11th Hellenic School and Workshops on Elementary Particle Physics and Gravity

1st School of ITN Unification in the LHC Era

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Flavour Physics

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New physics: effective lagrangian approach

$$L = L_{SM} + \sum_i c_i^5 \frac{O_i^5}{\Lambda} + \sum_i c_i^6 \frac{O_i^6}{\Lambda^2} + \ldots$$

O^d_i gauge invariant operators of dimension d

here: constraints from flavour physics on $|\Delta F|=2$ operators

FLAVOUR PROBLEM

Operator	Bounds on Λ in TeV ($c_{ij}=1$)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

TABLE I: Bounds on representative dimension-six $\Delta F = 2$ operators. Bounds on Λ are quoted assuming an effective coupling $1/\Lambda^2$, or, alternatively, the bounds on the respective c_{ij}'s assuming $\Lambda = 1$ TeV. Observables related to CPV are separated from the CP conserving ones with semicolons. In the B_s system we only quote a bound on the modulo of the NP amplitude derived from Δm_{B_s} (see text). For the definition of the CPV observables in the D system see Ref. [15].

[Isidori, Nir, Perez, 2010]

Minimal Flavour Violation

- either the scale of new physics is very large or flavour violation from New Physics is highly non-generic. Useful benchmark: a framework where the only source of flavour violation beyond the SM are the Yukawa coupling

- The Yukawa couplings Y_u and Y_d of the quark sector are promoted to non-dynamical fields (spurions) in such a way that the SM lagrangian is formally invariant under the flavour group G_q

$$G_q = SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{q_L} \qquad y_u = (3,1,\bar{3})$$

$$q_L = (1,1,3) \quad u_R = (3,1,1) \quad d_R = (1,3,1) \qquad y_d = (1,3,\bar{3})$$

- MFV assumes that new operators coming from New Physics do not involve any additional field/spurions and that they are still invariant under G_q
[additional assumption: no additional sources of CPV other than those in $y_{u,d}$]

- in this way operators that contribute to FC automatically carry some suppression from the small y_u/ y_d and V_{CKM} and one can hope to lower the allowed scale of New Physics.

Exercise: build the leading operator with $\Delta F=2$ in MFV

choose, e.g. the basis where

$$y_d = y_d^{Diag} \qquad y_u = y_u^{Diag} V_{CKM} \qquad y_{u,d}^{Diag} \text{ diagonal}$$

we can form the MFV invariant

$$\bar{q}_{Li}\gamma^{\mu}(y_u^{+}y_u)_{ij}q_{Lj}\,\bar{q}_{Lk}\gamma_{\mu}(y_u^{+}y_u)_{kl}q_{Ll}$$

looking at the down quark sector and selecting i=k=d,s and j=l=b
we get the MFV operator contributing to $\Delta B=2$

$$O_{MFV}(|\Delta B|=2) = \frac{c}{\Lambda_{NP}^2} y_t^4 (V_{tb}V_{tq}^*)^2\, \bar{q}_L\gamma^{\mu}b_L\, \bar{q}_L\gamma_{\mu}b_L \qquad (q=d,s)$$

where we used $y_u^{Diag} \approx \text{diag}(0,0,y_t)$

same CKM suppression as in the SM. Now the bound on the scale of New Physics reads

$$\Lambda_{NP} > 5.9\ TeV \qquad \Lambda_{NP} \leftrightarrow \frac{\Lambda_{NP}}{4\pi} \leftrightarrow \frac{4\pi}{g}\Lambda_{NP}$$

[this would modify M_{12} for B_d and B_s in the same way:
i.e Δ_d and Δ_s are identical and real in MFV]

New physics in ΔB=2 transitions ?

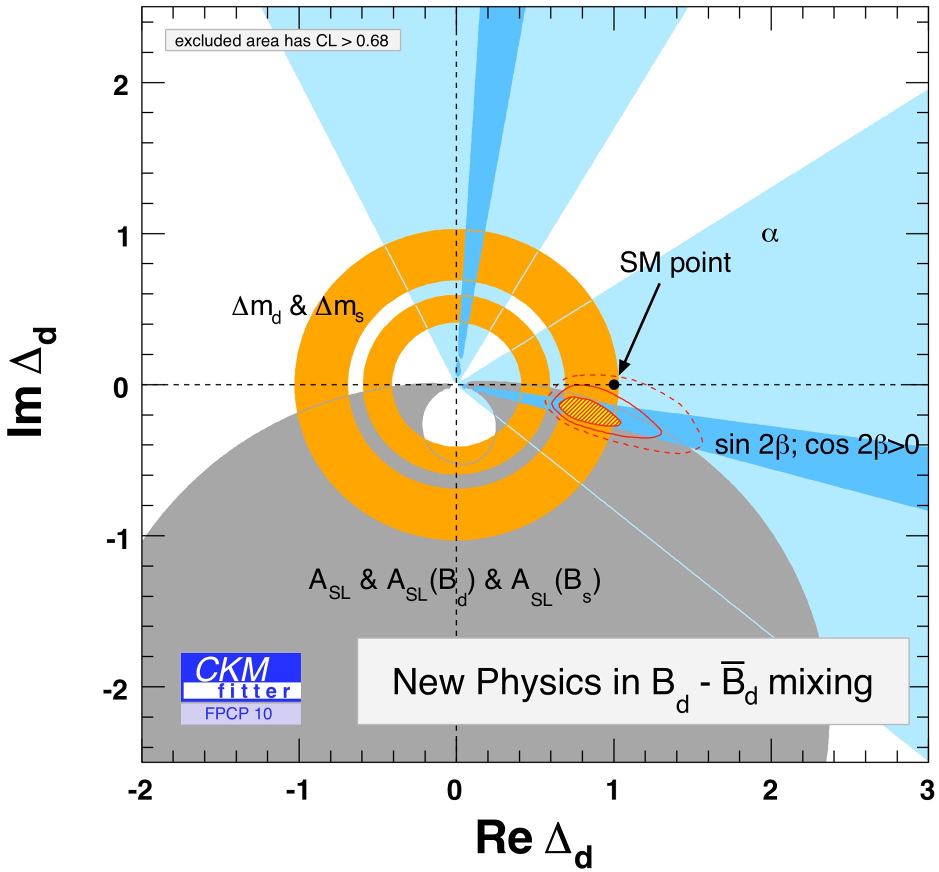
define 2 New Physics parameters

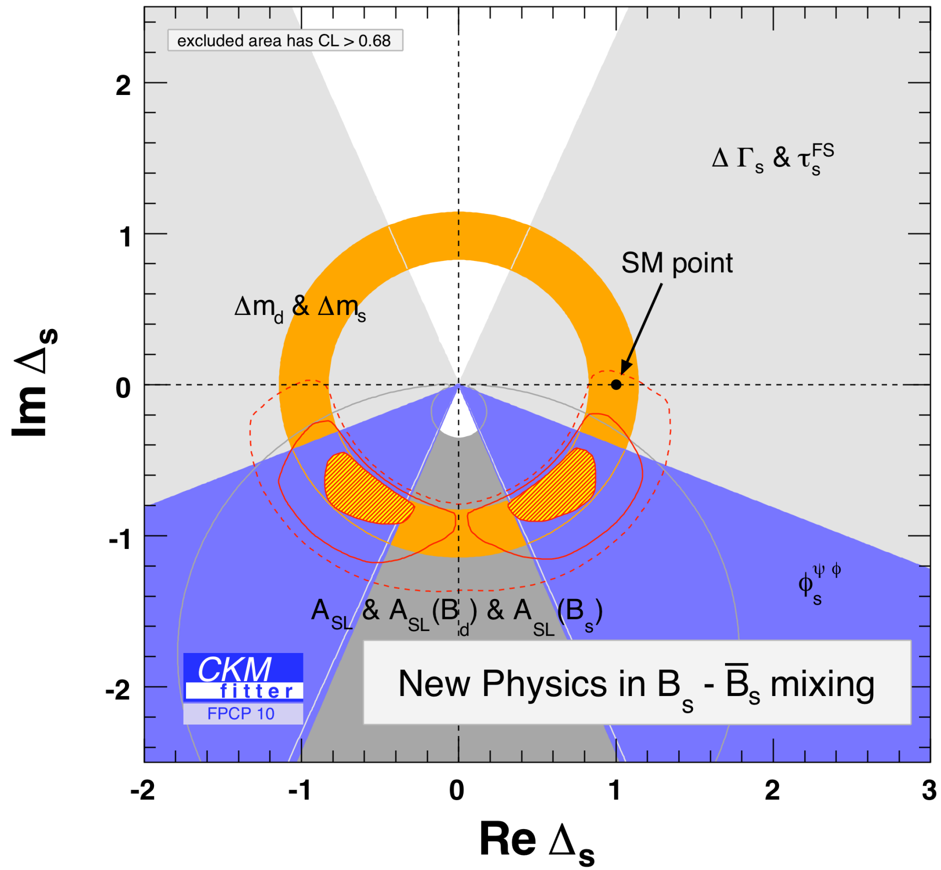
$$\Delta_q \equiv \frac{M_{12}^q}{M_{12}^{q,SM}} \qquad \Delta_q = |\Delta_q| e^{i\phi_q^\Delta}$$

(q=d,s)

2010 fits

large negative Φ_q^Δ preferred by both old D0 like-sign dimuon asymmetry and by Tevatron data on $B_s \to J/\psi\ \Phi$





SM OK within 3 σ...

SUMMARY

- The flavour sector brings many new parameters into the theory:
13 [in SM with vanishing neutrino masses and up to 22 for massive neutrinos].
Part of them displays a clear pattern calling for a more fundamental explanation. None of the explanations proposed so far is fully satisfactory.
We speak of a FLAVOUR MYSTERY.

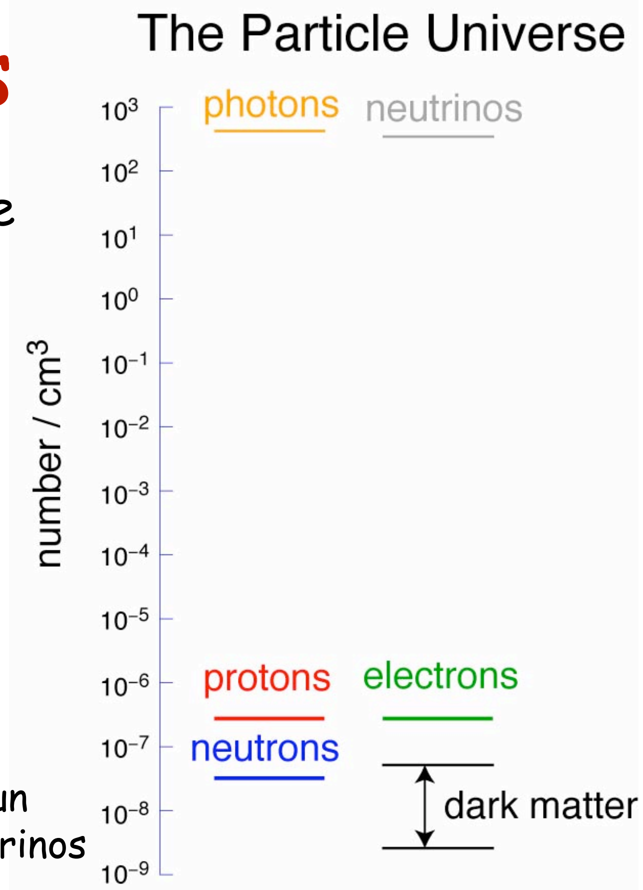
- One of the key property of the flavour sector of the SM is the absence of flavour changing neutral currents [FCNC].
Many FC transitions can only occur through electroweak loop, sensitive to New Physics at the TeV scale.

- In the quark sector there are many tests of the SM flavour picture.
The parameter space is over-constrained and the SM description is robust.
Only small deviations from the SM picture are allowed.

- This poses strong constraints on the flavour structure of most SM extensions.
Either the scale of New Physics is very large and the new contributions are decoupled or this scale is accessible, i.e. at the LHC, and the new contributions are highly non-generic, to avoid conflict with existing tests.
We speak of a FLAVOUR PROBLEM.

Lecture 3
Neutrino Masses, Mixing and Oscillations
the data

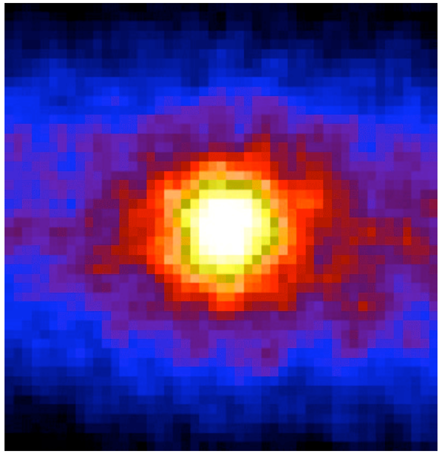
General remarks on neutrinos



from Murayama talk Aspen 2007

the more abundant particles in the universe after the photons: about 300 neutrinos per cm^3

produced by stars: sizeable fraction of the sun energy emitted in neutrinos. As I speak more than 1 000 000 000 000 solar neutrinos go through your bodies each second.

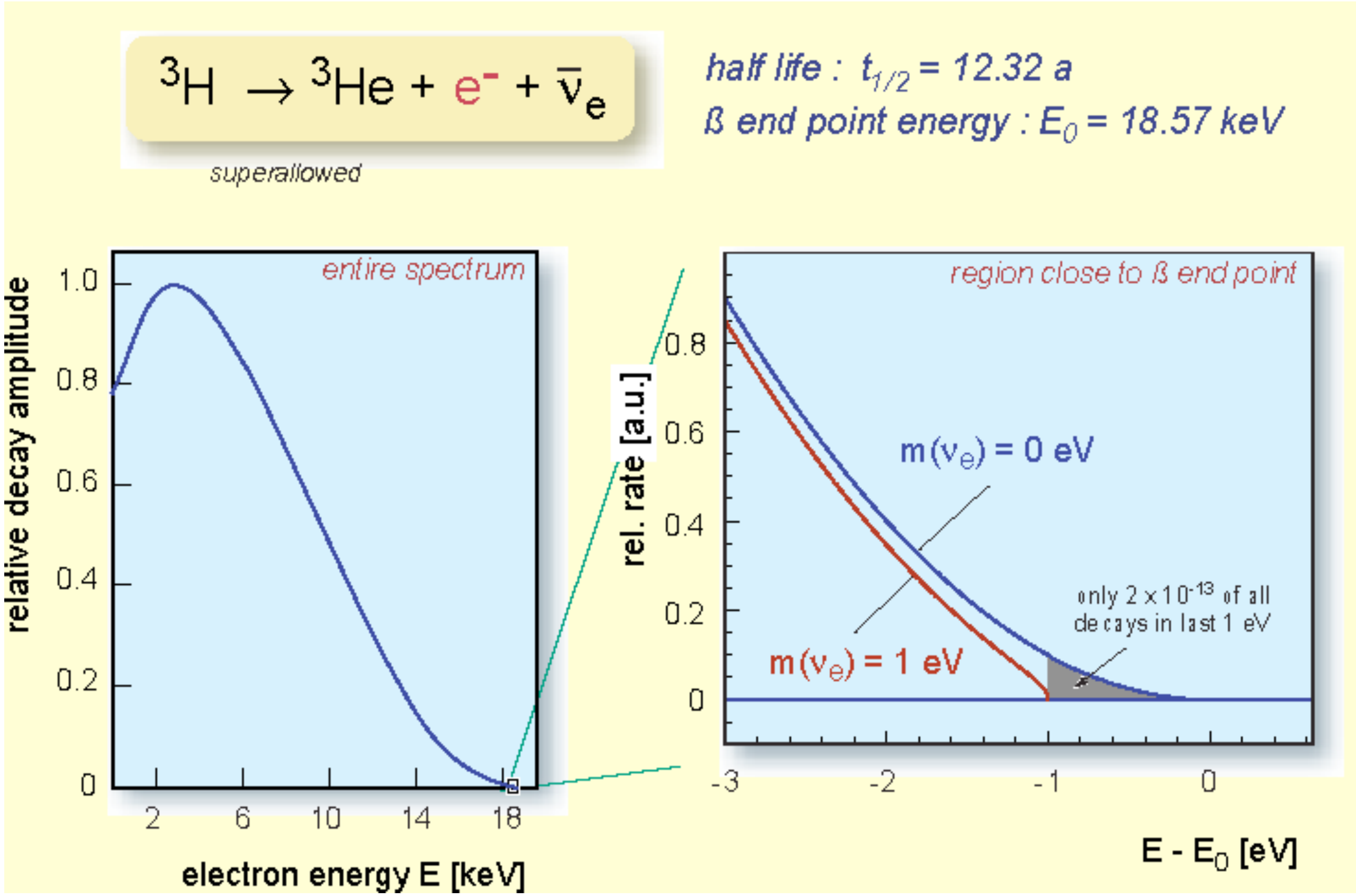


this is a picture of the sun reconstructed from neutrinos

electrically neutral and extremely light:
they can carry information about extremely large length scales
e.g. a probe of supernovae dynamics: neutrino events from a supernova explosion first observed 24 years ago

in particle physics:
they have a tiny mass (1 000 000 times smaller than the electron's mass)
the discovery that they are massive (1998) allows us to explore, at least in principle, extremely high energy scales, otherwise inaccessible to present laboratory experiments (more on this in the second part)

Upper limit on neutrino mass (laboratory)



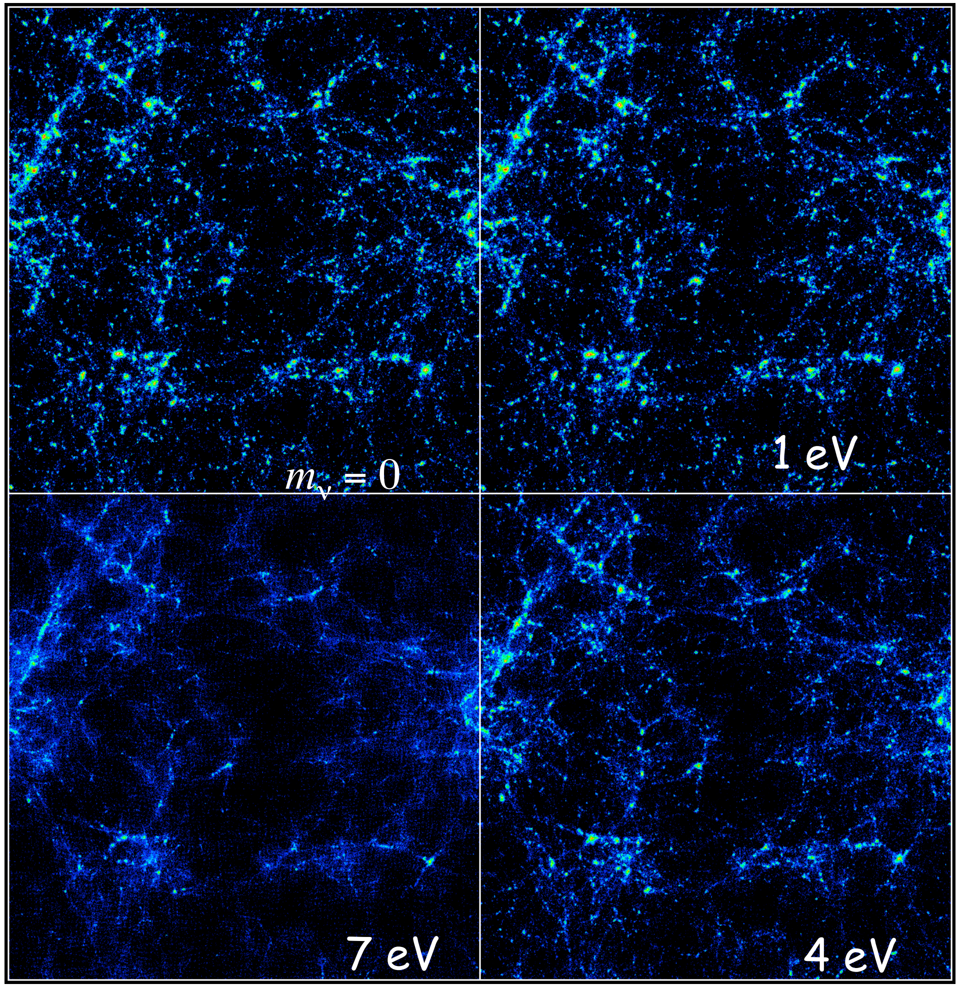
$$m_\nu < 2.2\, eV \quad (95\%\ CL)$$

Upper limit on neutrino mass (cosmology)

massive ν suppress the formation of small scale structures

$$\sum_i m_i < 0.2 \div 1 \quad eV$$

depending on
- assumed cosmological model
- set of data included
- how data are analyzed



$$k_{\rm nr} \approx 0.026 \left(\frac{m_\nu}{1\,{\rm eV}}\right)^{1/2} \Omega_m^{1/2} h\,{\rm Mpc}^{-1}.$$

The small-scale suppression is given by

$$\left(\frac{\Delta P}{P}\right) \approx -8\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\,{\rm eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right)$$

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)} P(\vec{k})$$

Two-flavour neutrino oscillations (ν_e, ν_μ)

here ν_e are produced with average energy E

L

source + - + detector

here we measure

$$P_{ee} \equiv P(\nu_e \to \nu_e)$$

neutrino interaction eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}}_{U} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$-\frac{g}{\sqrt{2}} W^-_\mu \bar{l}_L \gamma^\mu \nu_l$$

$$|\psi(t)\rangle = U^*_{e1} e^{-iE_1 t} |\nu_1\rangle + U^*_{e2} e^{-iE_2 t} |\nu_2\rangle$$

$$t \approx L$$

$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_2^2} \approx \frac{m_2^2}{2E} - \frac{m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E}$$

$$P_{ee} = |\langle \nu_e | \psi(L) \rangle|^2 = 1 - \underbrace{4|U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

no dependence on the phase α more on this later on ….

to see any effect, if Δm^2 is tiny, we need both θ and L large

regimes

$$P_{ee} = |\langle \nu_e | \psi(L) \rangle|^2 = 1 - \underbrace{4|U_{e1}|^2|U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

$$\frac{\Delta m^2 L}{4E} << 1 \qquad P_{ee} \approx 1$$

$$\frac{\Delta m^2 L}{4E} >> 1 \qquad \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \approx \frac{1}{2} \qquad P_{ee} \approx 1 - \frac{\sin^2 2\vartheta}{2}$$

by averaging over ν_e energy at the source

$$\frac{\Delta m^2 L}{4E} \approx 1 \qquad P_{ee} = P_{ee}(E)$$

useful relation $\frac{\Delta m^2 L}{4E} \approx 1.27 \left(\frac{\Delta m^2}{1\, eV^2}\right)\left(\frac{L}{1\, Km}\right)\left(\frac{E}{1\, GeV}\right)^{-1}$

source	L(km)	E(GeV)	$\Delta m^2 (eV^2)$
ν_e, ν_μ (atmosphere)	10^4 (Earth diameter)	1-10	10^{-4} - 10^{-3}
anti- ν_e (reactor)	1	10^{-3}	10^{-3}
anti- ν_e (reactor)	100	10^{-3}	10^{-5}
ν_e (sun)	10^8	10^{-3} - 10^{-2}	10^{-11} - 10^{-10}

neglecting matter effects

Three-flavour neutrino oscillations $(\nu_e, \nu_\mu, \nu_\tau)$

survival probability as before, with more terms

$$P_{ff} = P(\nu_f \to \nu_f) = \left| \langle \nu_f | \psi(L) \rangle \right|^2 = 1 - 4 \sum_{k<j} |U_{fk}|^2 |U_{fj}|^2 \sin^2\left(\frac{\Delta m_{jk}^2 L}{4E} \right)$$

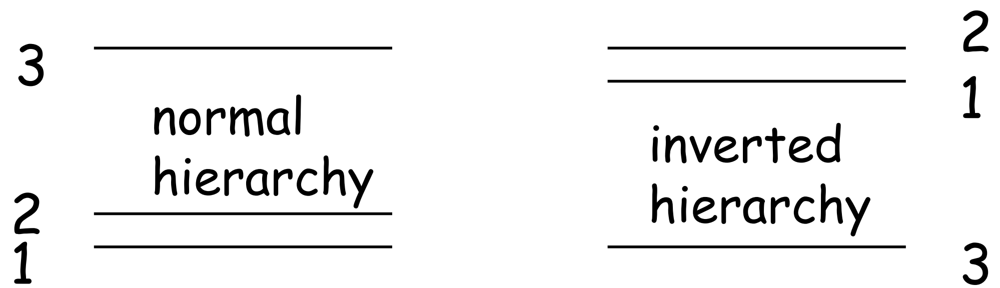
similarly, we can derive the disappearance probabilities $P_{ff'} = P(\nu_f \to \nu_{f'})$

conventions: $[\Delta m_{ij}^2 \equiv m_i^2 - m_j^2]$

$m_1 < m_2$

$\Delta m_{21}^2 < |\Delta m_{32}^2|, |\Delta m_{31}^2|$ i.e. 1 and 2 are, by definition, the closest levels

two possibilities:



we anticipatedthat $\Delta m_{21}^2 << |\Delta m_{32}^2|, |\Delta m_{31}^2|$

Mixing matrix U=U$_{PMNS}$ (Pontecorvo,Maki,Nakagawa,Sakata)

neutrino interaction eigenstates

$$\nu_f = \sum_{i=1}^{3} U_{fi}\nu_i \qquad (f = e,\mu,\tau)$$

neutrino mass eigenstates

U is a 3 x 3 unitary matrix
standard parametrization

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23}e^{-i\delta} + s_{12}s_{23} & -s_{12}s_{13}c_{23}e^{-i\delta} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

three mixing angles $\vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23}$

three phases (in the most general case) δ $\quad \underbrace{\alpha, \quad \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \rightarrow \nu_{f'})$

oscillations can only test 5 combinations

$\Delta m^2_{21}, \Delta m^2_{32}, \quad \vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23} \quad \delta$

θ_{13} is small

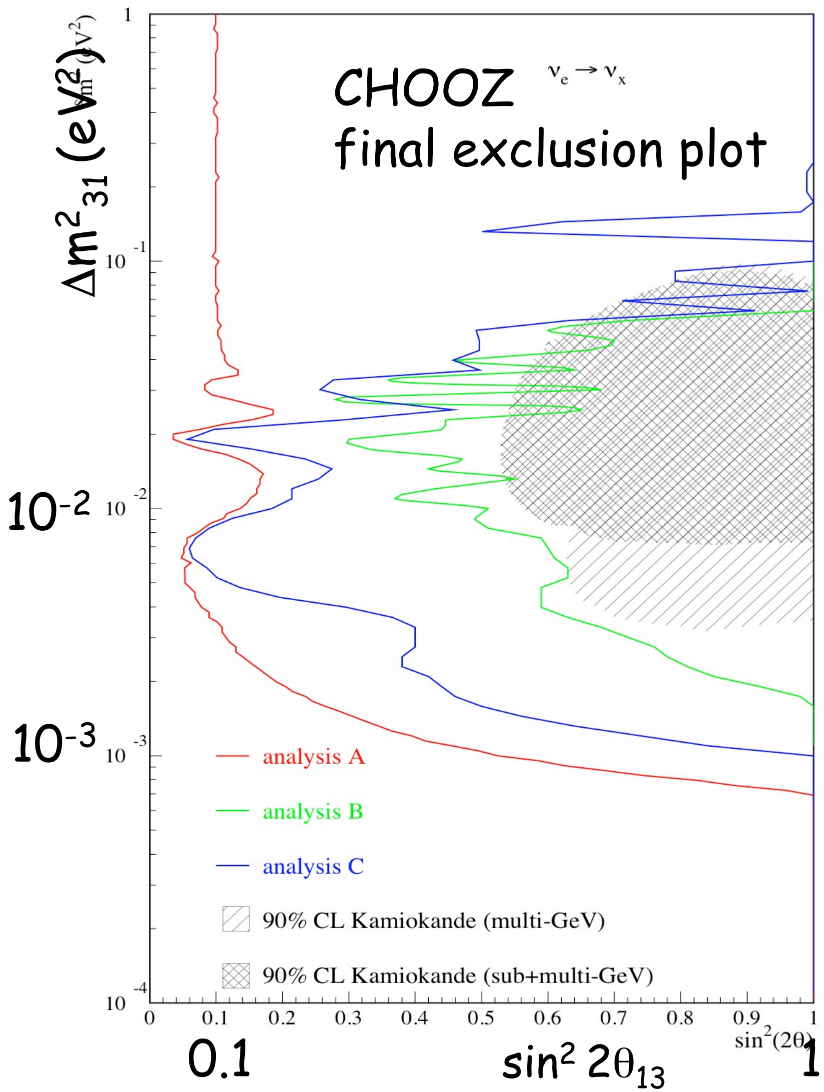
$\Delta m^2_{21} << |\Delta m^2_{32}|, |\Delta m^2_{31}|$ ⟶ set $\Delta m^2_{21} = 0$ in general formula for P_{ee}

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2 (1 - |U_{e3}|^2)}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

P_{ee} has been measured by the CHOOZ experiment that has not observed any sizeable disappearance. Electron anti-neutrinos are produced by a reactor ($E \approx 3$ MeV, $L \approx 1$ Km) and $P_{ee}^{reactor} \approx 1$ (by CPT the survival probability in vacuum is the same for neutrinos and antineutrinos and matter effects are negligible).

For a sufficiently large Δm_{31}^2 (above 10^{-3} eV^2), such that $P_{ee} = 1 - (\sin^2 2\theta_{13})/2$

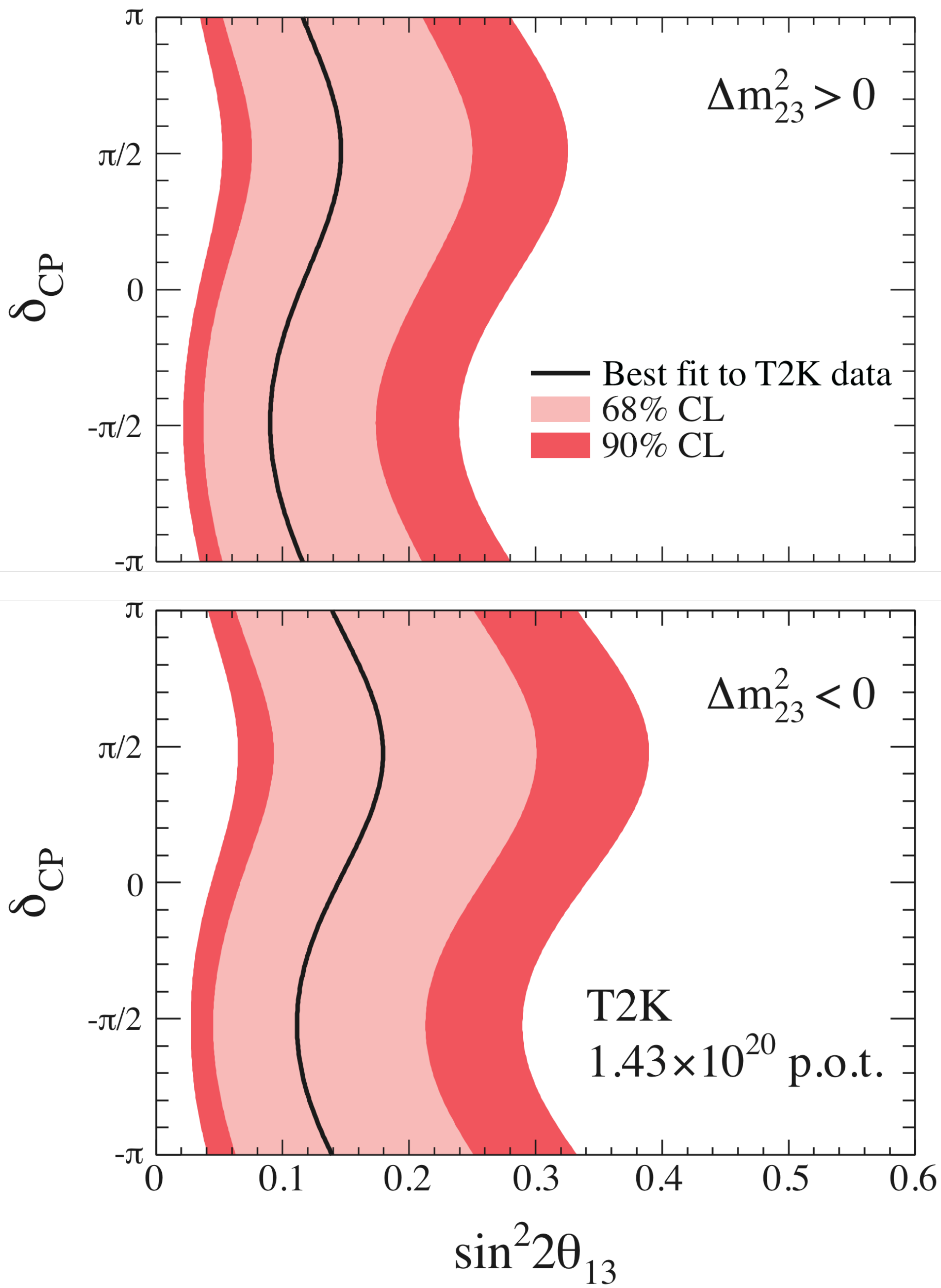
$$|U_{e3}|^2 \equiv |\sin^2 \vartheta_{13}|^2 < 0.05 \quad (3\sigma)$$



this year [1106.2822]

muon neutrino produced
at JPARC [Tokai]
E=0.6 GeV and sent to
SK 295 Km apart

6 electron neutrino events
seen [1.5 expected]
2.5 sigma away from θ_{13}



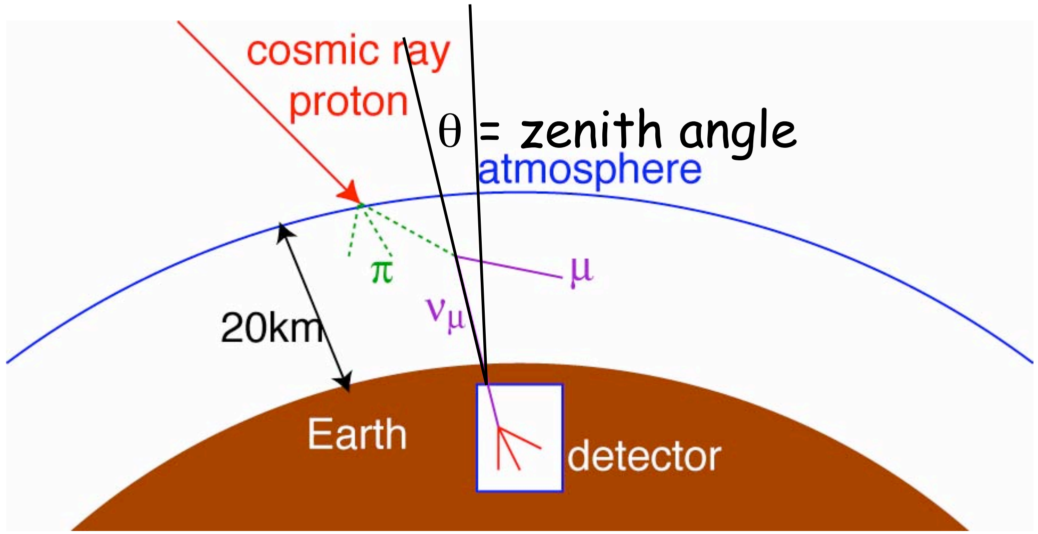
$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & \text{small} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

in what follows, for illustrative purposes, we will work in the approximation

$$U_{e3} = \sin\vartheta_{13} = 0$$

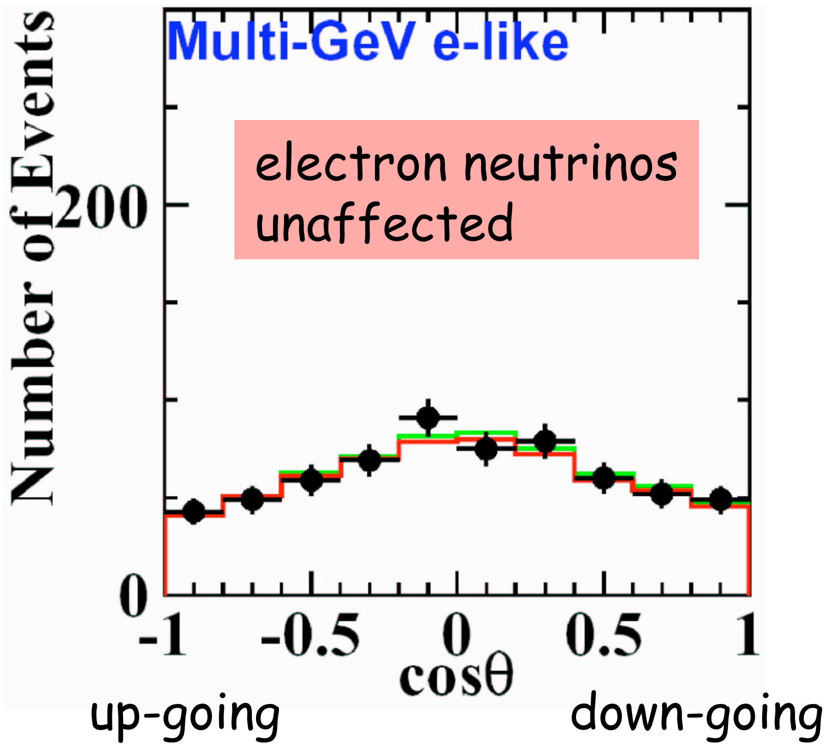
[dependence on CP violating phase δ is lost in this limit]

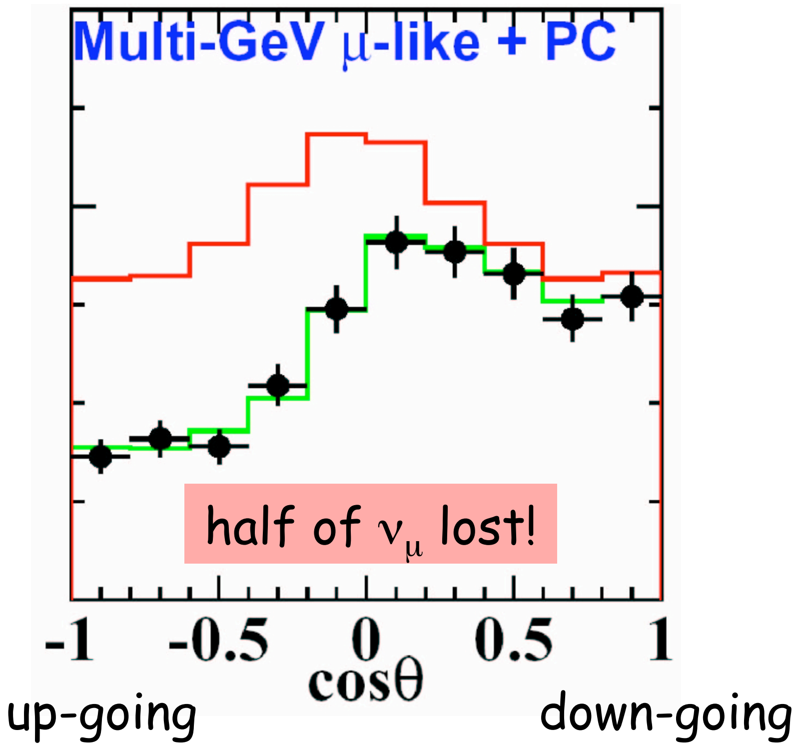
Atmospheric neutrino oscillations



Electron and muon neutrinos (and antineutrinos) produced by the collision of cosmic ray particles on the atmosphere

Experiment:
SuperKamiokande (Japan)





electron neutrinos do not oscillate

by working in the approximation $\Delta m^2_{21} = 0$

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \approx 1$$

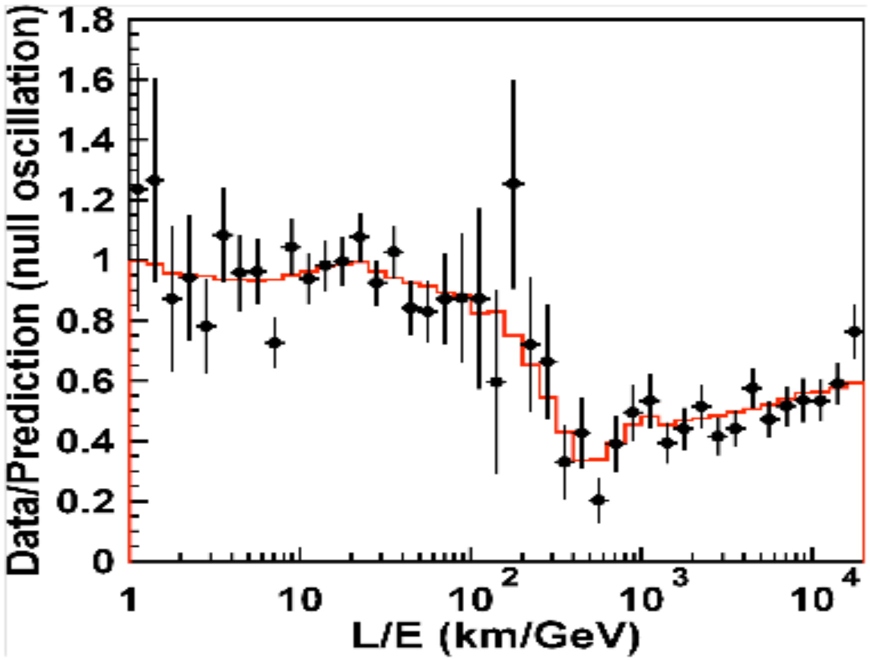
for $U_{e3} = \sin\vartheta_{13} \approx 0$

muon neutrinos oscillate

$$P_{\mu\mu} = 1 - \underbrace{4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2}_{\sin^2 2\vartheta_{23}} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$|\Delta m^2_{32}| \approx 2\cdot 10^{-3} \quad eV^2$$

$$\sin^2\vartheta_{23} \approx \frac{1}{2}$$



$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & -\frac{1}{\sqrt{2}} \\ \cdot & \cdot & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

maximal mixing!
not a replica of the quark mixing pattern

this picture is supported by other terrestrial esperiments such as
K2K (Japan, from KEK to Kamioka mine L ≈ 250 Km E ≈ 1 GeV)
and MINOS (USA, from Fermilab to Soudan mine L ≈ 735 Km E ≈ 5 GeV)
that are sensitive to Δm_{32}^2 close to 10^{-3} eV^2,

KamLAND

previous experiments were sensitive to Δm^2 close to 10^{-3} eV^2
to explore smaller Δm^2 we need larger L and/or smaller E

KamLAND experiment exploits the low-energy electron anti-neutrinos (E≈3 MeV) produced by Japanese and Korean reactors at an average distance of L≈180 Km from the detector and is potentially sensitive to Δm^2 down to 10^{-5} eV^2

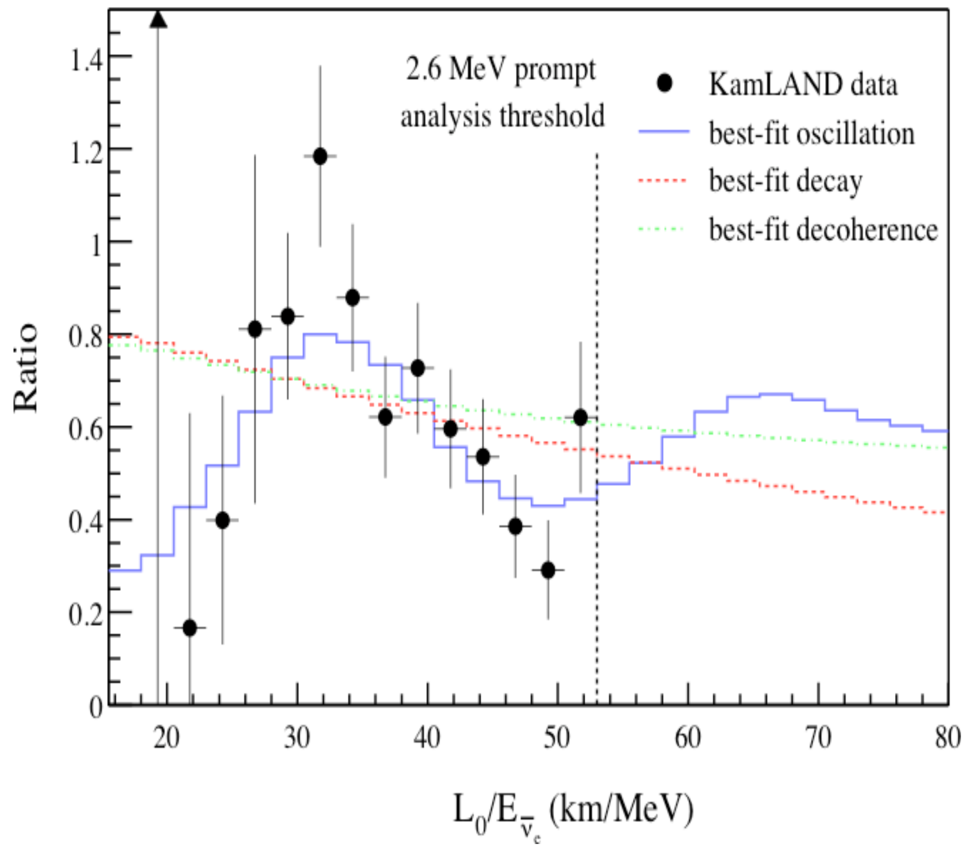
by working in the approximation

$U_{e3} = \sin\vartheta_{13} = 0$ we get

$$P_{ee} = 1 - \underbrace{4|U_{e1}|^2|U_{e2}|^2}_{\sin^2 2\vartheta_{12}} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

$$\Delta m^2_{21} \approx 8 \cdot 10^{-5} \quad eV^2$$

$$\sin^2\vartheta_{12} \approx \frac{1}{3}$$



$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

this pattern is called tri-bimaximal completely different from the quark mixing pattern: two angles are large

by unitarity

historically Δm_{21}^2 and $\sin^2\theta_{12}$ were first determined by solving the solar neutrino problem, i.e. the disappearance of about one third of solar electron neutrino flux, for solar neutrinos above few MeV. The desire of detecting solar neutrinos, to confirm the thermodynamics of the sun, was the driving motivation for the whole field for more than 30 years. Electron solar neutrinos oscillate, but the formalism requires the introduction of matter effects, since the electron density in the sun is not negligible. Experiments: SuperKamiokande, SNO

Summary of data

Summary of unkowns

$m_\nu < 2.2\,eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

absolute neutrino mass scale is unknown

$$\Delta m^2_{atm} \equiv \left|\Delta m^2_{32}\right| = (2.36^{+0.12}_{-0.10}) \times 10^{-3}\ \mathrm{eV}^2$$

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.54^{+0.25}_{-0.22}) \times 10^{-5}\ \mathrm{eV}^2$$

$\mathrm{sign}\,[\Delta m^2_{32}]$ unknown

[complete ordering (either normal or inverted hierarchy) not known]

$$\sin^2\vartheta_{13} = 0.014^{+0.009}_{-0.008}$$

$$\sin^2\vartheta_{23} = 0.42^{+0.09}_{-0.04}$$

$$\sin^2\vartheta_{12} = 0.307^{+0.018}_{-0.016}$$

δ, α, β unknown

[CP violation in lepton sector not yet established]

violation of individual lepton number implied by neutrino oscillations

violation of total lepton number not yet established

Lecture 4
Neutrino Masses, Mixing and Oscillations
the theory

Beyond the Standard Model

a non-vanishing neutrino mass is the first evidence of the incompleteness of the Standard Model [SM]

in the SM neutrinos belong to SU(2) doublets with hypercharge Y=-1/2
they have only two helicities (not four, as the other charged fermions)

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1,2,-1/2)$$

the requirement of invariance under the gauge group G=SU(3)xSU(2)xU(1) forbids pure fermion mass terms in the lagrangian. Charged fermion masses arise, after electroweak symmetry breaking, through gauge-invariant Yukawa interactions

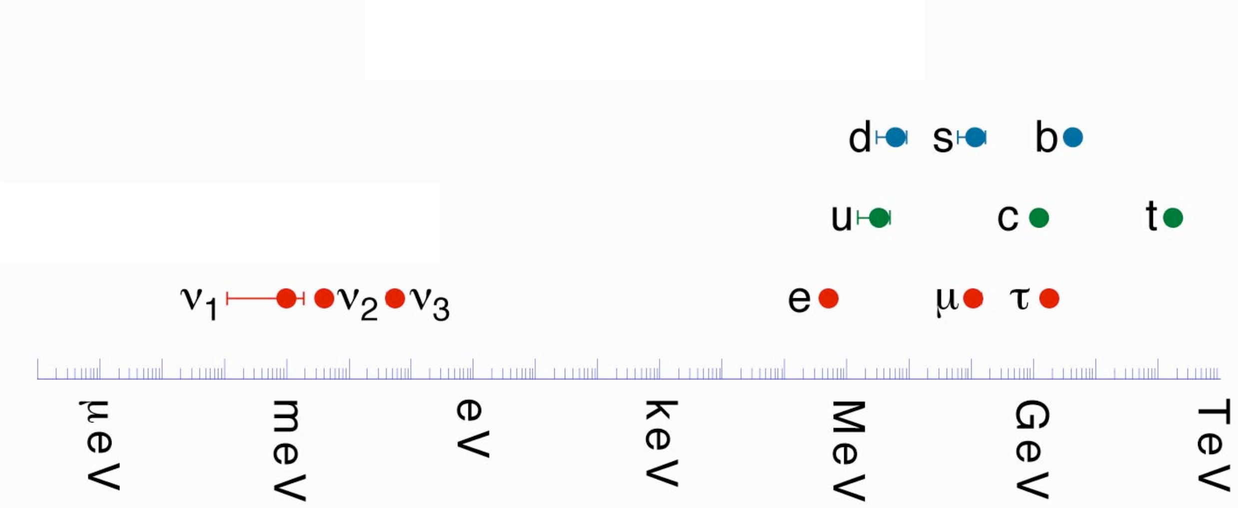
$$\Phi \quad \underbrace{\Psi\Psi'}_{\text{same helicity}}$$

not even this term is allowed for SM neutrinos, by gauge invariance

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angle are so different from those of the quark sector?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$$\lambda \approx 0.22$$

How to modify the SM?

the SM, as a consistent QFT, is completely specified by

0. invariance under local transformations of the gauge group G=SU(3)xSU(2)xU(1) [plus Lorentz invariance]

1. particle content — three copies of (q,u^c,d^c,l,e^c)
   one Higgs doublet Φ

2. renormalizability (i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i)\geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

(0.+1.+2.) leads to the SM Lagrangian, L_{SM}, possessing an additional, accidental, global symmetry: (B-L)

0. We cannot give up gauge invariance! It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory [remember: we need gauge invariance to eliminate the photon extra degrees of freedom required by Lorentz invariance]!
   We could extend G, but, to allow for neutrino masses, we need to modify 2. (and/or 3.) anyway…

First possibility: modify (1), the particle content

there are several possibilities
one of the simplest one is to mimic the charged fermion sector

Example 1

- add (three copies of) right-handed neutrinos $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)
- ask for (global) invariance under B-L (no more automatically conserved as in the SM)

the neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = d^c y_d (\Phi^+ q) + u^c y_u (\tilde{\Phi}^+ q) + e^c y_e (\Phi^+ l) + \nu^c y_\nu (\tilde{\Phi}^+ l) + h.c.$$

$$m_f = \frac{y_f}{\sqrt{2}} v \qquad f = u,d,e,\nu$$

with three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matrices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e} \sigma^\mu U_{PMNS} \nu + h.c.$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}

a generic problem of this approach

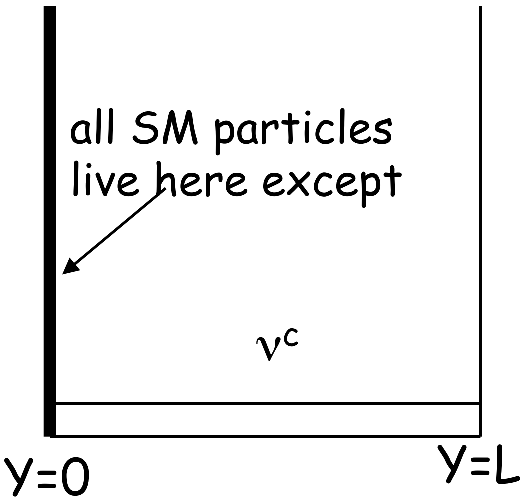
the particle content can be modified in several different ways in order to account for non-vanishing neutrino masses (additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet(s), SUSY particles,...). Which is the correct one?

a problem of the above example

if neutrinos are so similar to the other fermions, why are so light?

$$\frac{y_\nu}{y_{top}} \le 10^{-12}$$

Quite a speculative answer:
neutrinos are so light, because the right-handed neutrinos have access to an extra (fifth) spatial dimension



neutrino Yukawa coupling

$$\nu^c(y=0)(\tilde{\Phi}^+ l) = \text{Fourier expansion}$$
$$= \frac{1}{\sqrt{L}} \nu_0^c(\tilde{\Phi}^+ l) + \ldots \quad \text{[higher modes]}$$

if L>>1 (in units of the fundamental scale) then neutrino Yukawa coupling is suppressed

Second possibility: abandon (2) renormalizability

A disaster?

$$L = L^{SM}_{d\leq 4} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

a new scale Λ enters the theory. The new (gauge invariant!) operators L_5, L_6,… contribute to amplitudes for physical processes with terms of the type

$$\frac{L_5}{\Lambda} \rightarrow \frac{E}{\Lambda} \qquad \frac{L_6}{\Lambda^2} \rightarrow \left(\frac{E}{\Lambda}\right)^2 \qquad \ldots$$

the theory cannot be extrapolated beyond a certain energy scale $E \approx \Lambda$.
[at variance with a renormalizable (asymptotically free) QFT]

If $E \ll \Lambda$ (for example E close to the electroweak scale, 10^2 GeV, and $\Lambda \approx 10^{15}$ GeV not far from the so-called Grand Unified scale), the above effects will be tiny and, the theory will *look like* a renormalizable theory!

$$\frac{E}{\Lambda} \approx \frac{10^2\, GeV}{10^{15}\, GeV} = 10^{-13}$$

an extremely tiny effect, but exactly what needed to suppress m_ν compared to m_{top} !

Worth to explore. The dominant operators (suppressed by a single power of $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators

$$\frac{L_5}{\Lambda} = \frac{(\tilde{\Phi}^+ l)(\tilde{\Phi}^+ l)}{\Lambda} =$$

$$= \frac{v}{2}\left(\frac{v}{\Lambda}\right)\nu\nu + \ldots$$

a unique operator!
[up to flavour combinations]
it violates (B-L) by two units

it is suppressed by a factor (v/Λ) with respect to the neutrino mass term of Example 1:

$$\nu^c(\tilde{\Phi}^+ l) = \frac{v}{\sqrt{2}}\nu^c\nu + \ldots$$

it provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violations, is very large. How large? Up to about 10^{15} GeV

from this point of view neutrinos offer a unique window on physics at very large scales, inaccessible in present (and probably future) man-made experiments.

since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos … and indeed this was the case!

L_5 represents the effective, low-energy description of several extensions of the SM

Example 2: see-saw

add (three copies of) $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)

this is like Example 1, but without enforcing (B-L) conservation

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2} \nu^c M \nu^c + h.c.$$

mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out" the field ν^c

$$L_{eff}(l) = -\frac{1}{2} (\tilde{\Phi}^+ l) \left[y_\nu^T M^{-1} y_\nu \right] (\tilde{\Phi}^+ l) + h.c. + ...$$

terms suppressed by more powers of M^{-1}

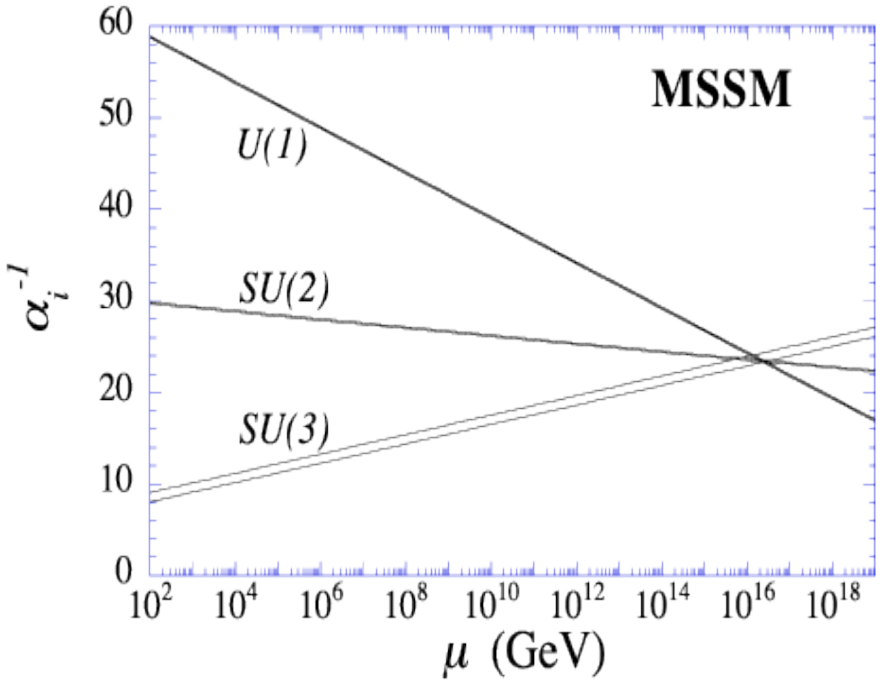
this reproduces L_5, with M playing the role of Λ. This particular mechanism is called (type I) see-saw.

Theoretical motivations for the see-saw

$\Lambda \approx 10^{15}$ GeV is very close to the so-called unification scale M_{GUT}.

an independent evidence for M_{GUT} comes from the unification of the gauge coupling constants in (SUSY extensions of) the SM.

such unification is a generic prediction of Grand Unified Theories (GUTs):
the SM gauge group G is embedded into a simple group such as SU(5), SO(10),...



Particle classification: it is possible to unify all SM fermions (1 generation) into a single irreducible representation of the GUT gauge group. Simplest example: G_{GUT}=SO(10)

$$16 = (q, d^c, u^c, l, e^c, \nu^c)$$

a whole family plus a right-handed neutrino!

quite a fascinating possibility. Unfortunately, it still lacks experimental tests. In GUT new, very heavy, particles can convert quarks into leptons and the proton is no more a stable particle. Proton decay rates and decay channels are however model dependent. Experimentally we have only lower bounds on the proton lifetime.

2 additional virtues of the see-saw

The see-saw mechanism can enhance small mixing angles into large ones

$$m_\nu = -\left[y_\nu^T M^{-1} y_\nu\right] v^2$$

Example with 2 generations

$$y_\nu = \begin{pmatrix} \delta & \delta \\ 0 & 1 \end{pmatrix}$$

δ<<1
small mixing

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

no mixing

$$y_\nu^T M^{-1} y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{M_2}$$

$$\approx \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} \qquad \text{for} \quad \frac{M_1}{M_2} << \delta^2$$

The (out-of equilibrium, CP-violating) decay of heavy right-handed neutrinos in the early universe might generate a net asymmetry between leptons and anti-leptons. Subsequent SM interactions can partially convert it into the observed baryon asymmetry

$$\eta = \frac{(n_B - n_{\bar{B}})}{s} \approx 6 \times 10^{-10}$$

weak point of the see-saw

full high-energy theory is difficult to test

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2} \nu^c M \nu^c + h.c.$$

depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those describing $(L_{SM})+L_5$:
3 masses, 3 mixing angles and 3 phases, as in lecture 3

few observables to pin down the extra parameters: η,...
[additional possibilities exist under special conditions, e.g. Lepton Flavor Violation at observable rates]

easier to test the low-energy remnant L_5

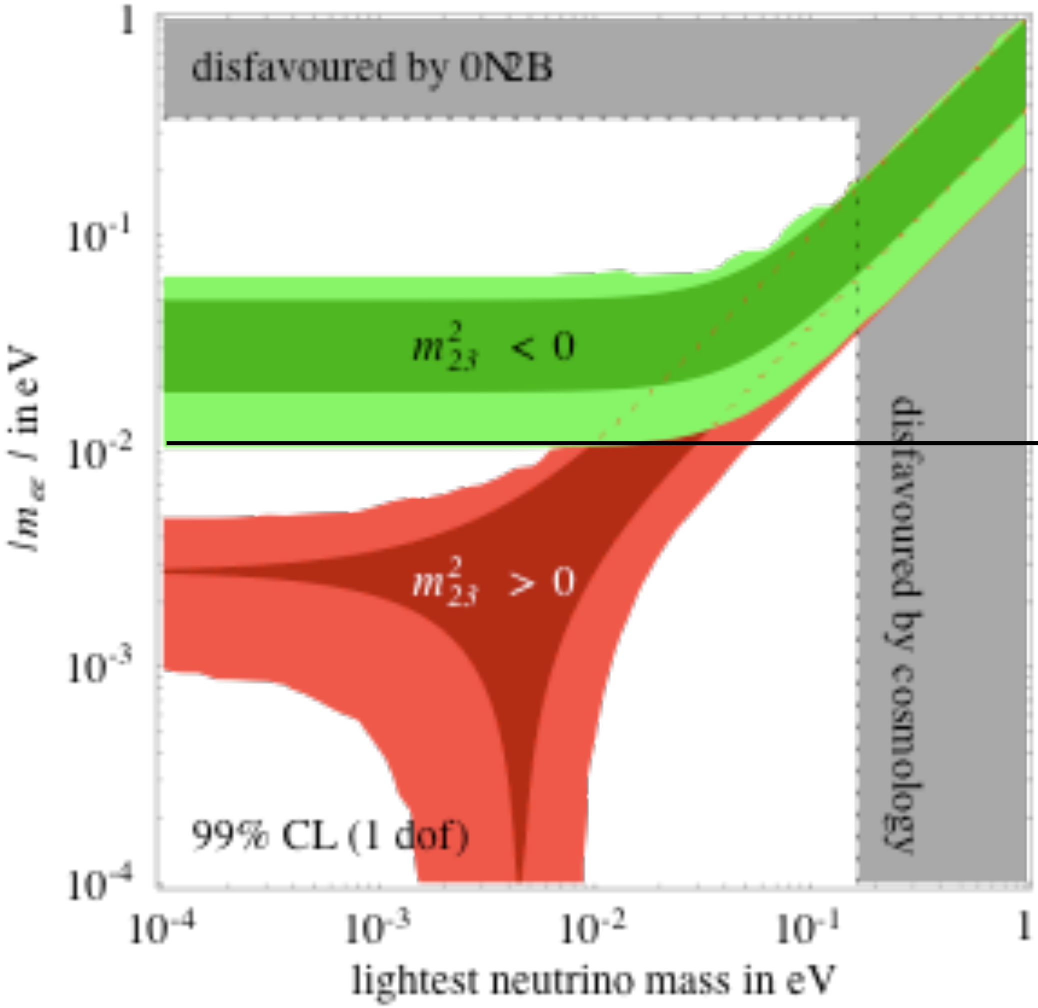
[which however is "universal" and does not implies the specific see-saw mechanism of Example 2]

look for a process where B-L is violated by 2 units. The best candidate is
$0\nu\beta\beta$ decay: $(A,Z) \rightarrow (A,Z+2)+2e^-$
this would discriminate L_5 from other possibilities, such as Example 1.

The decay in 0νββ rates depend on the combination $|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\ m_1 + \sin^2\vartheta_{12}e^{2i\alpha}\ m_2) + \sin^2\vartheta_{13}e^{2i\beta}\ m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



from the current knowledge of $(\Delta m_{ij}^2, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

future expected sensitivity on $|m_{ee}|$

10 meV

a positive signal would test both L_5 and the absolute mass spectrum at the same time!

Flavor symmetries I (the hierarchy puzzle)

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = (0.025 \div 0.049) \approx \lambda^2 << 1 \quad (2\sigma)$$

$$|U_{e3}| < 0.18 \leq \lambda \quad (2\sigma)$$

call ξ_i the generic small parameter. A modern approach to understand why ξ_i<<1 consists in regarding ξ_i as small breaking terms of an approximate flavour symmetry. When ξ_i=0 the theory becomes invariant under a flavour symmetry F

Example: why $y_e << y_{top}$? Assume F=$U(1)_F$

F(t)=F(t^c)=F(h)=0	$y_{top}(h+v)t^c t$	allowed
F(e^c)=p>0 F(e)=q>0	$y_e(h+v)e^c e$	breaks $U(1)_F$ by (p+q) units
if ξ=<φ>/Λ<1 breaks U(1) by one negative unit	$y_e \approx O(\xi^{p+q}) << y_{top} \approx O(1)$	

provides a qualitative picture of the existing hierarchies in the fermion spectrum

Flavor symmetries II (the lepton mixing puzzle)

why $U_{PMNS} \approx U_{TB} \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$?

[TB=TriBimaximal]

$$U_{PMNS} = U_e^{+} U_\nu$$

Consider a flavor symmetry G_f such that G_f is broken into two different subgroups: G_e in the charged lepton sector, and G_ν in the neutrino sector. m_e is invariant under G_e and m_ν is invariant under G_ν. If G_e and G_ν are appropriately chosen, the constraints on m_e and m_ν can give rise to the observed U_{PMNS}.

G_f

G_e $\quad$ G_ν

m_e diagonal $\quad$ $U_{TB}^T \, m_\nu \, U_{TB} = (m_\nu)_{diag}$

The simplest example is based on a small discrete group, $G_f=A_4$. It is the subgroup of SO(3) leaving a regular tetrahedron invariant. The elements of A_4 can all be generated starting from two of them: S and T such that

$$S^2 = T^3 = (ST)^3 = 1$$

S generates a subgroup Z_2 of A_4
T generates a subgroup Z_3 of A_4

simple models have been constructed where $G_e=Z_3$ and $G_\nu=Z_2$ and where the lepton mixing matrix U_{PMNS} is automatically U_{TB}, at the leading order in the SB parameters. Small corrections are induced by higher order terms.

the generic predictions of this approach is that θ_{13} and $(\theta_{23}-\pi/4)$ are very small quantities, of the order of few percent: testable in a not-so-far future.

Conclusion

theory of neutrino masses

it does not exist! Neither for neutrinos nor for charged fermions. We lack a unifying principle.

like weak interactions before the electroweak theory

$SU(2)_L \otimes U(1)_Y$ gauge invariance → all fermion-gauge boson interactions in terms of 2 parameters: g and g'

? → Yukawa interactions between fermions and spin 0 particles: many free parameters (up to 22 in the SM!)

only few ideas and prejudices about neutrino masses and mixing angles

caveat: several prejudices turned out to be wrong in the past!

- $m_\nu \approx 10$ eV because is the cosmologically relevant range
- solution to solar is MSW Small Angle
- atmospheric neutrino problem will go away because it implies a large angle