Constraints for quartic couplings in Inert Doublet Model

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collaboration with I.F. Ginzburg , K.A. Kanishchev and M. Krawczyk
[Phys.Rev.D82:123533,2010, arXiv:hep-ph/1009.5099, 1104.3326, 1107.1991]

Motivation

T.D. Lee, '73; Deshpande, Ma, '78

Barbieri, Hall, Rychkov '06; Cao, Ma, Rajasekaran '07

Two Higgs Doublet Model (2HDM):

- two scalar $SU(2)_W$ doublets Φ_S, Φ_D with the same hypercharge $Y = 1$
- rich phenomenology: CP violation in the scalar sector, heavy Higgs, ...
- **different types of extrema** (i.e. possible violation of $U(1)_{EM}$ or CP)
  → evolution of vacuum state in the past
- 2HDM with an exact Z_2 symmetry: Inert Doublet Model (IDM)
  → candidate for the dark matter
- Testing Inert Doublet Model:
  - properties of SM-like Higgs h_S (light and heavy)
  - properties of dark scalars $D_H, D_A, D^{\pm}$
  - collider constraints
  - astrophysical data

2HDM

Scalar potential V invariant under a D-transformation of Z_2 type:

$$D: \quad \Phi_S \to \Phi_S, \quad \Phi_D \to -\Phi_D, \quad \text{SM fields} \to \text{SM fields}$$

$$V = -\frac{1}{2}\left[m_{11}^2 \Phi_S^\dagger \Phi_S + m_{22}^2 \Phi_D^\dagger \Phi_D\right] + \frac{1}{2}\left[\lambda_1 \left(\Phi_S^\dagger \Phi_S\right)^2 + \lambda_2 \left(\Phi_D^\dagger \Phi_D\right)^2\right]$$
$$+\lambda_3 \left(\Phi_S^\dagger \Phi_S\right)\left(\Phi_D^\dagger \Phi_D\right) + \lambda_4 \left(\Phi_S^\dagger \Phi_D\right)\left(\Phi_D^\dagger \Phi_S\right) + \frac{1}{2}\lambda_5 \left[\left(\Phi_S^\dagger \Phi_D\right)^2 + \left(\Phi_D^\dagger \Phi_S\right)^2\right]$$

- explicit D-symmetry
- all parameters $\in \mathbf{R}$ – no CP violation
- Yukawa interaction: Model I, only Φ_S couples to fermions

The positivity constrains are required to have a stable vacuum:

$$\lambda_1 > 0\,, \quad \lambda_2 > 0, \quad R + 1 > 0, \quad R_3 + 1 > 0$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad R = \lambda_{345}/\sqrt{\lambda_1 \lambda_2}, \quad R_3 = \lambda_3/\sqrt{\lambda_1 \lambda_2}$$

Positivity constrains → extremum with the lowest energy is the global minimum (vacuum).

Other constraints: perturbativity, unitarity

Spontaneous Symmetry Breaking

The EW symmetric extremum:

$$\langle \Phi_S \rangle = \langle \Phi_D \rangle = 0$$

local minimum if $m^2_{11,22} < 0$.

Barroso et al., '05

The general type of EWSB v.e.v:

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v_S \end{array} \right), \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} u \\ v_D \end{array} \right)$$

$u \neq 0 \Longrightarrow U(1)_{EM}$ broken:

- charge breaking extremum (Ch)

$u = 0 \Longrightarrow U(1)_{EM}$ conserved:

- $v_{S,D} \neq 0$ neutral mixed extremum (M)
- $v_D = 0$ neutral inert extremum (I_1)

  Deshpande, Ma, '78; Barbieri, Hall, Rychkov, '06
- $v_S = 0$ neutral inertlike extremum (I_2)

Extrema: charge breaking and mixed

Charge breaking extremum Ch:

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \end{pmatrix}$$

- $U(1)_{EM}$ symmetry broken by $u \neq 0$ – massive photon
- not a case that is realized now, a possible vacuum in the past if

$$\lambda_4 \pm \lambda_5 > 0, \quad R_3 < 1$$

- requires a charged "dark matter" particle

  $\rightarrow$ evolution through Ch excluded

Ginzburg, Kanishchev, Krawczyk, Sokołowska '10

Mixed extremum M:

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_D \end{pmatrix}$$

- CP conserving, $\tan\beta = v_D/v_S$
- massive $Z^0, W^\pm$, massless photon, 5 physical Higgs bosons $H^\pm, A, H, h$, **no DM candidate**

$$R < 1, \quad \lambda_4 + \lambda_5 < 0, \quad \lambda_5 < 0$$

Extrema: inert and inertlike

Deshpande, Ma, '78, Barbieri et al., '06

Inert extremum I_1:

$$\langle\Phi_S\rangle = \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle\Phi_D\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Φ_S as in SM (SM-like Higgs boson h_S)

  Φ_D – "dark" or inert doublet with 4 dark scalars $(D_H, D_A, D^{\pm})$, no interaction with fermions
- exact D-symmetry – both in Lagrangian and in the extremum
- only Φ_D has odd D-parity

  $\rightarrow$ the lightest scalar is **a candidate for the dark matter**

Inertlike extremum I_2:

$$\langle\Phi_S\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle\Phi_D\rangle = \begin{pmatrix} 0 \\ v_D \end{pmatrix}$$

- Φ_S and Φ_D exchange roles
- fermions massless at tree-level (Model I, only Φ_S couples to fermions)
- **no DM candidate**

Evolution of the Universe

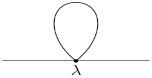
Ivanov '08; Ginzburg, Kanishchev, Ivanov '09
Ginzburg, Kanishchev, Krawczyk, Sokołowska '10

- assumption: today Inert Model is realized, however, in the past some other extrema could have been lower

- evolution of the Universe due to the thermal corrections to the potential

$$V_G(T) = Tr(Ve^{-H/T})/Tr(e^{-H/T}) \equiv V(T=0) + \Delta V(T)$$

- $\Delta V(T)$ – leading corrections $\propto T^2$ given by diagrams:



$\Longrightarrow$ fixed quartic terms, quadratic (mass) terms change with T

$$\Delta V(T) = \frac{1}{2}c_1 T^2 \Phi_S^\dagger \Phi_S + \frac{1}{2}c_2 T^2 \Phi_D^\dagger \Phi_D$$

- **change of potential → change of ground state**

Evolution of the Universe

From scalar, bosonic and fermionic contributions to $\Delta V \rightarrow m_{ii}^2(T)$:

$$m_{11}^2(T) = m_{11}^2 - c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2 - c_2 T^2$$

$$c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{6} + \frac{3g^2 + g'^2}{8} + \frac{g_t^2 + g_b^2}{2}, \quad c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{6} + \frac{3g^2 + g'^2}{8}$$

- fermionic contribution in c_1 (Model I)
- $c_1 + c_2 > 0$ from positivity constrains
- c_1 and c_2 positive to restore EW symmetry in the past

For a given T we determine:

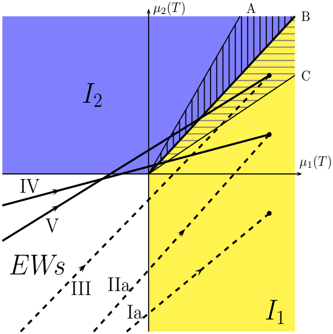
- sign of $v_i^2\big|_{I_1, I_2, M}$ $\rightarrow$ possible existence of a given extremum
- values of λ_i (fixed) $\rightarrow$ existence of a given local minimum
- value of extremum energy $\rightarrow$ global minimum

$\Rightarrow$ **sequences of possible phase transitions**

Possible rays

The possible sequences of phase transitions (**rays**) on $(\mu_1(T), \mu_2(T))$ plane:

$$\mu_1(T) = m_{11}^2(T)/\sqrt{\lambda_1}, \quad \mu_2(T) = m_{22}^2(T)/\sqrt{\lambda_2}.$$



$R > 1$

horizontal hatch – I_1 global, I_2 local min,
vertical hatch – I_2 global, I_1 local min;

$A: \ \mu_2(T) = \mu_1(T)R,$
$B: \ \mu_2(T) = \mu_1(T),$
$C: \ \mu_2(T) = \mu_1(T)/R$

$EWs \to I_1$

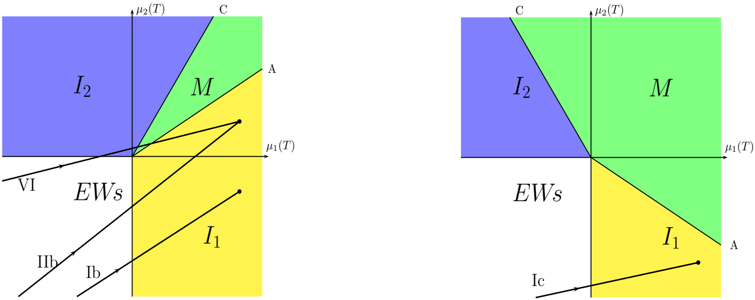
- ray **Ia** – I_2 is not an extremum
- ray **IIa** – I_2 is an extremum, but never was a (local) minimum
- ray **III** – I_2 is a **local** minimum, but never was a global minimum

$EWs \to I_2 \to I_1$

- ray **IV** – I_2 is not a local minimum, but was a global minimum in the past
- ray **V** – I_2 is a **local** minimum, it was a global minimum in the past

Possible rays

$0 < R < 1$ $-1 < R < 0$



$$EWs \to I_1$$

- rays **Ib, Ic** – I_2 is not an extremum
- rays **IIb** – I_2 is an extremum, but never was a (local) minimum

$$EWs \to I_2 \to M \to I_1$$

- ray **VI** – I_2, M were global minima in the past

Physical parameters

$$(\mu_1,\ \mu_2,\ R,\ c_1,\ c_2) \rightarrow (M_{h_S},\ M_{D^\pm},\ M_{D_A},\ M_{D_H},\ \lambda_{345},\ \lambda_2)$$

λ_{345} – **triple** and **quartic** coupling: $D_H D_H h_S$ and $D_H D_H h_S h_S$

- main annihilation channel $D_H D_H \rightarrow h_S \rightarrow f\bar{f}$:

  $\sigma \propto \lambda_{345}^2/(4M_{D_H}^2 - M_{h_S}^2)^2 \Rightarrow$ constraints from relic density data

- DM-nucleon elastic scattering via h_S:

  $\sigma_{DM,N} \propto \lambda_{345}^2/(M_{D_H} + M_N)^2 \Rightarrow$ direct detection experiments

λ_2 – **quartic** self-coupling $D_H D_H D_H D_H$

- no influence on DM relic density
- non-accesible in the colliders
- limits λ_{345} through positivity constraints

$(\lambda_{345}, \lambda_2)$ phase space

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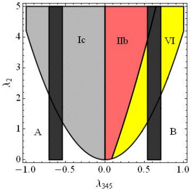
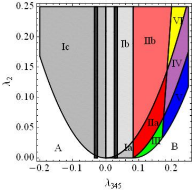
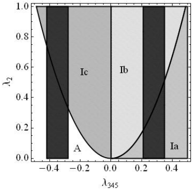
- **fixed** scalar masses, λ_{345} and λ_2 **vary**
- conditions for $(\mu_1, \mu_2) \rightarrow$ conditions for $(\lambda_{345}, \lambda_2)$
- each ray in separate region
- change of $M_{D_H} \rightarrow$ different rays possible



low DM mass



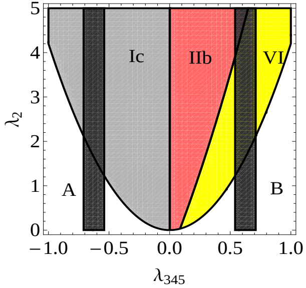
medium DM mass



high DM mass

Low DM mass region

- low DM mass $M_{D_H} \approx (3-8)$ GeV
- large mass splittings: $M_{D_A} \approx M_{D^\pm} \approx 100$ GeV
  $\rightarrow$ no coannihilation, mimics singlet DM
- direct detection measurements: excluded by XENON100, allowed by CRESST-II, DAMA/Libra and CoGent



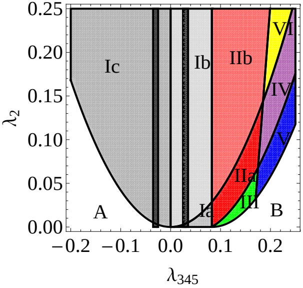
- limited number of rays
- large λ_2 needed, rather large λ_{345}
- T of final transition lower for large λ_{345}
  $\rightarrow T_{M,1} \approx M_{D_H}$
  $\rightarrow$ next order of correction to V needed

A – excluded by positivity constraints
B – I_2 is a global minimum
vertical bounds – WMAP-allowed region

$$M_{D_H} = 5 \text{ GeV},\ M_{D_A} = 105 \text{ GeV},\ M_{D^\pm} = 110 \text{ GeV},\ M_{h_S} = 120 \text{ GeV}$$

Medium DM mass region

- medium DM mass $M_{D_H} \approx (45 - 160)$ GeV
- large $(D_H, D^{\pm})$ mass splittings: $M_{D^\pm} - M_{D_H} \approx (50 - 90)$ GeV
  - **large** (D_H, D_A) mass splitting: $M_{D_A} - M_{D_H} \approx (50 - 90)$ GeV
    $\Rightarrow$ **no coannihilation**
  - **small** (D_H, D_A) mass splitting: $M_{D_A} - M_{D_H} < 8$ GeV
    $\Rightarrow$ **coannihilation**



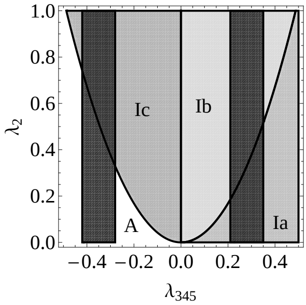
- all rays possible for all masses
- $\Omega_{DM} h^2$: strong dependence on M_{D_H} and $(M_{D_A} - M_{D_H})$
- for rays VI, IV, V – low T of final phase transition possible
- rays IV and V – 1st order phase transition

A – excluded by positivity constraints
B – I_2 is a global minimum
vertical bounds – WMAP-allowed region

$M_{D_H} = 50$ GeV, $M_{D_A} = 120$ GeV, $M_{D^\pm} = 120$ GeV, $M_{h_S} = 120$ GeV

High DM mass region

- high DM mass $M_{D_H} \approx (500 - 1000)$ GeV
- small mass splittings: $M_{D_H} \approx M_{D_A} \approx M_{D^\pm}$
- coannihilation between all dark particles
- only light Higgs h_S possible (EWPT)



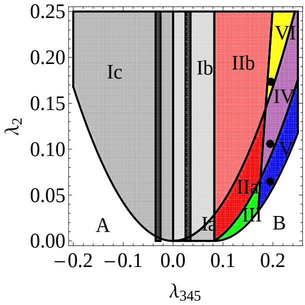
- only rays Ia, Ib, Ic
- other rays require $\lambda \approx O(20)$
- further limits from unitarity

A – excluded by positivity constraints
vertical bounds – WMAP-allowed region

$$M_{D_H} = 800 \text{ GeV}, \ M_{D_A} = 801 \text{ GeV}, \ M_{D^\pm} = 801 \text{ GeV}, \ M_{h_S} = 120 \text{ GeV}$$

Medium DM mass: example

Ray no.	λ_2 region
	$EWs \to I_2 \to I_1$
IV	$\alpha\lambda_{345} < \lambda_2 < \text{Min}\left(\tilde{\lambda}_2, \frac{\lambda_{345}^2 v^2}{M_{h_s}^2}\right)$
V	$\frac{\alpha^2 M_{h_S}^2}{v^2} < \lambda_2 < \text{Min}\left(\tilde{\lambda}_2, \alpha\lambda_{345}\right)$
	$EWs \to I_2 \to M \to I_1$
VI	$\frac{\lambda_{345}^2 v^2}{M_{h_s}^2} < \lambda_2 < \tilde{\lambda}_2$



$\tilde{\lambda}_2 = \tilde{\lambda}_2(M_i, g_i, \lambda_{345}), \quad \alpha = (v^2\lambda_{345} - 2M_{D_H}^2)/M_{h_s}^2$

$M_{D_H} = 50$ GeV, $M_{D_A} = 120$ GeV, $M_{D^\pm} = 120$ GeV, $M_{h_S} = 120$ GeV, $\lambda_{345} = 0.1945$

Rays may differ only by value of λ_2:

IV	$\lambda_2 = 0.1031$	$T_{EWSB} = 131.7$ GeV, $T_{2,1} = 107.5$ GeV
V	$\lambda_2 = 0.0684$	$T_{EWSB} = 134.8$ GeV, $T_{2,1} = 83.7$ GeV
VI	$\lambda_2 = 0.1672$	$T_{EWSB} = 126.7$ GeV, $T_{2,M} = 119.4$ GeV, $T_{M,1} = 119.0$ GeV

Conclusions

- Today – Inert Model (dark matter).
- Different types of extrema can be realized in the past.
- Possible sequences of phase transitions:

$$EWs \to I_2 \to M \to I_1$$
$$EWs \to I_2 \to I_1$$
$$EWs \to I_1$$

- λ_2 important for the evolution.
- Different behaviour for low, medium and high DM mass region.
- Need for the further corrections to V.