

Given that the axions

$$\beta_I(z) = \mathrm{Im}(\rho_I(z))$$

transform with shifts the three–form field strength $H_3$ of $B_2$ takes the form:

$$H_3 = \big(\mathrm{d}\beta_J + T_{IJ}\, a_1^I\big) f_2^J \ ,$$

where $a_1^I$ is the gauge field on the worldsheet.

In target space one has

$$H_3 = \mathrm{d}B_2 + \mathrm{CS}_3(\omega, \mathcal{R}) - \mathrm{CS}_3(\mathcal{A}, \mathcal{F})$$

non–vanishing, i.e. the compactification space is equipped with torsion: $H_3 = i(\bar\partial - \partial)J \neq 0$.

# Singular Fayet–Iliopoulos term

Let $\Psi$ a charged chiral superfield then

$$\Psi^a \to e^{q_I^a \Theta_I}\, \Psi^a \quad \Rightarrow \quad \ln \Psi^a \to \ln \Psi^a + q_I^a\, \Theta_I$$

Hence the logarithmic FI-term

$$W_{\text{log FI}} = \frac{1}{2\pi} \Big( \rho_J + T_{aJ} \ln \Psi^a \Big) F^J \ ,$$

can be used to cancel anomalies with $T_{IJ} = q_I^a\, T_{aJ}$.

Blaszczyk,SGN,Rühle'11, Quigley,Sethi'11

# Quantized log-FI terms

The quantities $T_{IJ}$ are subject to quantization conditions due to gauge instantons. Adams,Basu,Sethi'03

Flux quantization: $$(q_J)^b \int \frac{f_2^J}{2\pi} \in \mathbb{Z} \,.$$

Invariance under $z^a \sim z^a\, e^{2\pi i}$ of the action Blaszczyk,SGN,Rühle'11

$$\int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \, W_{\text{log FI}} + \text{h.c.} \supset i\, T_{aJ} \int \text{Im}(\ln z^a)\, \frac{f_2^J}{2\pi} \quad \Rightarrow$$

Anomaly coefficient quantization: $$T_{aJ} \int \frac{f_2^J}{2\pi} \in \mathbb{Z} \,.$$

# Interpretation of singular FI-terms

When the worldsheet FI–term become singular, a drastic modification of the target space geometry arises:

$$H_3 = \left(\mathrm{d}\beta_J + q_I^a T_{aJ}\, a_1^I\right) f_2^J$$

gives upon using the worldsheet anomaly cancellation conditions

$$\mathrm{d}H_3 = X_4 + \mathrm{tr}\mathcal{F}_2^2 - \mathrm{tr}\mathcal{R}_2^2 \ ,$$

as $X_4 = \mathrm{d}(\mathrm{d}\beta_J) f_2^J \neq 0$. Blaszczyk,SGN,Rühle'11

$$\text{E.g.:} \quad \beta_J = T_{aJ} \ln\left(\frac{z^a}{\bar{z}_a}\right) \quad \Rightarrow \quad \mathrm{d}(\mathrm{d}\beta_J) = 2\pi\, T_{aJ}\, \delta^2(z^a)\, \mathrm{d}\bar{z}_a \mathrm{d}z^a \ .$$

When $X_4$ defines an effective class, we may interpret this as an NS5–brane wrapping the corresponding curve.

# Backreaction on the geometry

Naively the geometry is given by the scalar potential:

$$V_{\text{D naive}} = \frac{e^2}{2} \Big( q^a \, |z^a|^2 + q \, |z|^2 - b \Big)^2 \,,$$

Suppose that all $q^a$ and $q$ are positive, then the naive potential shows that the target space geometry is compact.

The presence of the logarithmic FI–terms leads to the modification

Blaszczyk,SGN,Rühle'11, Quigley,Sethi'11

$$V_{\text{D}} = \frac{e^2}{2} \Big( q^a \, |z^a|^2 + q \, |z|^2 - b - T \, \ln |z|^2 \Big)^2 \,.$$

This has various consequences:

- The divisor $D = \{z = 0\}$ no longer exists,
- There are no instantons that wrap around $z = 0$, hence $z$ does not leads to a quantization condition any.

# Case $T < 0$: Gauge bundle subdominant

The modified D–term leads to the constraint:

$$\sum_a q^a \, |z^a|^2 = b + T \, \ln |z|^2 - q \, |z|^2 \, .$$

For $T < 0$ we may write:

$$\sum_a q^a \, |z^a|^2 = b - |T| \, \ln |z|^2 - q \, |z|^2 \, .$$

We can make the right–hand–side arbitrary large by taking $z$ very close to zero (no matter what the value of $b$ is).

In this case the target space geometry is no longer compact!

Hence we interpret the case $T < 0$ as having anti–NS5-branes.

# Conclusions

1. We associated a specific GLSM to a non–compact heterotic orbifold model with VEVs of twisted states:
   - The charges of the GLSM superfields are determined by the shifted momenta of the twisted non-oscillatory states.
2. The GLSM anomaly cancellation conditions are much stronger than the integrated Bianchi identities:
   - They seem to ensure that the Bianchi identities are satisfied in all possible phases (triangulations) simultaneously.
   - The mixed anomaly conditions is very constraining.
3. Field dependent FI–terms lead to a GS–mechanism:
   - Singular (logarithmic) FI–terms may be interpreted as NS5–branes,
   - and can substantially modify the target space geometry.