Quantum Nambu Geometry from D1-Strings in Large RR Flux

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Outline

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Quantized spacetime

- It is generally expected that the usual description of spacetime in terms of a Riemannian geometry would break down above the Planck energy scale.
- A possibility is that geometry is quantized and spacetime coordinates become quantum operators.
- In this case, traditional spacetime concepts such as locality and causality and even the fundamental nature of spacetime itself, will have to be re-examined.

Q. What can one learn from string theory?

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Q. What can one learn from string theory?

Lie algebra type NCG in string: $B - field$

- Example 1: The worldvolume of a D-brane become noncommutative when a constant NSNS B-field is turned on:

$$\text{Noncommutative geometry:} \quad [X^i, X^j] = i\theta^{ij}.$$

Connes, Douglas, Schwarz (1997); Douglas, Hull (1997); Chu, Ho (1998); Schomerus (1999); Seiberg, Witten (1999)

- This result can be derived by quantizing open string in NSNS B-field. One obtains:
  1. the NCG for the endpoint of the open string, i.e. the D-brane.
  2. moreover one obtains automatically the open string metric.

Chu, Ho (1998)

Lie algebra type NCG in string: Marix model

- Example 2: NCG culd be obtained as classical solution of equation of motion of BFSS or IKKT matrix models
- BFSS matrix model

$$S = \int dt (D_t X^I)^2 - [X^I, X^J]^2 + \text{fermions}$$

- EOM

$$D_t^2 X^I - [X^J, [X^I, X^J]] = 0$$

has time independent soln

$$[X^I, X^J] = i\theta^{IJ}.$$

These can be interpretrated as the spacetime coordinates of the worlvolume of a D-brane in B-field. Seiberg (2000)

Lie algebra type NCG in string: Myers effect

- Example 3: NCG also arises due to Myers effect
- In the presence of a RR-flux, a collection of D0-branes can expand and become a D2-brane whose worldvolume is described by a fuzzy sphere

$$[X^i, X^j] = i\epsilon^{ijk} X^k, \quad X^2 = R^2.$$

Myers (1999)

- also derived using open string without using Myers effective action

Alekseev, Recknagel, Schomerus (2000)

All very nice!
But ....

Q2. Any example which is not Lie-algebraic type?

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Q2. Any example which is not Lie-algebraic type?

3-bracket geometry: M5 brane with constant 3-form C

- Aim was achieved partially recently
- Consistency between the different descriptions of the M2-M5 intersecting branes system implies that the M5-brane geometry in the presence of a constant 3-form C-field takes the form of

$$[X^i, X^j, X^k] = i\theta\epsilon^{ijk}, \quad i, j, k = 2, 3, 4$$

Chu, Smith (2009)

- The reason why a Lie 3-bracket appears is because the geometry of the M5-brane was inferred from the boundary dynamics of the open M2-branes (BLG theory) which end on it
- In a QFT, necessary to understand the relation as an operator relation. However so far no representation of Lie 3-algebra have been constructed.

Need to consider different possibilities

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Quantum Nambu geometry from D1-branes in large RR-flux

- We will show that:
  the 3-bracket geometry arises as classical solution of matrix model of D1-strings in a backgrond of large RR 3-form field strength due to Myer effects
- The 3-bracket is given by

$$[f, g, h] := fgh + ghf + hfg - fhg - gfh - hgf \, ,$$

  defined on ordinary operators.
- This 3-bracket was originally proposed by Nambu (1973), so we will refer the geometry as Quantum Nambu geometry

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Exact IIB background with F_3 flux

- An exact IIB supergravity background with constant RR F_3 flux can be constructed Chu, Ho (2010)
- start with $AdS_5 \times S^5$ with

$$F_5 = \begin{cases} c\epsilon_5 & 01234, \\ c\epsilon_5' & 56789, \\ 0 & \text{otherwise} \end{cases}$$

and turns on a costant F_3 flux in the 123 diretion

$$F_3 = f\epsilon_{ijk}, \quad i,j,k = 1,2,3,$$

- Both F_3 and F_5 contributes a term like a cosmological constant to the Einstein equation.

Exact IIB background with F_3 flux

- Can adjust $f \sim c$ to balanced out the contribution of F_5 to the Einstein equation and flatten a 3d part of AdS_5 to $R^3 \times AdS_2$.
- The metric is $R^3 \times AdS_2 \times S^5$

$$ds^2 = \sum_{i=1}^{3} (dX^i)^2 + R^2 \left(\frac{-dt^2 + dU^2}{U^2} \right) + R'^2 d\Omega_{S^5}^2,$$

where

$$R^2 = 2e^{-2\Phi}/f^2, \qquad R'^2 = 40R^2,$$

- Nonsupersymmetric due to nonzero F_3.

D1-strings action

- The worldvolume action for the D1-branes is given by the Non-abelian Born-Infeld action plus the Chern-Simons term of the Myers type
- $$S_{CS} = \mu_1 \int \mathrm{Tr} P(e^{i\lambda \mathrm{i}_\Phi \mathrm{i}_\Phi} \sum_n C_n) e^{\lambda F}.$$

Myers (1999)

- For us,

$$\begin{aligned} S_{CS} &= \mu_1 \int \mathrm{Tr} \left[\lambda F \chi + P\, C_2 + i\lambda^2 F \mathrm{i}_\Phi \mathrm{i}_\Phi C_2 + i\lambda P\, \mathrm{i}_\Phi \mathrm{i}_\Phi C_4 - \frac{\lambda^3}{2} F \mathrm{i}_\Phi^4 C_4 \right] \\ &:= S_\chi + S_{C_2} + S_{C_4}, \end{aligned}$$

where, $C_2 = f \epsilon_{ijk} X^i dX^j dX^k$.

D1-strings action

- Nonabelian Born-Infeld in curved space is much less understood
- Two sources of difficulties:
  1, amguigity associated with ordering of F
  2. incoporation of a curved metric $G_{IJ}(X)$
- A natural proposal is to promote the metric to become a matrix $G_{IJ}(X)$ and to incooporate the effect of curved space with the action,

$$S_X/\mu_1 := \int d^{p+1}\sigma \sqrt{-\det G_{\alpha\beta}} \left(G_{IJ}(X) D_\alpha X^I D_\beta X^J G^{\alpha\beta} + \frac{1}{\alpha'} G_{IJ}(X) G_{KL}(X) [X^I, X^K][X^J, X^L] \right),$$

  It was proposed that the ambiguities in $G_{IJ}(X)$ could be resolved by requiring that the IR gravitational physics be correctly reproduced.
- For $p = 0$, it was proposed that the action gives the Matrix theory in curved space.

Douglas (1999)

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D1-strings action

- Full D1 strings action is

$$S := S_X + S_{CS} + S_{YM},$$

where the Yang-Mills term is

$$S_{YM}/\mu_1 = \alpha'^2 \int \sqrt{-\det G_{\alpha\beta}}\, F_{\alpha\beta} F_{\alpha'\beta'} G^{\alpha\alpha'} G^{\beta\beta'}$$

- Due to the specific form of our background, the scalars $X^i (i = 1, 2, 3)$ and $X^{i'} (i' = 5, 6, 7, 8, 9$, sphere directions) decoupled from each other.
- Consistent to set $X^{i'} = 0$ and focus on the sector of X^i and gauge fields.

Large F_3 double scaling limit

- Turns out there is a double scaling limit $\epsilon \to 0$:

$$\begin{aligned} \alpha' &\sim \epsilon, \\ f &\sim \epsilon^{-a}, \quad a > 0, \end{aligned}$$

with $1/2 < a < 2$, the low energy action of N D1-branes in a large F_3 background is given by the C_2 terms S_{C_2}:

$$\mu_1 f \int d^2\sigma \mathrm{Tr}(\frac{1}{2}\epsilon_{ijk} X^i D_\alpha X^j D_\beta X^k \epsilon^{\alpha\beta}) + \mu_1 f \int d^2\sigma \mathrm{Tr}(iFX^i X^j X^k \epsilon_{ijk})$$

such that the dominant terms in the EOM are reproduced.

- This matrix model described the physics of D1-strings in a large F_3 background.

Remarks:

- The dominance of the system by a topological term is similar to what happened in the discussions of Susskind etal (2001) where the effects of a Lorentz force term

$$L = \frac{\mu_0 H_3}{2} \epsilon_{ij} \mathrm{Tr} X^i D_t X^j, \quad i = 1, 2,$$

  on the physics of a system of N D0-branes dissolved in a D2-brane (whose spatial directions are $i = 1, 2$) was studied.
- The equation of motion of L is solved with any configuration $D_t X^i = 0$.
- A speciifc solution $[x^i, x^j] = i\theta\epsilon^{ij}$ which corresponds to a D2-brane charge density were considered.

Quantum Nambu geometry as classical solution

- The matrix model has classical solution

$$D_\alpha X^i = 0. \quad F = 0.$$

- Moyal type noncommutative geometry

$$[X^i, X^j] = i\theta^{ij}$$

is allowed. But there is also the new solution

$$[X^i, X^j, X^k] = i\theta\epsilon^{ijk},$$

where θ is a constant and the 3-bracket is given by the Quantum Nambu bracket.

Remark 1:

- The 3-bracket was originally introduced by Nambu as a possible candidate of the quantization of the classical Nambu bracket

$$\{f,g,h\} := \epsilon^{ijk}\partial_i f \partial_j g \partial_k h.$$

- Nambu was interested in generalizing the Hamiltonian mechanics to the form (Nambu mechanics)

$$\frac{df}{dt} = \{H_1, H_2, f\},$$

  which involves two "Hamiltonians" H_1, H_2. Fundamental identity was not needed in his consideration (in fact not satisfied).

- The concept of fundamental identity was introduced almost 20 years later by Takhtajan (and Baryen and Flato independently) as a natural condition for his definition of a Nambu-Poisson manifold.
- This allowed him to formulate the Nambu mechanics in an invariant geometric form similar to that of Hamiltonian mechanics.

Remark 2:

- In attempt to intrepret the M5-brane geometry in C-field

$$[X^i, X^j, X^k] = i\theta^{ijk}$$

The quantization of the Lie 3-bracket was considered recently:

- - Quantization of Nambu-Poisson manifold
DeBellis, Samann, Szabo (2011).
- Construction of representation of Lie 3-algebra in terms of a Cartan-Weyl basis Chu (2010).

For us here,
- no need to worry about fund identity.
- Quantization is made ready as 3-bracket is written in terms of operators.

Need only to construct representation of these operators

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Finite $SU(2)$ representations

- An intermediate observation is that if we start with the standard $SU(2)$ algebra
$$[l_i, l_j] = i\epsilon_{ijk} l_k,$$
then with $X^i = \alpha l_i$ for a constant α, we have
$$[X^i, X^j, X^k] = i\alpha^2 C_{\mathrm{R}},$$
where C_{R} is the quadratic Casmir for the representation R where X^i is in.
- For $N \times N$ matrices, $C_N = (N^2 - 1)/4$ and so if we choose $\alpha^2 = \theta/C_N$, then we can realize the NH relation with $N \times N$ matrices.

In what sense we have a new quantized geometry?

Finite $SU(2)$ representations

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$$[I_i, I_j] = i\epsilon_{ijk} I_k ,$$
then with $X^i = \alpha I_i$ for a constant α, we have
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- For $N \times N$ matrices, $C_N = (N^2 - 1)/4$ and so if we choose $\alpha^2 = \theta / C_N$, then we can realize the NH relation with $N \times N$ matrices.

In what sense we have a new quantized geometry?

- Turns out in the large N limit, there are new representations that cannot be considered as limits of the above form, i.e. not representations of $SU(2)$ or any Lie algebra.
- It is the existence of these new representations that demonstrates the indecomposable and fundamental nature of the Nambu-Heisenberg commutation relation.

Infinite dimesnional representation 1: $Z, \overline{Z}, X$

Two example of representations:

- Hermitian X^i's and introduce the complex coordinates

$$Z := X^1 + iX^2, \quad \overline{Z} := X^1 - iX^2, \quad X := X^3.$$

  Then $[X, Z, \overline{Z}] = 2\theta$.

- Consider an ansatz

$$\begin{aligned} Z|\omega\rangle &= f_1(\omega)|\omega+\beta\rangle + f_2(\omega)|\omega-\beta\rangle, \\ \overline{Z}|\omega\rangle &= f_2^*(\omega+\beta)|\omega+\beta\rangle + f_1^*(\omega-\beta)|\omega-\beta\rangle, \\ X|\omega\rangle &= g(\omega)|\omega\rangle, \end{aligned}$$

- The introduction of $Z, \overline{Z}$ is motivated by the creation and annhiliation operators for the Heisenberg commutation relation. Thus natural to consider the representation with $f_2 = 0$ or $f_1 = 0$. However this gives a constraint of the form

$$Z\overline{Z} + \overline{Z}Z = \mathcal{Z}(X)$$

  for some function $\mathcal{Z}$ and so describes at most a 2-dimensional spaces. Therefore we are prompt to the above ansatz.

Infinite dimesnional representation 1: $Z, \overline{Z}, X$

- Skipping the details, a solution can be stated

$$g(\omega) = \cos \alpha\omega,$$

$$|f_1(\omega)|^2 = |f_2(\omega)|^2 = k_0 - \frac{4\theta}{3} \cos \alpha\omega,$$

  where $k_0 > 4\theta/3$, $\alpha = \frac{\pi}{3\beta}(6n \pm 1)$, $n \in \mathbb{Z}$.
- The representation space is given by the 1-dimensional lattice

$$\{|\omega + n\rangle : n \in \mathbb{Z}\}$$

  and is of countably infinite dimension.

Infinite dimesnional representation 2: Z_3 symmetry

- Let $\rho = e^{2\pi i/3}$, cubic root of unity and consider

$$
\begin{aligned}
X_1|\omega\rangle &= (\omega + a)|\omega + 1\rangle, \\
X_2|\omega\rangle &= \rho^2(\omega + a\rho)|\omega + \rho\rangle, \\
X_3|\omega\rangle &= \rho(\omega + a\rho^2)|\omega + \rho^2\rangle.
\end{aligned}
$$

  it is

$$
[X_1, X_2, X_3]|\omega\rangle = 3(a^2 - a)(\rho - \rho^2)|\omega\rangle,
$$

  Thus so long as $a \neq 1$, we have a nonzero θ.

- In this representation the fields X_1, X_2, X_3 are not hermitian. They are however related through a unitary transformation,

$$
\begin{aligned}
X_1 &= U^\dagger X_3 U, \\
X_2 &= U^\dagger X_1 U, \\
X_3 &= U^\dagger X_2 U,
\end{aligned}
$$

  where

$$
U|\omega\rangle = |\rho^2\omega\rangle,
$$

Infinite dimesnional representation 2: Z_3 symmetry

- In this construction, the representation space is given by the 2-dimesnional lattice

$$\{|m + n\rho\rangle : m, n \in \mathbb{Z}\}$$

and is of countably infinite dimension.

Remarks:

1. Infinite dimesnional representation has also been constructed by Takhtajan. However his representation is complex as the operators X^i are not represented as Hermitian operators there. So his representation is for a deformation of a 6 real dimenional space.
2. There is probably infinite number of inequivalent infinite dimensional representations. We gave two examples here.

   Precisely which representation is to be used is a question that depends on the physics under consideration.

Outline

Small fluctuations around the Nambu geometry

Recall

$$\begin{aligned} S_{C_2} &= \mu_1 f \int d^2\sigma \mathrm{Tr}\left(\frac{1}{2}\epsilon_{ijk} X^i D_\alpha X^j D_\beta X^k \epsilon^{\alpha\beta}\right) + \mu_1 f \int d^2\sigma \mathrm{Tr}(iFX^i X^j X^k \epsilon_{ijk}) \\ &= \mu_1 f (L_1 + L_2) \end{aligned}$$

- Consider fluctuation around the classical solution $X^i_{cl} = x^i$

$$[x^i, x^j, x^k] = i\epsilon^{ijk}\theta.$$

parametrized as

$$X^i = \frac{x^i}{\theta}\mathbf{1}_{K\times K} + \epsilon^{ijk} B_{jk}(\sigma^\alpha, x^i),$$

- The two-form B_2 is analagous to the one-form gauge field A used in the perturbative solution to non-commutative Yang-Mills.

Small fluctuations around the Nambu geometry

- We obtain

$$L_1 = \int_x \mathrm{tr}\left(\epsilon^{l_1 l_2 l_3}\epsilon^{m_1 m_2 m_3}\epsilon^{\alpha\beta}(B_{l_1 m_1} D_\alpha B_{l_2 m_2} D_\beta B_{l_3 m_3})\right),$$

Here $\int_x$ is an integral on the quantum Nambu geometry which can be constructed from a representation of the geometry.
This term has a form analogous to the kinetic piece of a Chern-Simons theory.

- For L_2, we get

$$\begin{aligned}[X^2, X^3, X^4] &= \frac{i}{\theta^2} + \frac{1}{2\theta^2}[x^i, x^j, B_{ij}] - \frac{1}{4\theta}\epsilon^{ijk}[x^l, B_{li}, B_{jk}] + \frac{1}{3}[B_{ii'}, B_{jj'}, B_{kk'}]\epsilon^{ijk}\epsilon^{i'} \\ &:= \frac{i}{\theta^2} + H\end{aligned}$$

and so

$$L_2 = i\int_x \mathrm{tr}(FH).$$

- Not clear if the 5-dimensional theory has an interpretation as worldvolume theory of some branes.
- A theory of non-abelian self-dual two forms reduced to 5-dimensions?

Quantum field theory on quantum Nambu geometry

- Interesting question to construct quantum field theory on the quantum Nambu geometry.
- An integral on the space can be constructed from the representation as a trace.
- The Nambu-Heisenberg relation implies a kind of minimal volume relation. What is its manifestation in a quantum field theory?
- work in progress

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Conclusions

- We have obtained Quantum Nambu geometry in string theory
- We have studied representation of the geometry and demonstrated that the geometry is not reducible to known Lie-algebraic type.
- Physics on this kind of quantum spaces may be interesting.
- Small fluctuation of D1-brane around quantum Nambu geometry suggests a formulation of "Non-Abelian" self-dual tensor multiplet on multiple M5-branes?

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