Nongeometric fluxes, asymmetric strings and nonassociative geometry

Erik Plauschinn

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Corfu — 09.09.2011

Based on ...

This talk is based on:

- *Nonassociative Gravity in String Theory?* with R. Blumenhagen

  arXiv:1010.1263
  J. Phys. A: Math. Theor. **44** (2011) 015401

- *Non-geometric Fluxes, Asymmetric Strings and Nonassociative Geometry* with R. Blumenhagen, A. Deser, D. Lüst, and F. Rennecke

  arXiv:1106.0316
  J. Phys. A: Math. Theor. **44** (2011) 385401

Motivation :: Noncommutativity in string theory

String theory provides a unified framework for:

gauge theories
(open strings)

gravity
(closed strings)

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UV finite

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noncommutative geometry?

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String theory provides a unified framework for:

gauge theories

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$\downarrow$ UV finite

renormalization

gravity

(closed strings)

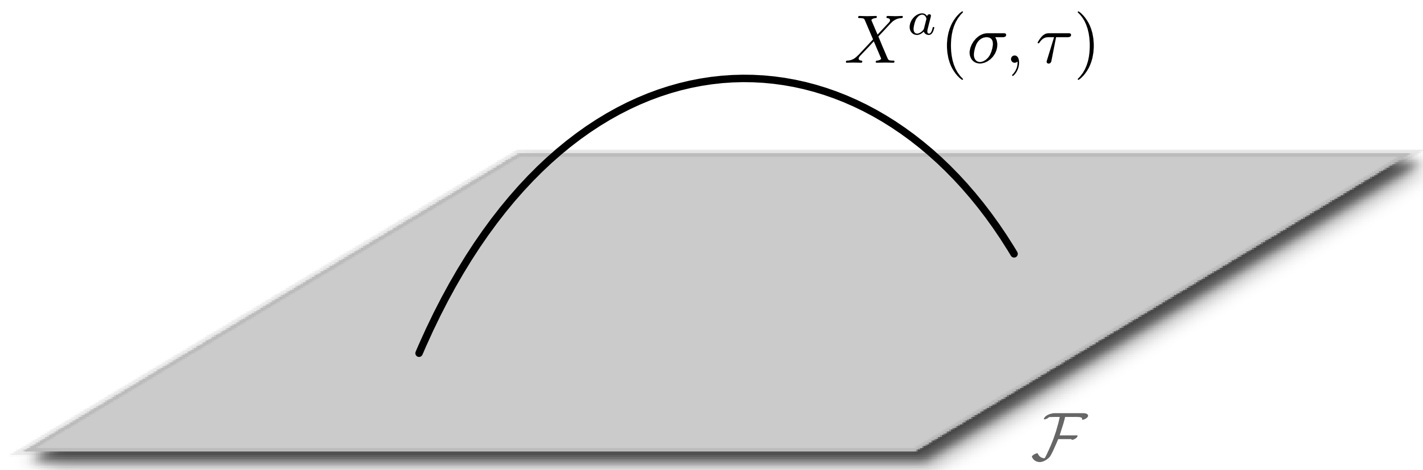
UV finite $\downarrow$

noncommutative geometry?

But, noncommutativity first appeared in the gauge theory sector to which open strings belong to ...

Motivation :: Noncommutativity for open strings

Consider open strings ending on a D-brane with 2-form flux $\mathcal{F}$.



The open string coordinates satisfy (where $\theta^{ab} = 2\pi\alpha' \left[(1-\mathcal{F}^2)^{-1}\mathcal{F}\right]^{ab}$)

$$\left[X^a(\sigma_1,\tau)\,,\,X^b(\sigma_2,\tau)\right] = \begin{cases} +i\,\theta^{ab} & \sigma_1=\sigma_2=0\,, \\ -i\,\theta^{ab} & \sigma_1=\sigma_2=\pi\,, \\ 0 & \text{else}\,. \end{cases}$$

Chu, Ho - 1998

Correlation functions of vertex operators $T = :e^{i\,p\cdot X(\sigma,\tau)}:$ satisfy (with $\epsilon(\tau) = \pm 1$)

$$\langle\, T_1\, T_2 \rangle = \exp\Big[-\tfrac{i}{2}\, p_{1,a}\, \theta^{ab}\, p_{2,b}\, \epsilon(\tau_1 - \tau_2)\Big] \times \langle\, T_1\, T_2 \rangle_{\theta=0} \;.$$

A product reproducing this phase is the Moyal-Weyl star-product

$$f_1(x) \star f_2(x) := \exp\Big[\tfrac{i}{2}\, \theta^{ab}\, \partial_a^{x_1}\, \partial_b^{x_2}\Big]\, f_1(x_1)\, f_2(x_2)\Big|_{x_1 = x_2 = x} \;.$$

From correlators such as above

- an action can be constructed using the star-product
- which describes noncommutative gauge theories.

Seiberg, Witten - 1999

Motivation :: Agenda for closed strings

Can this structure be generalized to closed strings?

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1. Compute correlation functions

2. Determine an algebra of functions

3. Construct a theory of gravity using such an algebra

Motivation :: Agenda for closed strings

Can this structure be generalized to closed strings?

1. Compute correlation functions $\rightarrow$ which background? $\rightarrow$ how to do computations?

2. Determine an algebra of functions $\rightarrow$ mathematical properties? $\rightarrow$ correct physical properties? $\rightarrow$ ...

3. Construct a theory of gravity using such an algebra $\rightarrow$ ...

Blumenhagen, EP - 2010
Blumenhagen, Deser, Lüst, EP, Rennecke - 2011

Plan of the talk

1. Motivation
2. Background
3. CFT
4. Some results

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1. Motivation

2. Background

   a) Preliminaries

   b) Three-bracket

   c) Summary

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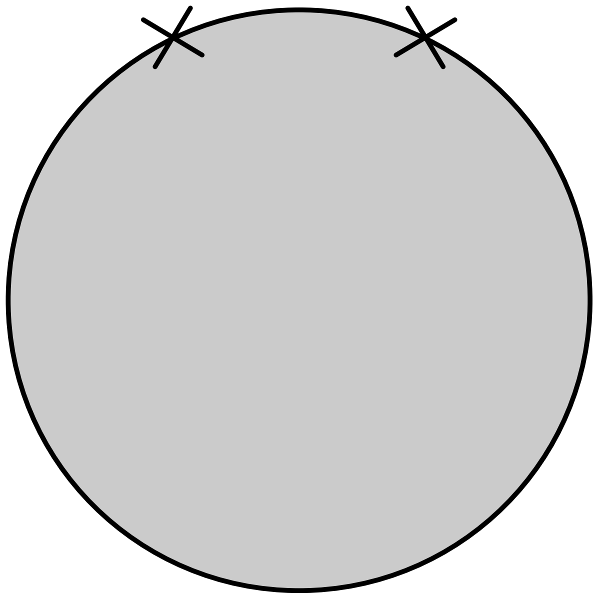
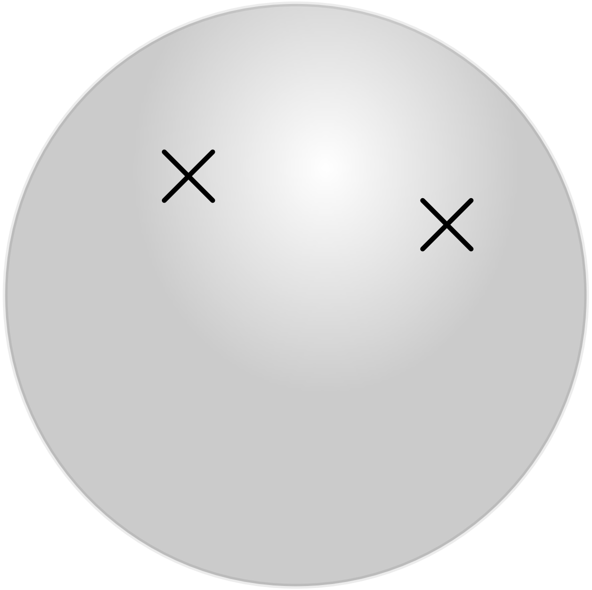
Origin of noncommutativity for open strings:

- Two vertex operators are inserted on the boundary of a disk.
- The two-form flux $\mathcal{F}$ is sensitive to the ordering.





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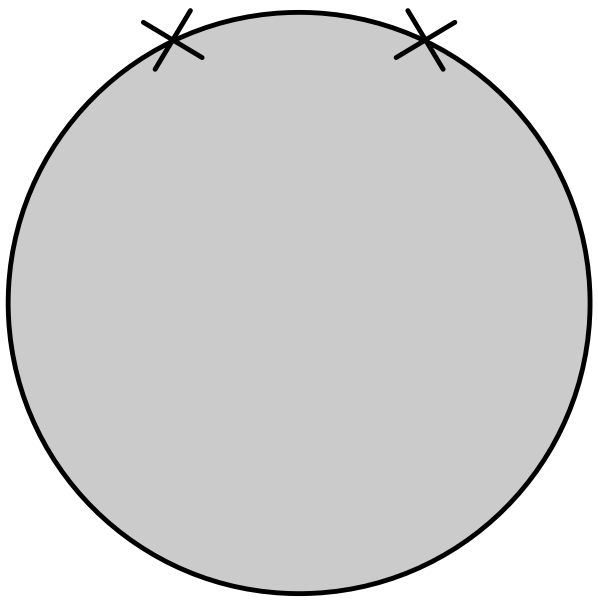
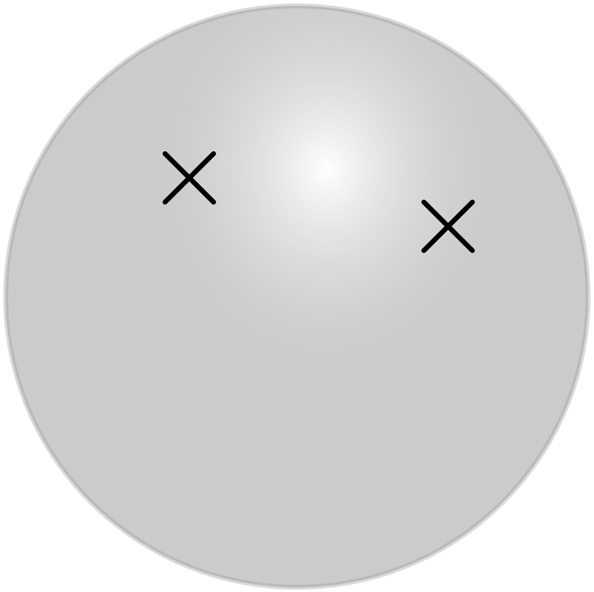


Noncommutativity for closed strings?

- Two vertex operators on a sphere cannot be ordered.



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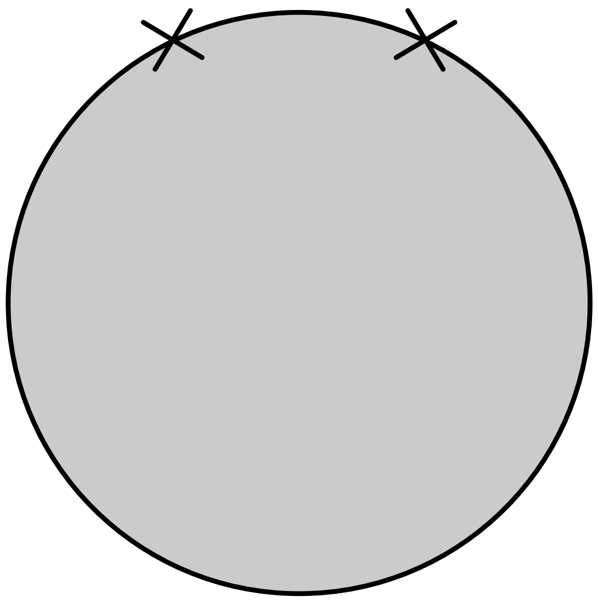
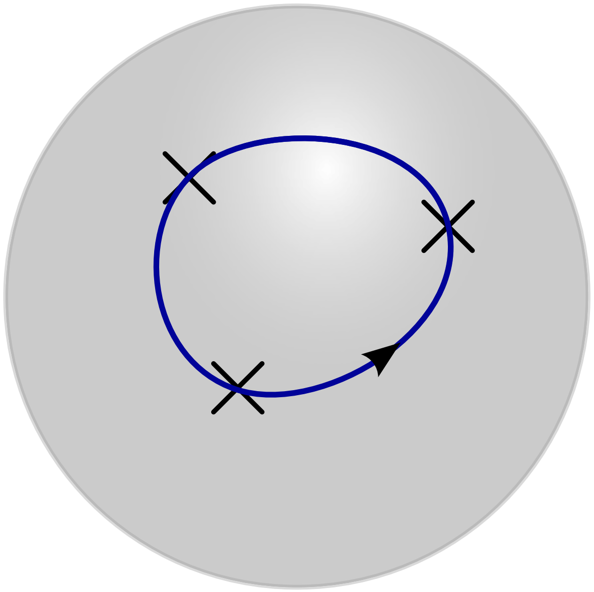


Noncommutativity for closed strings?

- Two vertex operators on a sphere cannot be ordered.
- Though, a loop connecting three nearby points has an orientation.
- A three-form flux can be sensitive to this ordering.



Therefore, for closed strings one might consider

- not a single commutator involving two fields,
- but a **double commutator** involving **three fields**

$$\big[[X^a(\sigma_1,\tau),X^b(\sigma_2,\tau)],X^c(\sigma_3,\tau)\big] + \text{cyclic}\,.$$

Therefore, for closed strings one might consider

- not a single commutator involving two fields,
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$$\big[X^a, X^b, X^c\big] := \lim_{\sigma_i \to \sigma} \big[[X^a(\sigma_1,\tau), X^b(\sigma_2,\tau)], X^c(\sigma_3,\tau)\big] + \text{cyclic}\,.$$

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The corresponding three-bracket

- vanishes for an associative product (Jacobi identity),
- while a non-zero result indicates a

noncommutative and nonassociative (NCA) structure.

The three-form flux(es) of interest should originate in the NS-NS sector:

- H-flux H_{xyz}
- geometric flux ${\omega_{xy}}^{z}$
- nongeometric flux ${Q_x}^{yz}$
- R-flux R^{xyz}

These fluxes are related by T-duality:

$$H_{xyz} \xleftrightarrow{T_z} {\omega_{xy}}^{z} \xleftrightarrow{T_y} {Q_x}^{yz} \xleftrightarrow{T_x} R^{xyz}$$

Shelton, Taylor, Wecht - 2005

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$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ \text{flux background} & \textit{twisted torus} & \text{T-fold} & \text{not even locally geometric} \end{array}$$

Shelton, Taylor, Wecht - 2005

Goal :

- Determine properties of backgrounds for closed string noncommutativity.

$\rightarrow$ Compute a cyclic double-commutator in an H-flux background.

Plan of the talk

1. Motivation

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Consider backgrounds with H-flux.

- Due to Einstein's equation, these spaces are curved.
- Solvable examples of such NLSMs are WZW models.

A WZW model for a group *G* is described by

$$\begin{aligned}\mathcal{S} = \quad & \frac{k}{16\pi}\int_{\partial\Sigma} d^2x\, \mathrm{Tr}\left[(\partial_\alpha g)(\partial^\alpha g^{-1})\right] \\ & - \frac{ik}{24\pi}\int_{\Sigma} d^3y\, \epsilon^{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}}\, \mathrm{Tr}\left[(g^{-1}\partial_{\tilde{\alpha}} g)(g^{-1}\partial_{\tilde{\beta}} g)(g^{-1}\partial_{\tilde{\gamma}} g)\right] ,\end{aligned}$$

- where Σ is an euclidean three-manifold with boundary $\partial\Sigma$,
- $k \in \mathbb{Z}^+$ denotes the level,
- and $g \in G$.

Witten - 1984
Gepner, Witten - 1986

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The WZW model on *SU(2)* with level *k* corresponds to

- a NLSM on S^3 with radius $R = \sqrt{k}$
- and H-flux through the sphere.

Witten - 1984
Gepner, Witten - 1986

The conserved currents are ($z = \exp(x^1 + ix^2)$, σ^a... Pauli matrices)

$$J = J^a \frac{\sigma^a}{\sqrt{2}} = -k\left(\partial_z g\right) g^{-1}\,, \qquad \overline{J} = \overline{J}^a \frac{\sigma^a}{\sqrt{2}} = +k\, g^{-1}\left(\partial_{\overline{z}} g\right)\,.$$

The eom require these currents to be (anti-)holomorphic.

Therefore, one can perform Laurent expansions

- of the form

$$J^a(z) = \sum_{n\in\mathbb{Z}} j^a_n\, z^{-n-1}\,, \qquad \overline{J}^a(\overline{z}) = \sum_{n\in\mathbb{Z}} \overline{j}^a_n\, \overline{z}^{-n-1}\,,$$

- where the modes satisfy Kac-Moody algebras

$$\left[j^a_m, j^b_n\right] = i f^{ab}{}_c\, j^c_{m+n} + k\, m\, \delta_{m+n}\, \delta^{ab}\,,$$
$$\left[\overline{j}^a_m, \overline{j}^b_n\right] = i f^{ab}{}_c\, \overline{j}^c_{m+n} + k\, m\, \delta_{m+n}\, \delta^{ab}\,, \qquad f^{abc} = \sqrt{2}\,\epsilon^{abc}\,.$$

Similarly as in flat space, one can introduce fields $X^a(z,\overline{z}) = X^a(z) + \overline{X}^a(\overline{z})$ via

$$J^a(z) \;=\; -i\sqrt{k}\,\partial_z X^a(z,\overline{z})\;, \qquad\qquad \overline{J}^a(z) \;=\; -i\sqrt{k}\,\partial_{\overline{z}} X^a(z,\overline{z})\;.$$

Integrating the mode expansion of the current $J^a(z)$, one finds

$$X^a(z) = \underbrace{\frac{i}{\sqrt{k}}\,x_0^a}_{\mathbf{x}^a} - \underbrace{\frac{i}{\sqrt{k}}\,j_0^a \log z}_{\mathbf{p}^a(z)} + \underbrace{\frac{i}{\sqrt{k}}\sum_{n\neq 0}\frac{j_n^a}{n}\,z^{-n}}_{\mathbf{j}^a(z)}\;.$$

- The algebra involving x_0^a is not known.
- The modes j_n^a satisfy a Kac-Moody algebra.

Background // Three-bracket :: Computation I

Compute the holomorphic double-commutator (at equal times $|z_1| = |z_2| = |z_3|$)

$$\left[X^a(z_1), X^b(z_2), X^c(z_3)\right] = \left[\,\left[X^a(z_1), X^b(z_2)\right]\,,\; X^c(z_3)\right] + \text{cyclic}\;.$$

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- The contribution from x_0^a is so far undetermined

$$\left[\mathbf{x}^a, \mathbf{x}^b, \mathbf{x}^c\right] + \left[\mathbf{x}^a, \mathbf{x}^b, \,\cdot\,\right] + \left[\mathbf{x}^a, \,\cdot\,, \,\cdot\,\right] + \ldots = \mathcal{P}^{abc}(z_1, z_2, z_3)\ .$$

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- The bracket involving only j_0^a vanishes via the Jacobi identity

$$\big[\mathbf{p}^a(z_1), \mathbf{p}^b(z_2), \mathbf{p}^c(z_3)\big] \sim \big(f^{ab}{}_u f^{uc}{}_v + f^{bc}{}_u f^{ua}{}_v + f^{ca}{}_u f^{ub}{}_v\big)\, j_0^v = 0\;,$$

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- The bracket involving only j_n^a can be written as

$$\big[\mathbf{j}^a(z_1), \mathbf{j}^b(z_2), \mathbf{j}^c(z_3)\big] = -\frac{f^{abc}}{\sqrt{k}} \sum_{\substack{n,m\neq 0\\ n+m\neq 0}} \frac{1}{n\,m} \left(\frac{z_3}{z_1}\right)^n \left(\frac{z_3}{z_2}\right)^m + \text{cyclic}\ .$$

To evaluate the sum, recall $z_i = \exp(\tau + i\sigma_i)$ and compute

$$\Gamma(\sigma_1, \sigma_2, \sigma_3) = -\sum_{\substack{n,m\neq 0\\ n+m\neq 0}} \frac{1}{n\,m} \left(\frac{z_3}{z_1}\right)^n \left(\frac{z_3}{z_2}\right)^m + \text{cyclic}$$

$$= \begin{cases} -\pi^2 & \sigma_1 = \sigma_2 = \sigma_3\ , \\ 0 & \text{else}\ . \end{cases}$$

Combining these results with the anti-holomorphic sector, one finds

$$\big[X^a(z_1, \overline{z}_1), X^b(z_2, \overline{z}_2), X^c(z_3, \overline{z}_3)\big]$$
$$= \mathcal{P}^{abc}(z_1, z_2, z_3) + \overline{\mathcal{P}}^{abc}(\overline{z}_1, \overline{z}_2, \overline{z}_3) + 2\,\frac{f^{abc}}{\sqrt{k}}\,\Gamma(\sigma_1, \sigma_2, \sigma_3)\ .$$

Background // Three-bracket :: Final result

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- the equal-time double commutator has to vanish.

Combining these arguments, one then arrives at

$$\big[X^a(z_1,\overline{z}_1), X^b(z_2,\overline{z}_2), X^c(z_3,\overline{z}_3)\big] = \begin{cases} -\dfrac{2\pi^2}{\sqrt{k}}\, f^{abc} & \sigma_1 = \sigma_2 = \sigma_3\ , \\ \quad 0 & \text{else}\ . \end{cases}$$

Plan of the talk

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   a) Preliminaries
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Background :: Summary

To study noncommutativity for closed strings,

- one may consider backgrounds with H-, ω-, Q- or R-flux.
- Noncommutative effects can be seen already at linear order in the flux.

Background :: Agenda

1. Compute correlation functions
   - → which background?
   - → how to do computations?

2. Determine an algebra of functions
   - → mathematical properties?
   - → correct physical properties?
   - → ...

3. Construct a theory of gravity using such an algebra
   - → ...

Blumenhagen, EP - 2010
Blumenhagen, Deser, Lüst, EP, Rennecke - 2011

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CFT :: Setting

Consider the following background (as part of a bosonic string construction)

$$ds^2 = \sum_{a=1}^{3} (dX^a)^2 \,, \qquad\qquad H = \frac{2}{\alpha'^2}\, \theta_{abc}\, dX^a \wedge dX^b \wedge dX^c \,.$$

At linear order in H (and for constant dilaton Φ), the β-functions are vanishing

$$\begin{aligned} \beta^G_{ab} &= \alpha' R_{ab} - \frac{\alpha'}{4}\, H_a{}^{cd} H_{bcd} + 2\alpha' \nabla_a \nabla_b \Phi + O(\alpha'^2) \,, \\ \beta^B_{ab} &= \ldots \,. \end{aligned}$$

Goal: construct a CFT describing this background (at linear order in H).

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Goal: construct a CFT describing this background (at linear order in H).

→ Using conformal perturbation theory,

→ determine two- and three-point functions.

- The sigma-model action for the closed string reads

$$\mathcal{S} = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z \left(g_{ab} + B_{ab} \right) \partial X^a \, \overline{\partial} X^b \ ,$$

which (in a convenient gauge) can be written as

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1 \qquad \text{with} \qquad \mathcal{S}_1 = \frac{1}{2\pi\alpha'} \frac{H_{abc}}{3} \int_\Sigma d^2z \, X^a \partial X^b \, \overline{\partial} X^c \ .$$

- Correlation functions of operators $\mathcal{O}_i$ are computed as

$$\big\langle \mathcal{O}_1 \ldots \mathcal{O}_N \big\rangle = \frac{1}{\mathcal{Z}} \int [dX] \, \mathcal{O}_1 \ldots \mathcal{O}_N \, e^{-\mathcal{S}[X]} \ ,$$

which can be expanded as

$$\big\langle \mathcal{O}_1 \ldots \mathcal{O}_N \big\rangle = \big\langle \mathcal{O}_1 \ldots \mathcal{O}_N \big\rangle_0 - \big\langle \mathcal{O}_1 \ldots \mathcal{O}_N \, \mathcal{S}_1 \big\rangle_0 + \mathcal{O}\big(H^2\big) \ .$$

CFT :: Two- and three-point functions

Suitable holomorphic and antiholomorphic conformal fields are

$$\mathcal{J}^a(z) = i\,\partial X^a(z,\overline{z}) - \tfrac{1}{2}\,H^a{}_{bc}\,\partial X^b(z,\overline{z})\,X_R^c(\overline{z}) \ ,$$
$$\overline{\mathcal{J}}^a(\overline{z}) = i\,\overline{\partial} X^a(z,\overline{z}) - \tfrac{1}{2}\,H^a{}_{bc}\,X_L^b(z)\,\overline{\partial} X^c(z,\overline{z}) \ .$$

The nonvanishing (holomorphic) two- and three-point functions read

$$\big\langle \mathcal{J}^a(z_1)\mathcal{J}^b(z_2)\big\rangle = \frac{\alpha'}{2}\,\frac{1}{(z_1-z_2)^2}\,\delta^{ab} \ ,$$
$$\big\langle \mathcal{J}^a(z_1)\,\mathcal{J}^b(z_2)\,\mathcal{J}^c(z_3)\big\rangle = -i\,\frac{{\alpha'}^2}{8}\,H^{abc}\,\frac{1}{z_{12}\,z_{23}\,z_{13}} \ .$$

The OPE derived from these correlators reads (up to linear order in H)

$$\mathcal{J}^a(z_1)\,\mathcal{J}^b(z_2) = \frac{\alpha'}{2}\,\frac{\delta^{ab}}{(z_1-z_2)^2} - \frac{\alpha'}{4}\,\frac{i\,H^{ab}{}_c}{z_1-z_2}\,\mathcal{J}^c(z_2) + \text{reg.}\ .$$

The energy-momentum tensor (at linear order in H) has the form

$$\mathcal{T}(z) = \frac{1}{\alpha'}\,\delta_{ab}\,{:}\mathcal{J}^a\mathcal{J}^b{:}(z)\ .$$

→ The central charge is that of the free theory.
→ The fields $\mathcal{J}^a$ are primary with dimension (1,0).

Nahm - 1989

Consider the integrated currents $\mathcal{X}_L^a(z)$ (and $\mathcal{X}_R^a(\overline{z})$) with OPE

$$\mathcal{J}^a(z_1)\,\mathcal{X}_L^b(z_2) = -i\,\frac{\alpha'}{2}\,\frac{\delta^{ab}}{z_1 - z_2} + \frac{\alpha'}{4}\,H^{ab}{}_c\,\mathcal{J}^c(z_2)\,\log(z_1 - z_2) + \text{reg.}\ .$$

Define vertex operators for the CFT at linear order in H as

$$\mathcal{V}(z,\overline{z}) = :\exp\big(ik_L\cdot\mathcal{X}_L + ik_R\cdot\mathcal{X}_R\big):\,, \qquad k^a_{L/R} = p^a \pm \frac{w^a}{\alpha'}\ .$$

For the tachyon with $k^2_{L/R} = \frac{4}{\alpha'}$, the vertex operator is primary and has conformal dimensions (1,1).

CFT :: Summary

For a flat background with H-flux

- we have constructed a CFT at linear order in the flux,
- and defined a vertex operator for the tachyon (of the bosonic string).

Plan of the talk

1. Motivation
2. Background
3. CFT
4. Some results

Results :: T-duality

The CFT is defined for H-flux – other flux-backgrounds are related via T-duality.

T-duality can be realized on the world-sheet by

$$\begin{matrix} \mathcal{X}^a_L(z) \\ \mathcal{X}^a_R(\overline{z}) \end{matrix} \quad \xrightarrow{\text{T-duality}} \quad \begin{matrix} +\mathcal{X}^a_L(z) \\ -\mathcal{X}^a_R(\overline{z}) \end{matrix}$$

Vertex operators with momentum $p^a \neq 0$ and winding $w_a = 0$ are only well-defined for H- and R-flux.

Results :: Three-tachyon correlator

First, define a function $\mathcal{L}(z)$ in terms of the Rogers dilogarithm $L(z)$

$$\mathcal{L}(z) = L(z) + L\big(1 - \tfrac{1}{z}\big) + L\big(\tfrac{1}{1-z}\big) \;, \qquad L(z) = \mathrm{Li}_2(z) + \tfrac{1}{2}\log(z)\log(1-z) \;.$$

The three-tachyon correlator (at linear order in the flux) then reads

$$\big\langle\, \mathcal{V}_1\, \mathcal{V}_2\, \mathcal{V}_3 \,\big\rangle^{H/R} = \exp\Big[-i\theta^{abc}\, p_{1,a}p_{2,b}p_{3,c} \big[\mathcal{L}\big(\tfrac{z_{12}}{z_{13}}\big) \mp \mathcal{L}\big(\tfrac{\overline{z}_{12}}{\overline{z}_{13}}\big) \big] \Big]_\theta \big\langle\, \mathcal{V}_1\, \mathcal{V}_2\, \mathcal{V}_3 \,\big\rangle_0^{H/R} \;.$$

Using momentum conservation, the flux-dependent part vanishes (at linear order).

Results :: Permutations and tri-product

When permuting two vertex operators,

- in the case of H-flux, tachyon correlators are invariant;
- in the case of R-flux, one finds

$$\big\langle\, \mathcal{V}_2 \mathcal{V}_1 \mathcal{V}_3 \big\rangle^R = \exp\Big[\, i\,\pi^2\,\theta^{abc}\, p_{1,a}\, p_{2,b}\, p_{3,c} \Big]_\theta \big\langle \mathcal{V}_1\, \mathcal{V}_2\, \mathcal{V}_3 \big\rangle^R \;.$$

One can then define a tri-product as

$$\begin{aligned} &f_1(x) \vartriangle f_2(x) \vartriangle f_3(x) := \\ &\qquad \exp\Big(\tfrac{\pi^2}{2}\, \theta^{abc}\, \partial_a^{x_1}\, \partial_b^{x_2}\, \partial_c^{x_3} \Big)\, f_1(x_1)\, f_2(x_2)\, f_3(x_3) \Big|_{x_1=x_2=x_3=x} \;. \end{aligned}$$

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The tri-product correctly reproduces the three-bracket via

$$\big[X^a, X^b, X^c\big] = \sum_{\sigma\in P_3} \mathrm{sign}(\sigma)\; X^{\sigma(a)} \vartriangle X^{\sigma(b)} \vartriangle X^{\sigma(c)} \,.$$

Results :: Summary

Employing the previously defined CFT up to linear order in the flux,

- from tachyon correlation functions
- a tri-product on functions has been defined for the case of R-flux.

$$\boxed{\begin{aligned} & f_1(x) \vartriangle f_2(x) \vartriangle f_3(x) := \\ & \qquad \exp\Big(\tfrac{\pi^2}{2}\, \theta^{abc}\, \partial^{x_1}_a\, \partial^{x_2}_b\, \partial^{x_3}_c \Big)\, f_1(x_1)\, f_2(x_2)\, f_3(x_3) \Big|_{x_1=x_2=x_3=x} \,. \end{aligned}}$$

Results :: Agenda

1. Compute correlation functions	→ which background? → how to do computations?
2. Determine an algebra of functions	→ mathematical properties? → correct physical properties? → ...
3. Construct a theory of gravity using such an algebra	→ ...

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Further results :: Asymmetric strings

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- revealing new tachyons in the spectrum (of the bosonic string).

Conjecture:

- The instability indicated by the tachyons points towards a relation
- between left-right-asymmetric string and R-flux backgrounds.

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