Proton decay in local F-theory GUTs

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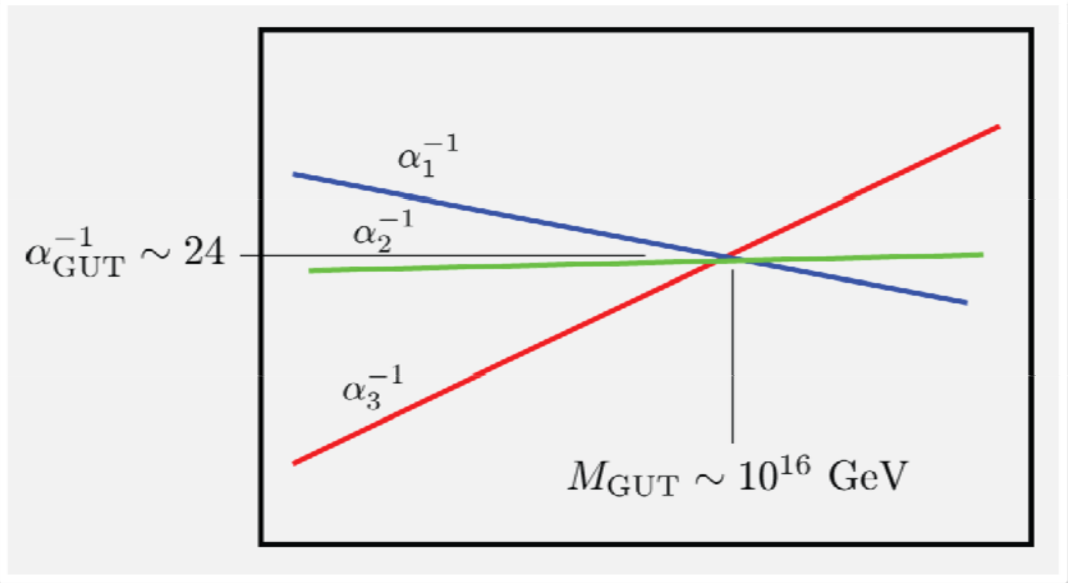
1. Motivation

SUSY GUTs are extremely interesting candidates for physics beyond the Standard Model. Experimentally well-motivated :

- Gauge coupling unification
- SM fields nicely fit in SU(5) representations
- SU(5) Weak mixing angle prediction $\sin^2\theta_W \simeq 0.23$



$$SU(5) \rightarrow SU(3) \times SU(2)_L \times U(1)_Y$$

$$\bar{\mathbf{5}}_M \rightarrow (\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus (\mathbf{1},\mathbf{2})_{-1/2}$$

$$\mathbf{10}_M \rightarrow (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{3},\mathbf{2})_{1/6} \oplus (\mathbf{1},\mathbf{1})_1$$

$$\bar{\mathbf{5}}_H \rightarrow (\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus (\mathbf{1},\mathbf{2})_{-1/2}$$

doublet – triplet splitting

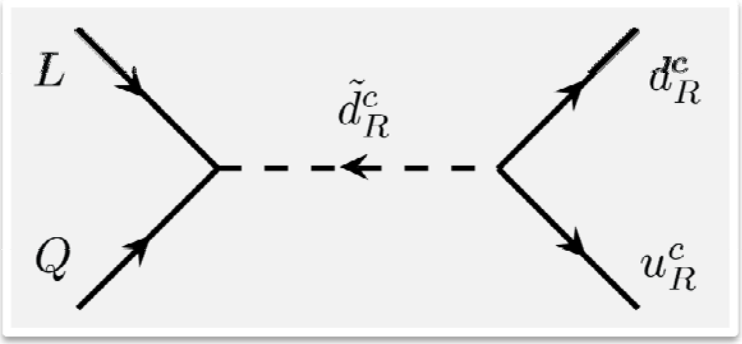
1. Motivation

Proton decay is the classic constraint on SUSY GUTs:

$$\tau_p \sim 10^{31} - 10^{33} \text{ yr}$$

- Dim. 4 proton decay:

$$\mathbf{10}_M \times \bar{\mathbf{5}}_M \times \bar{\mathbf{5}}_M \; = \; u^c_R\, d^c_R\, d^c_R \; + \; L\, L\, e^c_r \; + \; Q\, L\, d^c_R$$



Dim. 4 proton decay can be forbidden by anomay-free discrete symmetries (e.g. matter parity).

In String Theory they arise from massive U(1) gauge symmetries.

[Berasaluce, Ibanez, Soler, Uranga ‘11]

1. Motivation

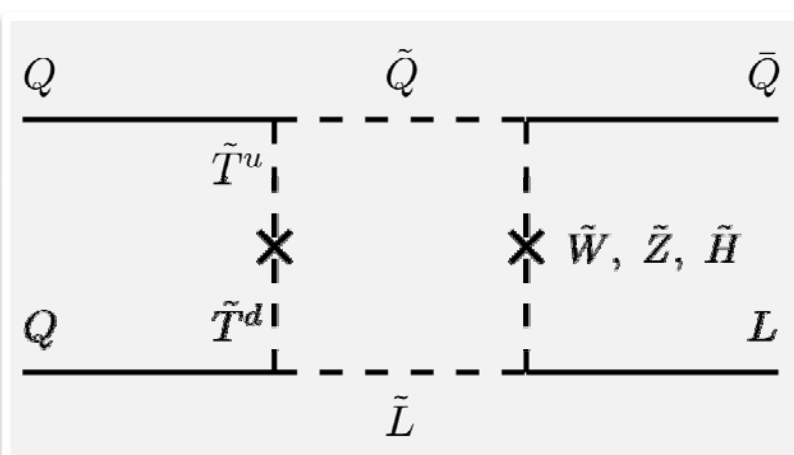
- Dim. 5 proton decay more difficult to avoid:

$$\mathbf{10}_M \times \bar{\mathbf{5}}_M \times \bar{\mathbf{5}}_H = Y^d_{ij}(L e^c_R + Q d^c_R)_{ij} H^d \; + \; \hat{Y}^d_{ij}(QL + d^c_R u^c_R)_{ij} T^d$$

$$\mathbf{10}_M \times \mathbf{10}_M \times \mathbf{5}_H = Y^u_{ij}(Q u^c_R)_{ij} H^u + \hat{Y}^u_{ij}(QQ + u^c_R e^c_R) T^u$$

doublet – triplet splitting: $W \supset M_{\mathrm{GUT}} T^u T^d$

$$W_{\mathrm{eff.}} = \frac{\hat{Y}^u_{ij}\hat{Y}^d_{kl}}{M_{\mathrm{GUT}}}(Q_i Q_j Q_k L_l + u^c_{R\,i} d^c_{R\,j} u^c_{R\,k} e^c_{R\,l})$$



$p \to K^+ + \bar{\nu}$

$p \to \ldots$

$$\hat{Y}^u_{ij}\hat{Y}^d_{kl} < 10^{-11}$$

1. Motivation

Anomaly-free discrete gauge symmetries forbidding dim. 5 proton decay (baryon triality, hexality...) do not commute with $SU(5)_{GUT}$

In String Theory the problem might be alleviated due to the GS mechanism ($U(1)_{PQ}$ symmetries), however still in conflict with usual GUT breaking mechanisms (hypercharge flux, Wilson lines...)

⇩

Exotics in incomplete GUT multiplets

[Marsano, Saulina, Schafer-Nameki '09]
[Dudas, Palti '10]
[Marsano '10]

F-theory is a theory of flavour: framework for computing $Y_{ij}^{u,d}$, $\hat{Y}_{ij}^{u,d}$

Can F-theory explain the smallness of $\hat{Y}_{ij}^{u}\hat{Y}_{kl}^{d}$??

[I will say nothing about dim 4 proton decay in this talk]

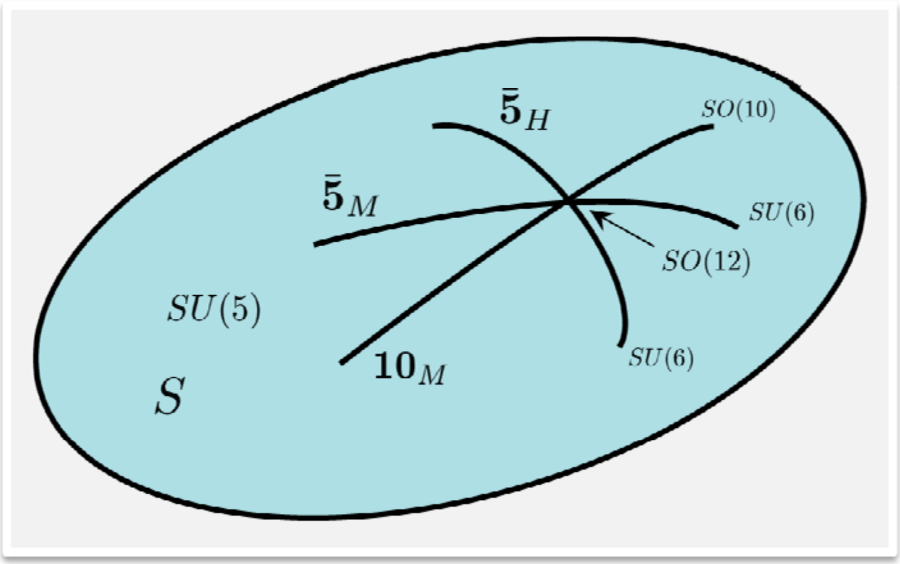
C.f. also Leontaris' talk...

2. F-theory GUTs

[Beasly, Heckman, Vafa '08], [Donagi, Wijholt '08]

$SU(5)_{GUT}$ d.o.f. localized on a stack of 7-branes wrapping a 4-cycle S

(codimension-2 singularity of elliptically fibered CY 4-fold)



- Matter localized at complex curves
- Interactions localized at codimension-3 singularities

Gauge symmetry enhanced at localization defects:

$\mathbf{10}_M \times \bar{\mathbf{5}}_M \times \bar{\mathbf{5}}_H$ ⇨ SO(12) enhancement, down-type Yukawas

$\mathbf{10}_M \times \mathbf{10}_M \times \mathbf{5}_H$ ⇨ E_6 enhancement, up-type Yukawas

2. F-theory GUTs

In the IR the d.o.f. of the GUT 7-brane are described by a twisted 8d N = 1 SYM theory with support on $\mathbb{R}^{1,3} \times S$

$$\mathbf{A}_{\bar{m}} = (A_{\bar{m}}, \psi_{\bar{m}}, \mathcal{G}_{\bar{m}})$$
$$\mathbf{\Phi}_{mn} = (\varphi_{mn}, \chi_{mn}, \mathcal{H}_{mn})$$
$$\mathbf{V} = (\eta, A_{\mu}, \mathcal{D})$$

Varying Higgs field ⇨ gauge symmetry enhancement at vanishing loci (matter curves)

UV cutoff: $\langle \partial A \rangle, \ \langle \partial \varphi \rangle \ll M_*^2$ In the type IIB limit related to string scale

4d theory from Kaluza-Klein reduction of the 8d theory:

$$\Psi_{8\mathrm{d}} = \sum_i \phi^{(i)}_{4\mathrm{d}}(x) \times \psi^{(i)}_{\mathrm{int.}}(y)$$

wavefunctions localize at gauge enhancement points

3. Local F-theory GUTs

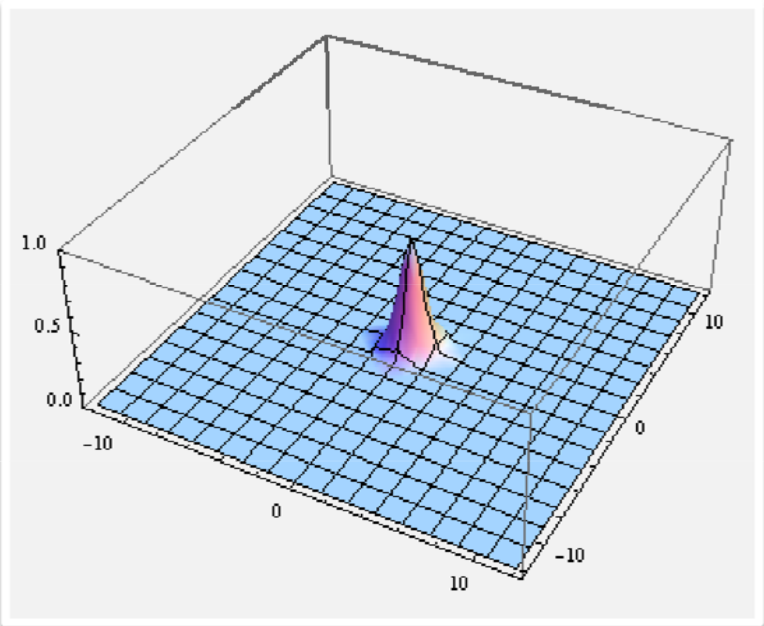
4d 3-point couplings as overlap of internal wavefunctions:

$$W_{3-\text{point}} = \int_S \text{Tr}\left[\mathbf{A} \wedge \mathbf{A} \wedge \Phi\right]$$

Due to the exponential localization of the wavefunctions the integral only receives contributions from a small patch around the intersection point of the matter curves

Can study locally at the interaction point

[Beasly et al. '08]
[Font, Ibanez '08]
[Heckman, Vafa '08]
[Hayashi, Kawaro, Tatar, Watari '09]
[Conlon,Palti '09]
...



One therefore expands:

$$\omega = \frac{i}{2}\left(dz_1 \wedge d\bar{z}_{\bar{1}} + dz_2 \wedge d\bar{z}_{\bar{2}}\right) + \ldots$$

$$\langle A \rangle = -\frac{M_*}{R_{\|}} \,\text{Im}(M^a_{ij} z_i d\bar{z}_j) Q_a + \ldots$$

$$\langle \varphi \rangle = M_* R_{\perp}\, m^a_i z_i \, Q_a dz_1 \wedge dz_2 + \ldots$$

Two local scales $R_{\|}$ & $R_{\perp}$ (longitudinal and transverse lengths in M_* units)

3. Local F-theory GUTs

In the local patch e.o.m's for internal wavefunctions become:

$$\mathbb{D}^\dagger \mathbb{D} \Psi = |m_\lambda|^2 \Psi$$

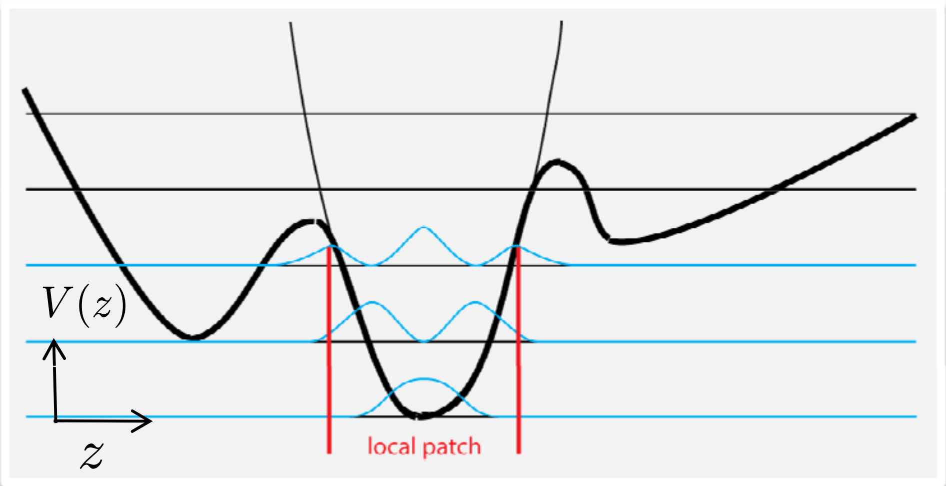
$$\mathbb{D} = \begin{pmatrix} 0 & D_1 & D_2 & D_3 \\ -D_1 & 0 & D_3^\dagger & -D_2^\dagger \\ -D_2 & -D_3^\dagger & 0 & D_1^\dagger \\ -D_3 & D_2^\dagger & -D_1^\dagger & 0 \end{pmatrix} \qquad \Psi = \begin{pmatrix} \sqrt{2}\eta \\ \psi_{\bar{1}} \\ \psi_{\bar{2}} \\ \chi \end{pmatrix}$$

$$D_i = \frac{M_*}{R_\parallel}\left(\partial_i - \frac{1}{2} q_a M_{ij}^a \bar{z}_j\right)$$

$$D_3 = -M_* R_\perp \, m_i^a \bar{z}_i$$

Equivalent to a Hamiltonian system of 3 quantum harmonic oscillators with shifted vacuum energy

[Marchesano, McGuirk, Shiu '10]
[Aparicio, Font, Ibanez, Marchesano '11]



$$\tilde{\mathbb{D}}^\dagger \tilde{\mathbb{D}} = -\sum_{i=1,2,3} a_i^\dagger a_i \, \mathbb{I} + \text{diag}(-\lambda_1, 0, \lambda_2 - \lambda_1, \lambda_3 - \lambda_1)$$

Local approach valid if:

$$R_\parallel R_\perp \gg 1 \gg \frac{R_\perp}{R_\parallel}$$

Large number of excitations within the local patch, everything well below M_* .

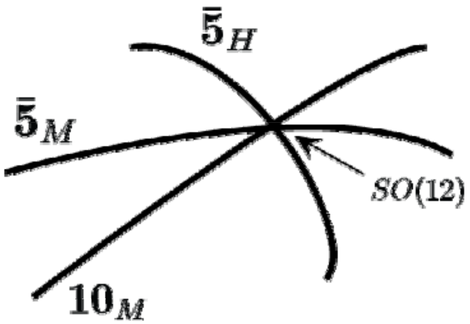
4. The SO(12) enhancement point

We have computed massless and massive wavefunctions and their triple overlaps for a SO(12) enhancement point.

SO(12) 8d SYM theory with varying Higgs and flux:

$$\langle \varphi \rangle = M_* R_\perp \left(\frac{z_1}{v_1} Q_1 + \frac{z_2}{v_2} Q_2 \right)$$

$$F = \frac{2iM_*^2}{R_\parallel^2}(dz_1 \wedge d\bar{z}_1 - dz_2 \wedge d\bar{z}_2)(-M_1 Q_1 + M_2 Q_2 + \beta Q_Y)$$



$$SO(12) \supset SU(5) \times U(1)_1 \times U(1)_2$$

$$\mathbf{66} \to \mathbf{24}^{(\mathbf{0},\mathbf{0})} \oplus \mathbf{1}^{(\mathbf{0},\mathbf{0})} \oplus \mathbf{1}^{(\mathbf{0},\mathbf{0})} \oplus \left(\mathbf{5}^{(-\mathbf{1},\mathbf{0})} \oplus \mathbf{5}^{(\mathbf{1},\mathbf{1})} \oplus \mathbf{10}^{(\mathbf{0},\mathbf{1})} \oplus \ \mathbf{c.c.} \right)$$

- M_1, M_2 and β are taken such that there are localized massless modes trasforming in the $\bar{\mathbf{5}}_M^{(\mathbf{1},\mathbf{0})}$, the $\mathbf{10}_M^{(\mathbf{0},\mathbf{1})}$ and the $(\mathbf{1},\mathbf{2})_{-1/2} \subset \bar{\mathbf{5}}_H^{(-\mathbf{1},-\mathbf{1})}$

4. The SO(12) enhancement point

There are 4 towers of complex fermions localized at each matter curve, corresponding to the fermionic d.o.f. of a broken N = 4 supermultiplet.

Only the ground state of a single tower is massless. Higher Landau excitations obtained by acting on ground state wavefunctions with raising operators.

$$\Psi^i_{p,(n,m,l)} = \frac{1}{\sqrt{m!n!l!}\,(-\lambda_1)^{n/2}\,(\lambda_2)^{m/2}\,(\lambda_3)^{l/2}}(\tilde{D}_1^\dagger)^n(\tilde{D}_2)^m(\tilde{D}_3)^l\Psi^i_p$$

$$\Psi^i_p = \frac{\xi_p}{N_i}\,(-k_2 z_1 + k_1 z_2)^{3-i}\,e^{-p_1|z_1|^2-p_2|z_2|^2+p_3\bar{z}_1 z_2+p_4\bar{z}_2 z_1}$$

$$M^2_{\Psi_{0,(n,m,l)}} = \left(\frac{M_*}{R_\parallel}\right)^2[-(n+1)\lambda_1 + m\lambda_2 + l\lambda_3]$$

$$M^2_{\Psi_{1,(n,m,l)}} = \left(\frac{M_*}{R_\parallel}\right)^2(-n\lambda_1 + m\lambda_2 + l\lambda_3)$$

$$M^2_{\Psi_{2,(n,m,l)}} = \left(\frac{M_*}{R_\parallel}\right)^2[-(n+1)\lambda_1 + (m+1)\lambda_2 + l\lambda_3]$$

$$M^2_{\Psi_{3,(n,m,l)}} = \left(\frac{M_*}{R_\parallel}\right)^2[-(n+1)\lambda_1 + m\lambda_2 + (l+1)\lambda_3]$$

4. The SO(12) enhancement point

Down Yukawa couplings from overlaps of 3 massless wavefunctions

$$Y_{ij}^{d} \sim \int_S \Psi^{\bar{\mathbf{5}}_H} \Psi^{\bar{\mathbf{5}}_M, i} \Psi^{\mathbf{10}_M, j}$$

Couplings to massive triplets from overlaps of 2 massless and one massive wavefunctions

$$Y_{(n,m,l)}^{(i,j)} \sim \int_S \Psi^{\bar{\mathbf{5}}_H}_{(n,m,l)} \Psi^{\bar{\mathbf{5}}_M, i} \Psi^{\mathbf{10}_M, j}$$

Geometric selection rules:

Global U(1) charges associated to local rotations $\quad z_1 \to e^{i\theta} z_1 \;, \quad z_2 \to e^{i\theta} z_2$

$$Q_{U(1)} \left[\Psi^{i}_{p,(n,m,l)} \right] = i + n - m - l$$

4d couplings should be neutral

4. The SO(12) enhancement point

For Yukawa couplings selection rules impliy rank 1:

$$Y^d \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y^{33}_{1,(0,0,0)} \end{pmatrix}$$

Rank > 1 by IASD fluxes and/or non-perturbative effects

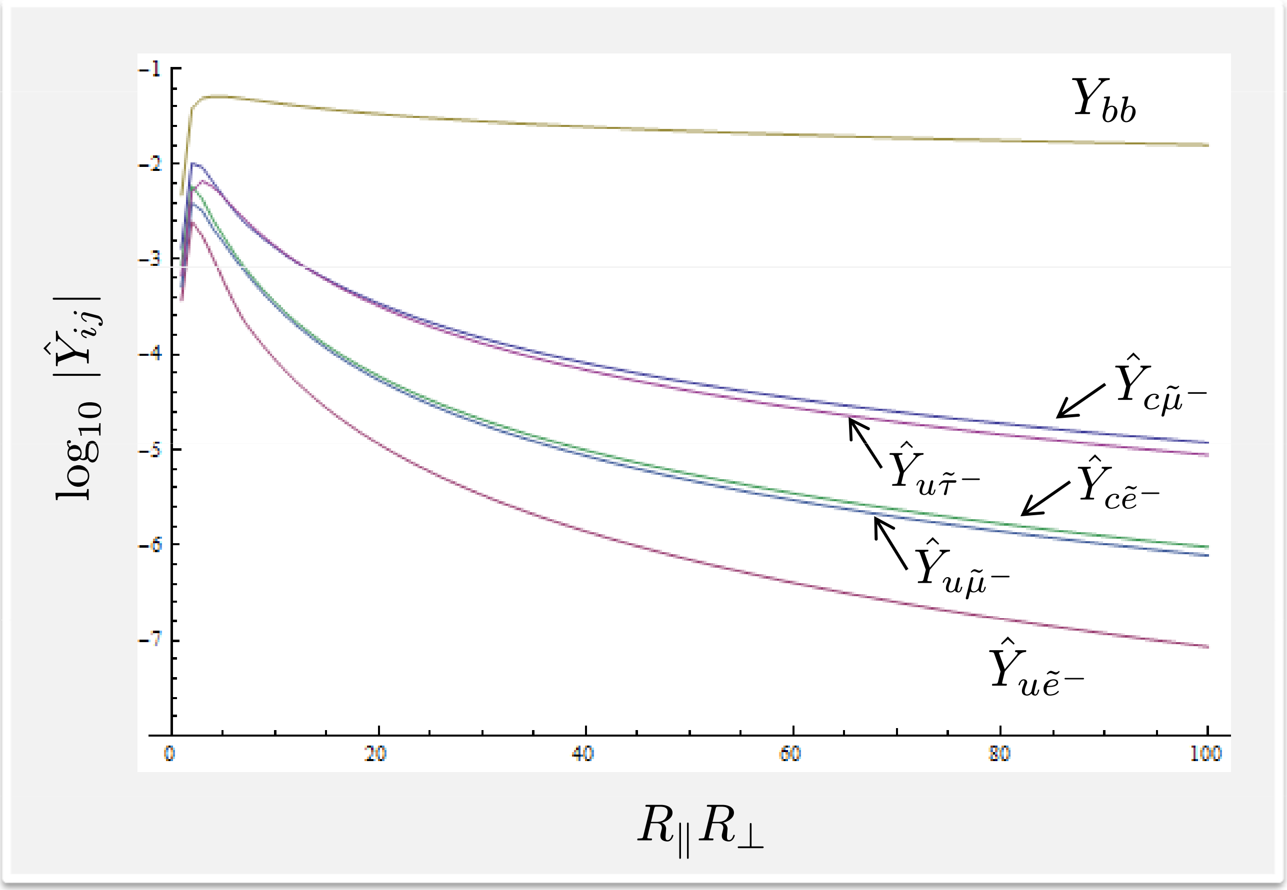
[Cecotti, Cheng, Heckman, Vafa '09]
[Marchesano, Martucci '09]

For couplings to heavy triplets impliy dominant couplings to small sets of massive modes:

$$\hat{Y}^d \sim \left(\begin{array}{c|c|c} [0,4,0]+[0,3,1]+[0,2,2]+[0,1,3]+[0,0,4] & [0,3,0]+[0,2,1]+[0,1,2]+[0,0,3] & [0,2,0]+[0,1,1]+[0,0,2] \\ \hline [0,3,0]+[0,2,1]+[0,1,2]+[0,0,3] & [0,2,0]+[0,1,1]+[0,0,2] & [0,1,0]+[0,0,1] \\ \hline [0,2,0]+[0,1,1]+[0,0,2] & [0,1,0]+[0,0,1] & [0,0,0] \end{array}\right)$$

We have solved for the triple overlaps analytically. Expressions are large and cumbersome but can be easily evaluated.

5. Results for the SO(12) enhancement point



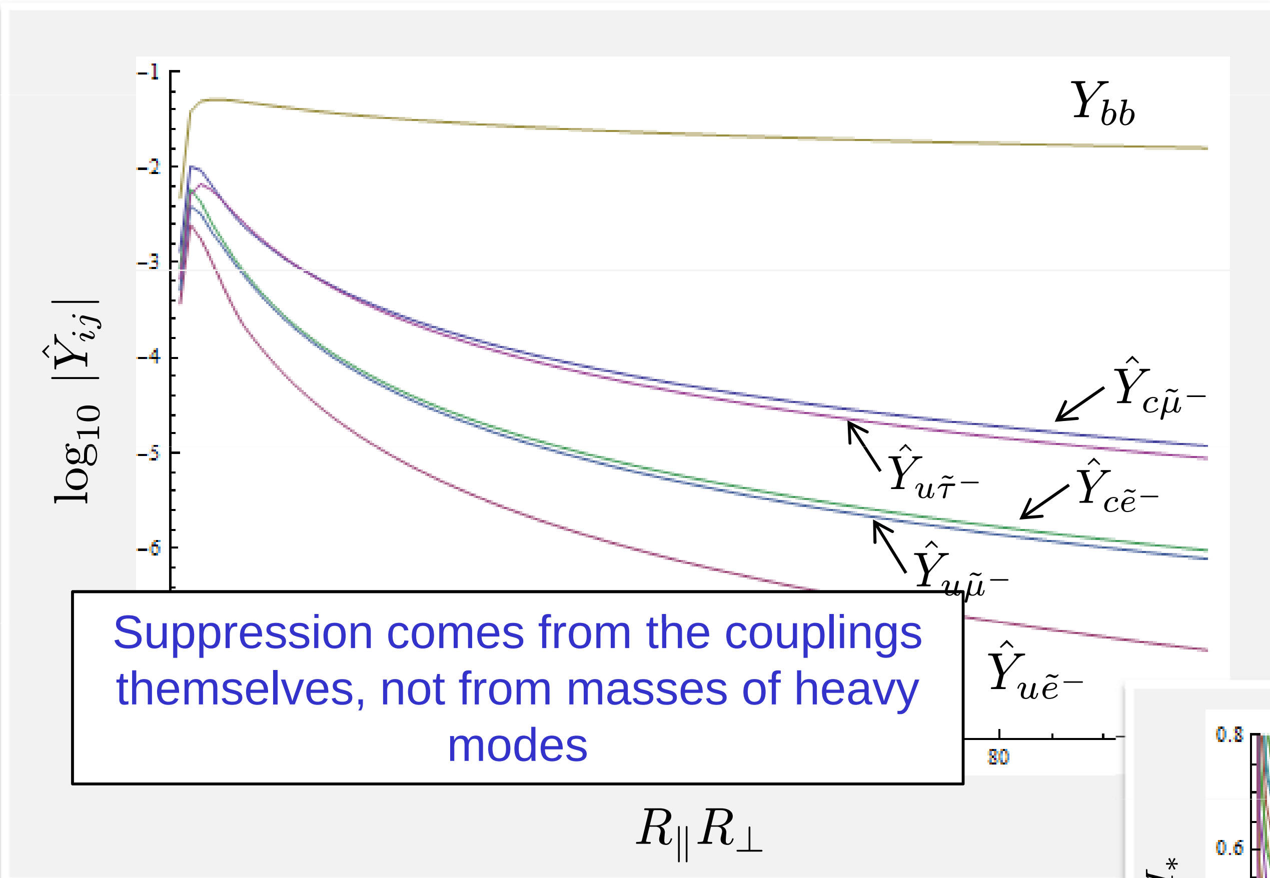
We vary $R_{\|}R_{\perp}$ while keeping $R_{\perp}/R_{\|}$ fix and small, for O(1) fluxes.

For too small $R_{\|}R_{\perp}$ the local approximation breaks down, and there are substantial α' corrections to the Kahler metrics.

$$R_{\|}R_{\perp} \gg 1 \gg \frac{R_{\perp}}{R_{\|}}$$

Heavy triplets couple very differently from the massless Higgs doublets! Large suppression for $R_{\|}R_{\perp} \sim 1 - 100$. Dimension 5 proton decay constraints are essentially eliminated since there is no substantial coupling to mediating modes.

5. Results for the SO(12) enhancement point



Suppression comes from the couplings themselves, not from masses of heavy modes

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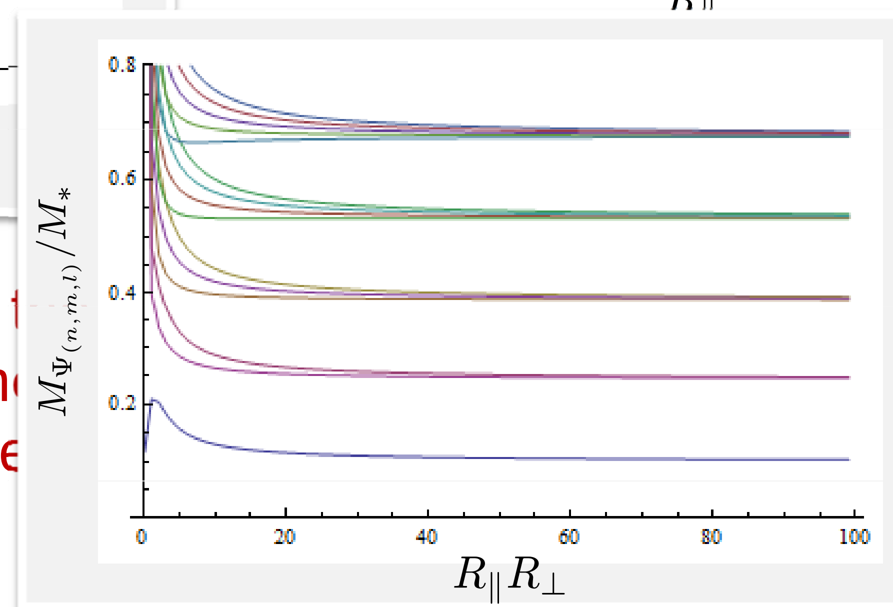

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5. Results for the SO(12) enhancement point

(c.f. also [Ludeling, Nilles, Stephan ‘11])

The local approach has its own drawbacks: in general difficult to relate the local scales $R_\perp$ and $R_{||}$ with compactification moduli. If *S* is homogeneous and contractible $R_{||} \sim \alpha_{\mathrm{GUT}}^{-\frac{1}{4}}$ and $R_\perp$ related to the winding scale.

In this sense our results show a large suppression of dim. 5 proton decay with the winding scale.

Also interesting the comparison with heterotic strings. In that case dim 5 operators are produced by worldsheet instantons

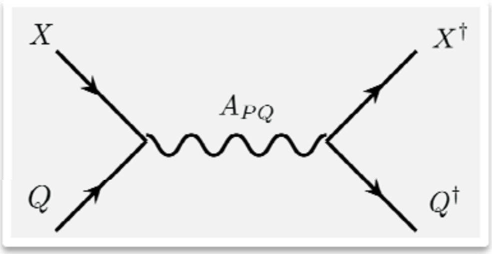
$$\sim \exp\left(-\frac{M_*}{M_{\mathrm{compact.}}}\right)$$

In local F-theory GUTs the detachment between global and local scales gives some freedom to suppress dim 5 proton decay.

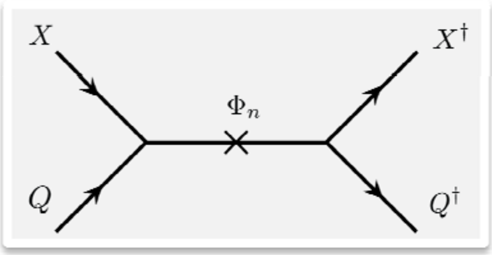
6. Further applications

Coupling to heavy modes also important for studying other higher-dimensional operators. E.g. contribution to soft SUSY breaking masses from dim 6 operators in the Kahler potential:

$$K \supset K_{ijX\bar{X}} \int d^4\theta \; X^\dagger X Q_i^\dagger Q_j \quad \Rightarrow \quad m_{ij}^2 = K_{ijX\bar{X}} |F_X|^2$$



Flavor universal [Heckman, Vafa '08]



Can be computed by wavefunction overlaps, generically lead to FCNC

Severe experimental bounds in flavor violating soft-masses [Gabbiani et al. '96] [Choudhury et al. '94]

In toy models we find again a large suppression with $R_{||} R_{\perp}$

7. Final comments

• We have computed the couplings to localized heavy triplets for SO(12) enhancement points in local F-theory GUTs.

• For some regions of the parameter space it is possible to supress the couplings substantially. This is a sufficient condition to avoid dim. 5 proton decay.

• We expect similar results for E_6 enhancement points. However, that is a much harder computation because of local monodromies.

• Coupling to heavy modes also important for studying other higher-dimensional operators: particularly interesting those inducing FCNC