Can Gravity be Localized ?

& new AdS_4-$SCFT_3$ duality

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based on : CB, J. Estes, *arXiv:1103.2800 [hep-th]*

B. Assel, CB, J. Estes, J. Gomis, *1106.4253 [hep-th]*

and in preparation

also: O. Aharony, L. Berdichevsky, M. Berkooz, I. Shamir, *arXiv:1106.1870 [hep-th]*

This is closely related to the questions:

Can the graviton have mass?

Can it be a resonance?

Are sectors “hidden” from gravity possible ?

Other IR modifications of Einstein equations ?

The subject has a long history, to which I will not try to do justice here .....

In Minkowski spacetime, the answer seems to be NO

An important obstruction is the vDVZ discontinuity

van Dam, Veltman, Zakharov '70

Notice that for the photon the answer is YES

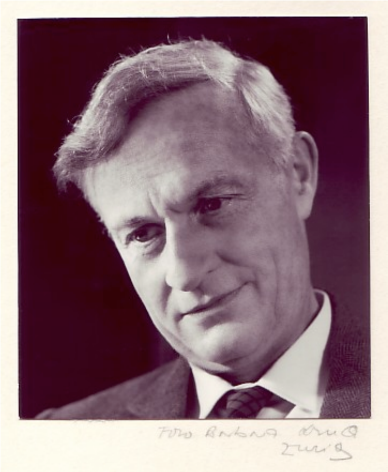
Indeed, the particle data group quotes the experimental bound:

$$m_\gamma < 10^{-18} eV$$

$\text{range} > 10^9\, km \sim 1\,\text{light hour}$ but could be finite!

Now repeat the exercise for a massive spin-2 field.

The (ghost-free) massive Pauli - Fierz Lagrangian is:



$$\mathcal{L}_{\mathrm{PF}} = \mathcal{L}_{\mathrm{EH}} - \frac{m^2}{2}\left(h^{\nu\lambda}\, h_{\nu\lambda} - (h^{\rho}{}_{\rho})^2\right)$$

where

$$\mathcal{L}_{\mathrm{EH}} = -\frac{1}{2}\partial_\mu h^{\nu\lambda}\,\partial^\mu h_{\nu\lambda} + \partial^\mu h^{\nu\lambda}\,\partial_\nu h_{\mu\lambda} - \partial_\mu h^{\mu\nu}\,\partial_\nu h^{\lambda}{}_{\lambda} + \frac{1}{2}\partial_\nu h^{\lambda}{}_{\lambda}\partial^\nu h^{\rho}{}_{\rho} + h_{\mu\nu}T^{\mu\nu}$$

with $$\partial_\mu T^{\mu\nu} = 0$$

Introduce again compensators to restore gauge invariance:

$$h_{\mu\nu} = h'_{\mu\nu} + \frac{1}{m}(\partial_\mu A_\nu + \partial_\nu A_\mu) + \frac{2}{m^2}\partial_\mu\partial_\nu\phi$$

invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$
$$\delta A_\mu = -m\xi_\mu + \partial_\mu \Lambda$$
$$\delta\phi = -m\Lambda$$

Inserting in $\mathcal{L}_{\rm PF}$ gives a free massless spin-1 field, and a two-derivative Lagrangian mixing ϕ and $h'_{\mu\nu}$.

PF was precisely devised for this, i.e. the 4-derivative term drops out

Redefining fields to remove the mixing ($h'_{\mu\nu} = h''_{\mu\nu} + \eta_{\mu\nu}\phi$) finally gives:

$$\mathcal{L}_{\rm PF} = \mathcal{L}_{\rm EH} - \frac{1}{2} F_{\mu\nu}F^{\mu\nu} - 3\,\partial_\mu\phi\,\partial^\mu\phi + \boxed{\phi\,T^\rho{}_\rho}$$

The residual coupling is different for light, than for massive matter;

thus the Pauli-Fierz theory does not give Einstein's theory when $m \longrightarrow 0$

If we set Newton's law to its measured form,

light bending = 3/4 of measured effect

.... so however tiny the mass, it is ruled out !

The story is more complicated than this, because strong coupling sets in at intermediate scales

Hotly debated whether this can cure the discontinuity;

(some) consensus that the issue cannot be settled without **UV completion**.

The story looks more promising in AdS:

- The vDVZ discontinuity is absent if $m_{\rm gr} < 1/L_{\rm AdS}$

  Kogan - Mouslopoulos - Papazoglou; Porrati

- $\exists$ a simple “model”, possibly embed-*able* in string theory

  Karch-Randall

- Supersymmetry can protect the required hierarchy

Of course, we don’t seem to live in AdS spacetime !

OK, take attitude that anything one can learn about IR gravity is interesting, and proceed.

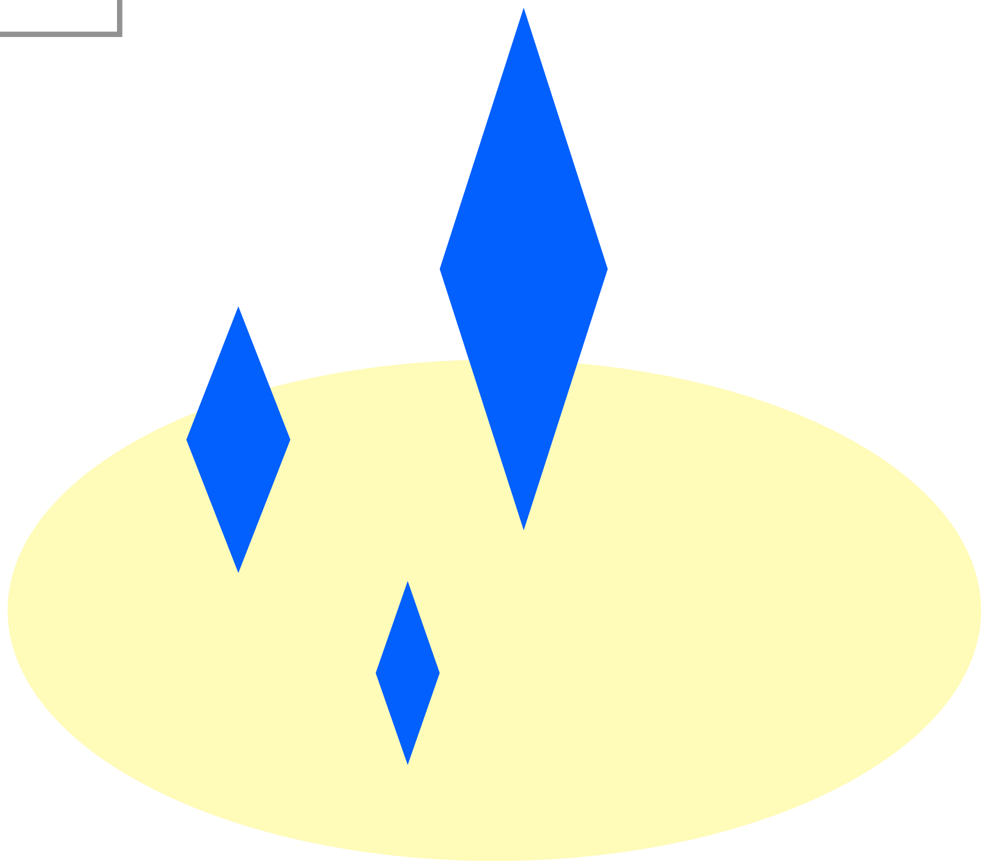
KK reduction for spin 2

Interested in *warped*-(A)dS geometries,

$$\widehat{ds^2} = e^{2A(y)} \bar{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + \hat{g}_{ab}(y)\, dy^a dy^b$$

$\bar{\mathcal{M}}_4 = \mathrm{AdS}_4,\ \mathbb{M}_4,\ \mathrm{dS}_4$

$k = -1, 0, 1$



Consider (consistent reduction to) metric perturbations

$$ds^2 = e^{2A}\,(\bar{g}_{\mu\nu} + h_{\mu\nu})\,dx^\mu dx^\nu + \hat{g}_{ab}\,dy^a dy^b \ ,$$

with $\qquad h_{\mu\nu}(x,y) \ = \ h^{[\mathrm{tt}]}_{\mu\nu}(x)\,\psi(y)$

where $\qquad (\bar{\Box}^{(2)}_x - \lambda)\,h^{[\mathrm{tt}]}_{\mu\nu} = 0 \qquad \text{and} \qquad \bar{\nabla}^\mu h^{[\mathrm{tt}]}_{\mu\nu} = \bar{g}^{\mu\nu} h^{[\mathrm{tt}]}_{\mu\nu} = 0 \ .$

Pauli-Fierz $\qquad (\lambda = m^2 + 2k)$

Linearizing the Einstein equations $R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}$

leads to the Schrodinger problem :

$$-\frac{e^{-2A}}{\sqrt{[\hat{g}]}} \left(\partial_a \sqrt{[\hat{g}]}\, \hat{g}^{ab} e^{4A} \partial_b\right) \psi = m^2 \psi$$

This is equivalent to a *scalar-Laplace* equation in d dimensions :

$$\frac{1}{\sqrt{\hat{g}}} \left(\partial_M \sqrt{\hat{g}}\, \hat{g}^{MN} \partial_N\right) h_{\mu\nu}(x, y) \;=\; 0 \;.$$

Important: the linearized equation depends only on the geometry, not on the detailed matter-field backgrounds.

Brandhuber, Sfetsos
Csaki, Erlich, Hollowood, Shirman
CB, JE

⇒ **Localization of spin-2 can only come from geometry**

The wavefunction norm is

$$||\psi||^2 \equiv \int d^{d-4}y \sqrt{[\hat{g}]}\, e^{2A}\, |\psi|^2$$

The would-be massless graviton has $\psi(y) = \text{constant}$

It is normalizable iff the transverse volume is finite

For infinite transverse volume, either the spin-2 spectrum has a continuum, or the **lowest-lying state must be massive.**

In the latter case we may talk about **localization of the graviton** provided m_0 can be made arbitrarily small, and in any case much smaller than the typical KK scale.

Wait a minute: Why can't the warp factor "help"?

When it does, *infinity* is an **apparent horizon**, which can be reached in finite proper time. This can be shown for flat brane-worlds as follows:

For a particle moving in a flat transverse dimension y : $e^{2A} = C\sqrt{e^{2A} - \dot{y}^2}$

As y goes to infinity, we need $e^A \longrightarrow 0$, so that $\dot{y} \simeq e^A \to 0$

The total proper time $\int d\tau = \int dt\, C^{-1} e^{2A} \simeq \int dy\, e^A$ must be infinite, for geodesic completeness.

If we request $\int dy\, e^{2A}$ finite for a normalizable zero mode, then

$$A \simeq -\nu \log y \qquad \text{with} \qquad 1 > \nu > 1/2$$

This is ruled out by the "holographic c-theorem" $A'' \leq 0$ which follows from the energy conditions in the flat-brane case *QED*.

Girardello et al, Freedman et al '98-99

Given an apparent horizon,

we need to supplement the *quantum* theory with boundary conditions at the horizon (“IR brane”)

This is an effective compactification.

For $\mathcal{M}_4 = \mathrm{AdS}_4$ the warp factor need not be monotonic.

Thus it can approach zero and then turn around and diverge, so as to create a graviton “trap”. The almost constant lowest mode goes to zero near the warp-factor minimum, giving the graviton a tiny mass.

The Karch-Randall model illustrates this point.

Karch-Randall model

Starting point is 5D Einstein action plus a thin 3-brane

$$I_{\rm KR} = -\frac{1}{2\kappa_5^2}\int d^4x\,dy\,\sqrt{g}\left(R+\frac{12}{L^2}\right)+\lambda\int d^4x\,\sqrt{[g]_4}\ ,$$

The solution is:

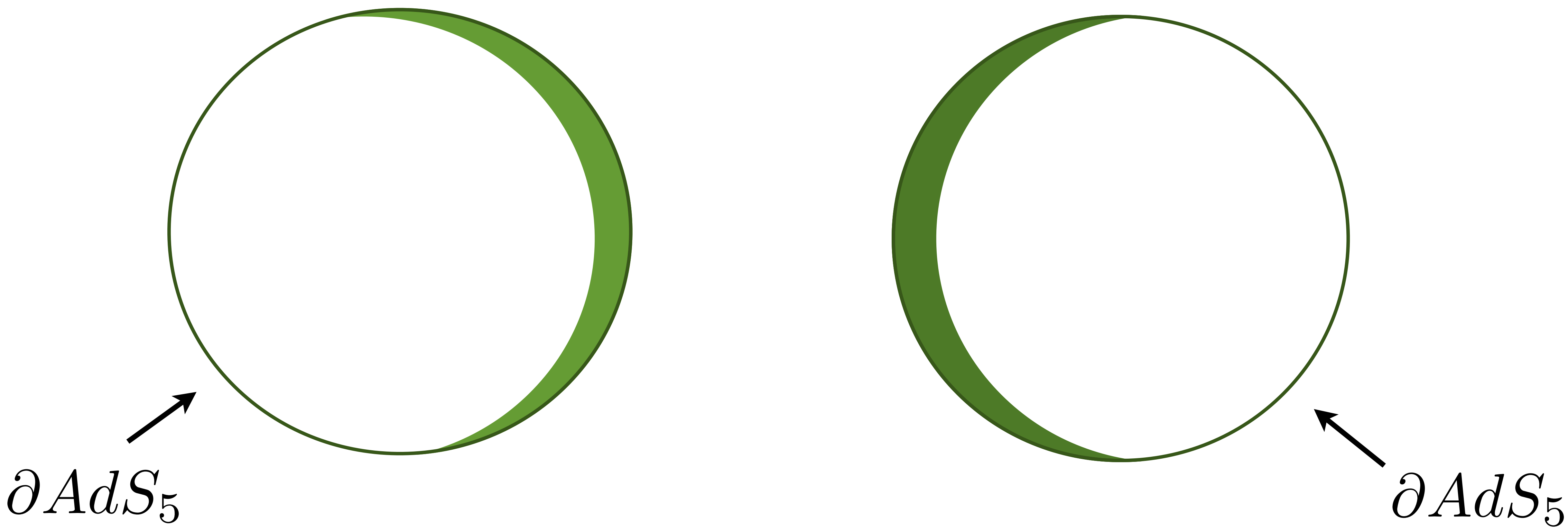
$$ds^2 = L^2\cosh^2\left(\frac{y_0-|y|}{L}\right)\bar{g}_{\mu\nu}dx^\mu dx^\nu + dy^2\,,$$ *where* $$y_0 = L\,\mathrm{arctanh}\left(\frac{{\kappa_5}^2\lambda L}{6}\right)$$

It describes two (large) slices of AdS_5 glued along a AdS_4 brane

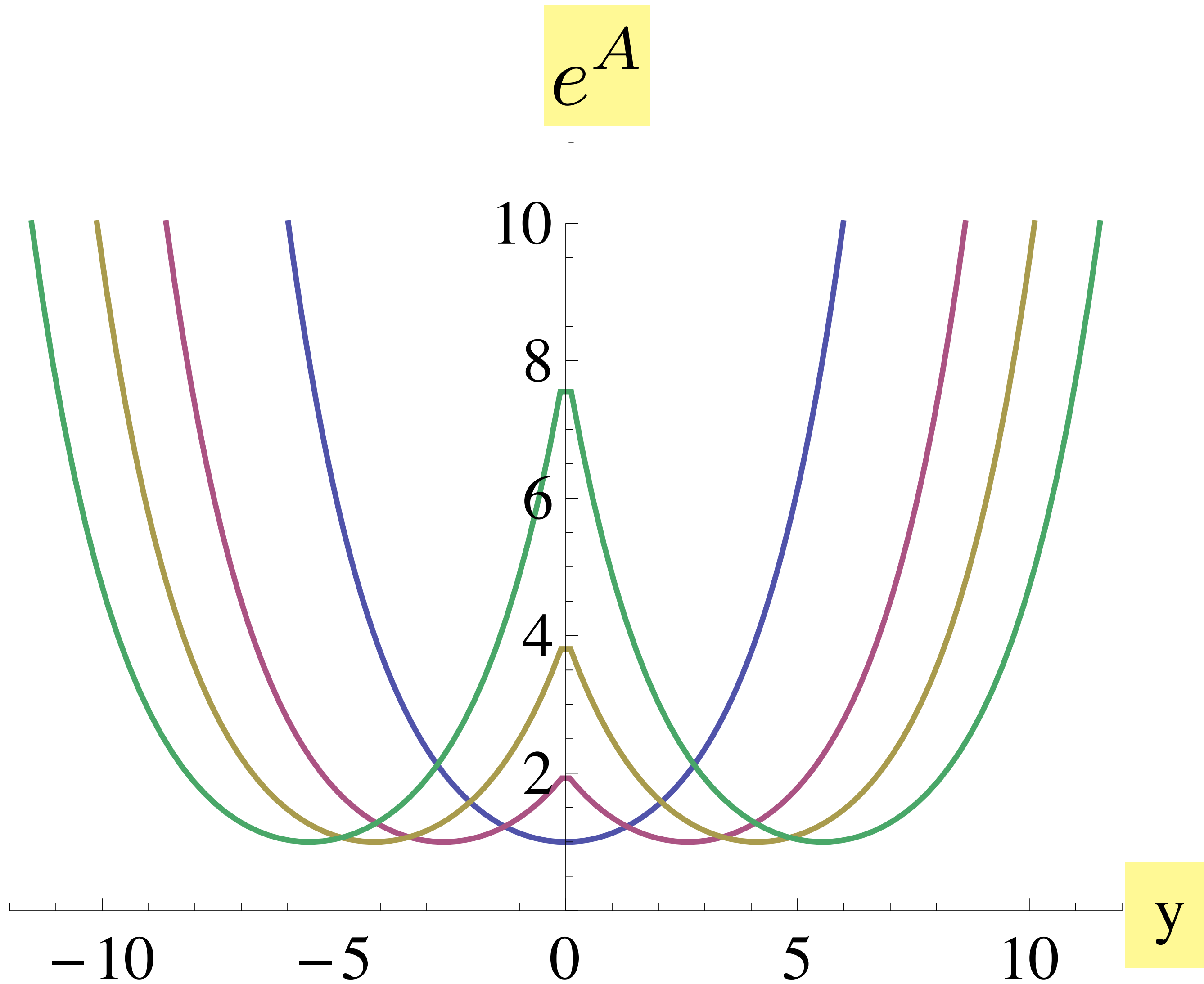
with radius

$$\ell^2 = e^{2A(0)} = L^2\cosh^2\left(\frac{y_0}{L}\right)\ .$$

One can tune λL so that $\frac{\ell}{L} \gg 1$



Cut away green slices, then glue the white ones in a symmetric fashion. Gives two 4D boundaries glued across two 3D defects (domain walls).



Warp factor $$e^{2A} = L^2 \cosh^2\left(\frac{y_0 - |y|}{L}\right)$$

as ℓ/L is gradually tuned up

4D parameters: $8\pi G_N \simeq \kappa_5^2/L$ ← *as in usual KK*

$$V_{\text{Newton}} + \Delta V \simeq -\frac{G_N m_1 m_2}{r}\left(1+\gamma\frac{L^2}{r^2}+\cdots\right)$$

↑ *unlike standard KK*

so $$\frac{\ell}{L} \sim 10^{31} - 10^{62}$$

<u>Spectrum</u> : - a nearly-constant, nearly massless mode $m_0^2 \simeq \dfrac{3L^2}{2\ell^2}$

- two towers of AdS_5 modes

$$m^2 \simeq (2n+1)(2n+4) \qquad n = 0, 1, \cdots$$

These masses are expressed in units of the AdS_4 radius

so states with $m^2 \simeq o(1)$ mediate long-range interactions.

What "saves the day" is that the AdS_5 states live at the

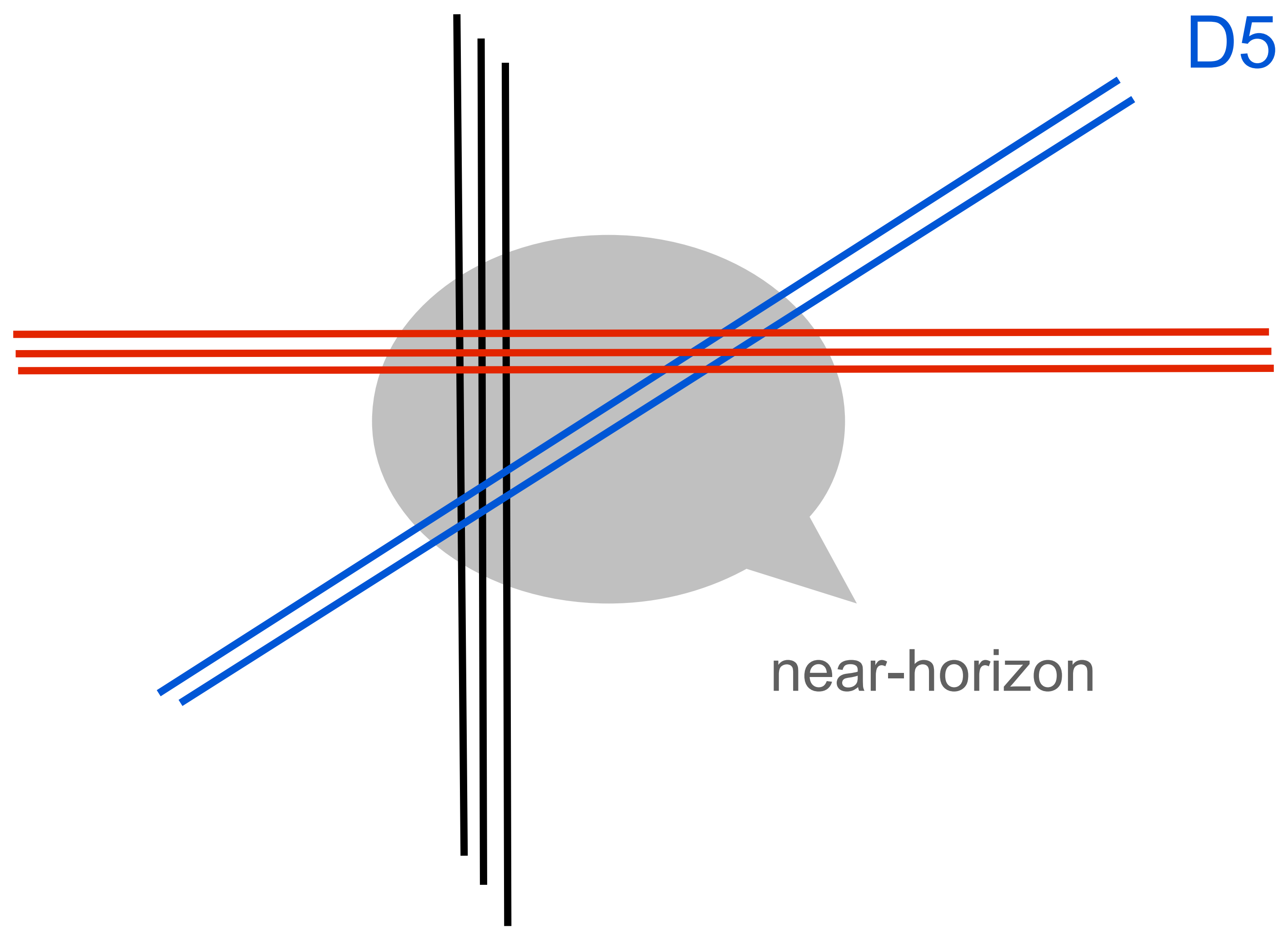
bottom of the warp-factor well . Their wavefunctions are

exponentially suppressed at the brane position

Furthermore, $$\int \psi_0 \psi^\dagger \psi \neq \text{universal}$$

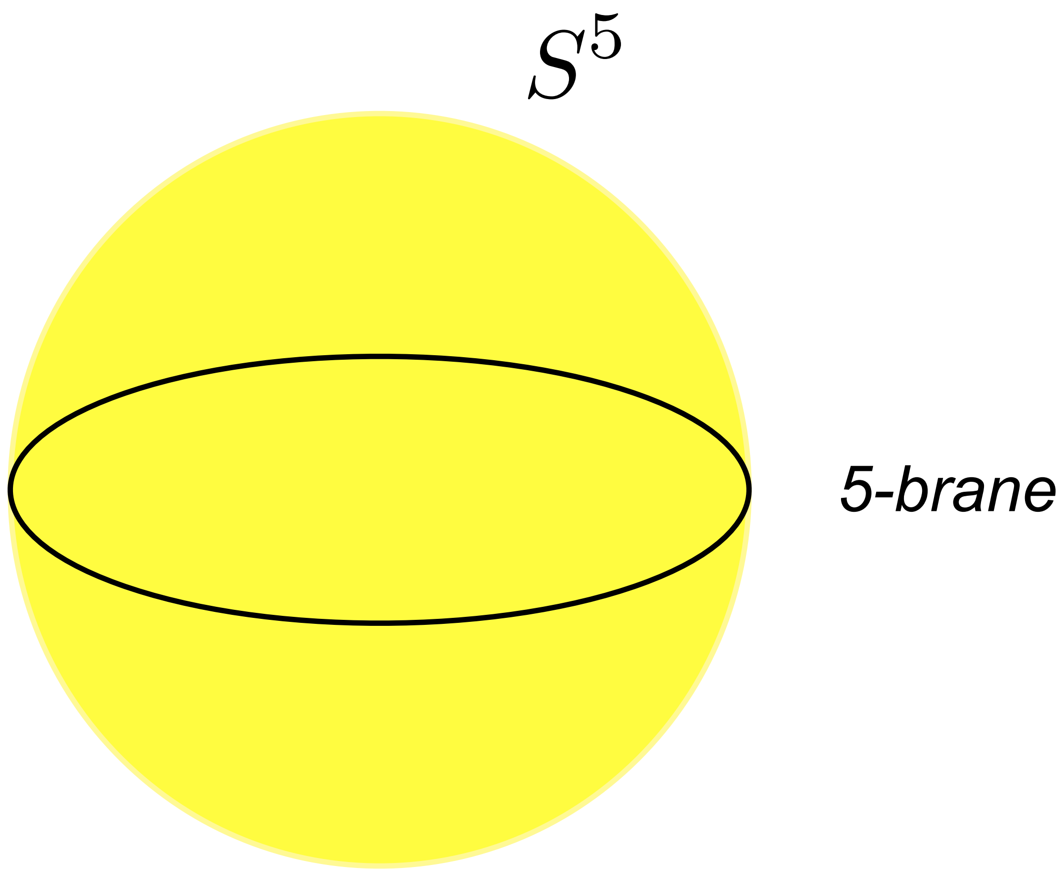
so the nearly-massless graviton has non-universal couplings
to the other fields !

The String-theory embedding



Karch and Randall proposed to embed their model in IIB string theory, by inserting 5-branes in the $AdS_5 \times S^5$ geometry of D3-branes.

The geometry of a 5-brane in the probe limit is $AdS_4 \times S^2$



The exact geometry of these configurations was discovered some years ago by *D'Hoker, Estes and Gutperle*

Q: Is the graviton in these geometries "localized" ?

A: No; but it does obtain an arbitrarily-small mass in a curved 10d spacetime.

The ensuing geometries are interesting for other reasons:

- holographic duals of N=4 $SCFT_3$ of Gaiotto-Witten
- (first?) IIB compactifications without moduli

The EDG solutions are $AdS_4 \times S^2 \times S^2$ fibrations over a surface $\sum$.

They depend on **two harmonic functions** h_1, h_2 subject to certain global consistency conditions.

metric :
$$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{\mathrm{S}_1^2} + f_2^2 ds^2_{\mathrm{S}_2^2} + 4\rho^2 dz d\bar{z} \ ,$$

$$f_4^8 = 16 \frac{N_1 N_2}{W^2} \ , \quad f_1^8 = 16\, h_1^8 \frac{N_2 W^2}{N_1^3} \ , \quad f_2^8 = 16\, h_2^8 \frac{N_1 W^2}{N_2^3}$$

dilaton :
$$e^{4\phi} = \frac{N_2}{N_1}$$

where :
$$W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 = \partial \bar{\partial}(h_1 h_2) \ ,$$

$$N_1 = 2h_1 h_2 |\partial h_1|^2 - h_1^2 W \ , \qquad N_2 = 2h_1 h_2 |\partial h_2|^2 - h_2^2 W \ .$$

There are also *3-form* and *5-form* backgrounds, and 1/4 unbroken supersymmetry.

The solutions of interest have Σ = infinite strip with h_1, h_2 obeying N or D conditions, possibly with isolated singularities on the boundary, e.g.

The harmonic functions for this choice are:

$$h_1 = \left[-i\alpha_1 \sinh(z-\beta_1) - \gamma_1 \ln\left(\tanh(\frac{i\pi}{4} - \frac{z-\delta_1}{2}) \right) \right] + \text{c.c.} \; ,$$

$$h_2 = \left[\alpha_2 \cosh(z-\beta_2) - \gamma_2 \ln\left(\tanh(\frac{z-\delta_2}{2}) \right) \right] + \text{c.c.} \; .$$

Reduction of eigenmode equation: $\psi(y^a) = Y_{l_1 m_1} Y_{l_2 m_2}\, \psi_{l_1 l_2}(z,\bar z)$

leads to a Laplace-Beltrami spectral problem on Σ :

$$\frac{2h_1h_2}{\partial\bar\partial(h_1h_2)}\,\partial\bar\partial\,\tilde\psi_{00} \;=\; (m^2+2)\tilde\psi_{00}\ , \qquad \text{where} \qquad \tilde\psi_{00} \equiv h_1h_2\psi_{00}\ .$$

The norm is $\|\psi\|^2 \;=\; \int_\Sigma d^2z\,|Wh_1h_2|\,|\psi_{l_1l_2}|^2 \;=\; \int_\Sigma d^2z\,\Big|\frac{W}{h_1h_2}\Big|\,|\tilde\psi_{l_1l_2}|^2$

and the b.conditions for ψ_{00} are *Neumann*.

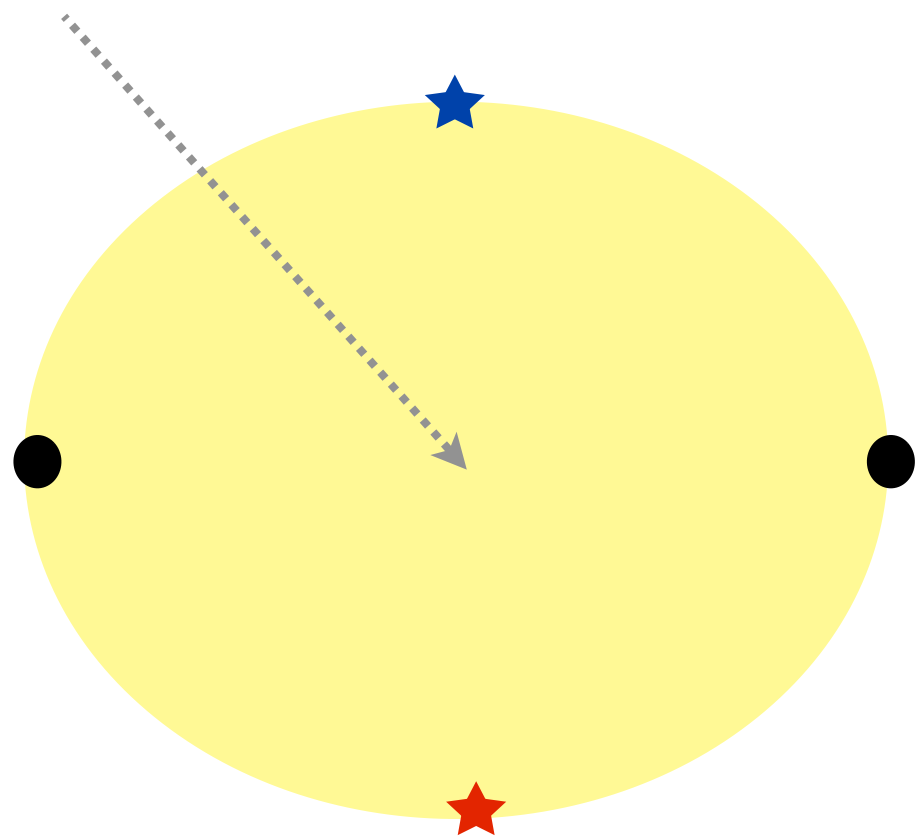
Too hard to solve analytically, except in the simplest case of the (dilaton domain wall) Janus solution where the equation reduces to Heun's equation.

Janus cannot localize gravity because the dilaton has no (super)potential, so its domain wall tends to spread to infinite thickness.

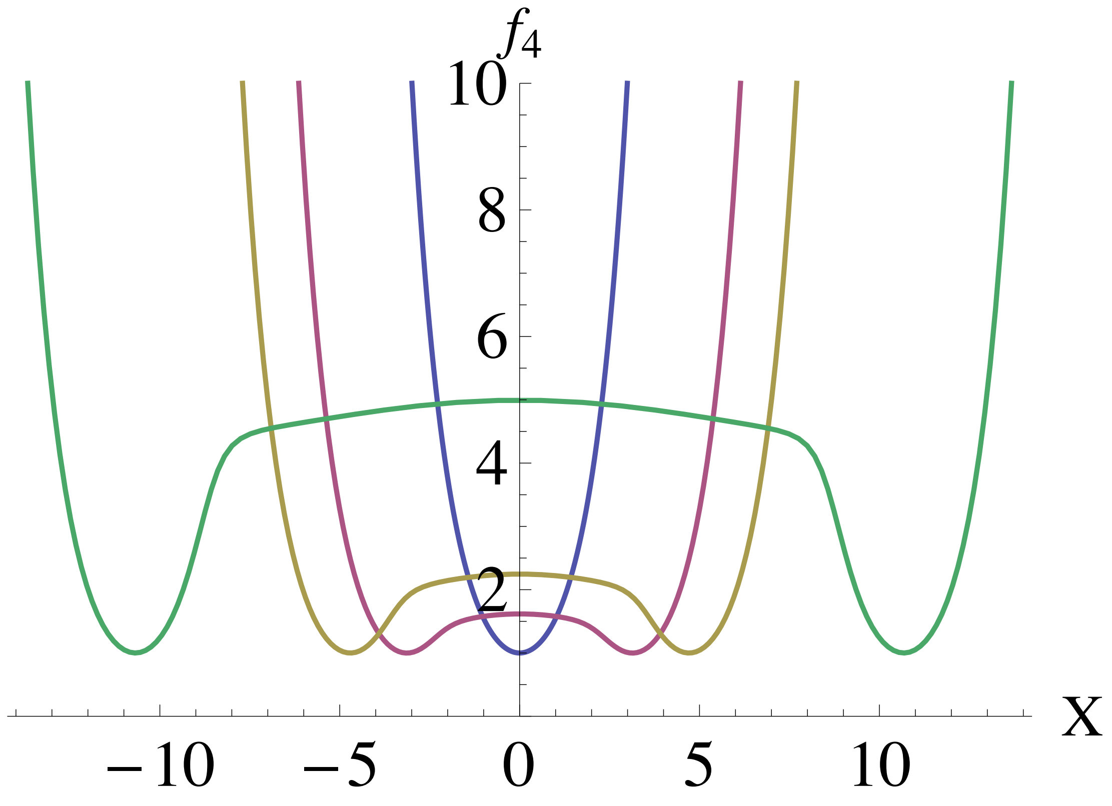
Adding one type of 5-branes does not help: the dilaton adjusts to (∞ ly) small or large value, so as to minimize the 5-brane tension.

The only interesting limit is one with both NS5 and D5 charges, and with $\frac{Q_5}{Q_3} \gg 1$.

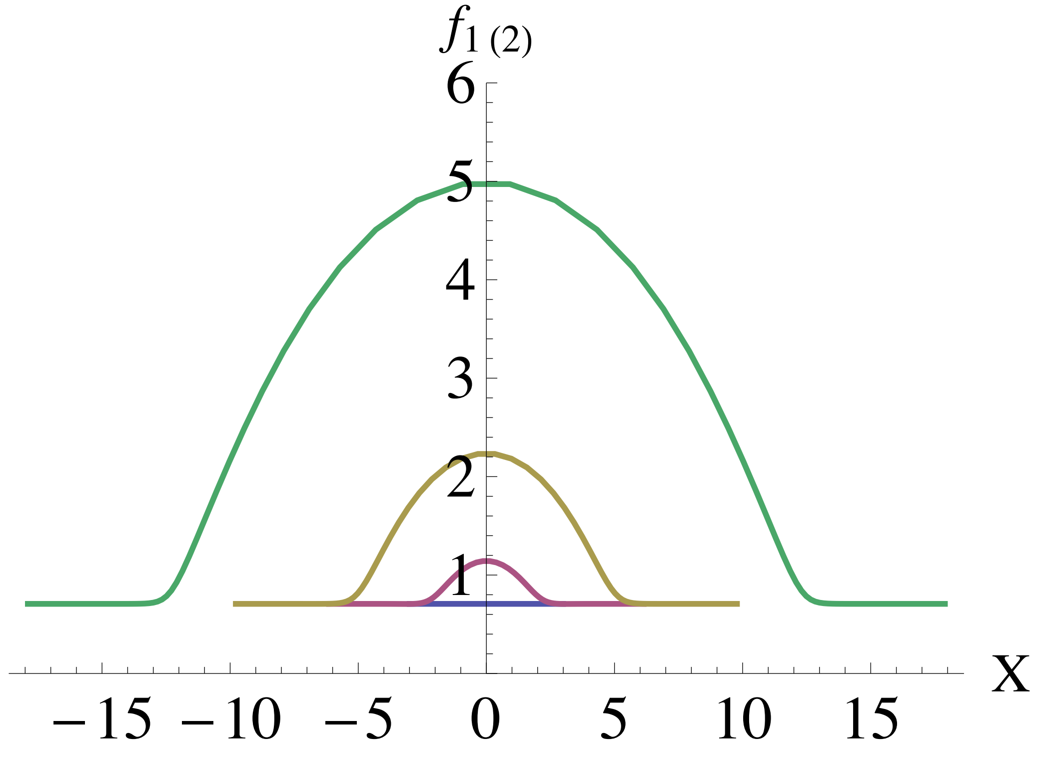
Inspection of the geometry shows that this creates a bubble of almost factorized $AdS_4 \ltimes K$ geometry in the central region.



The $AdS_5 \ltimes S^5$ regions are much more curved

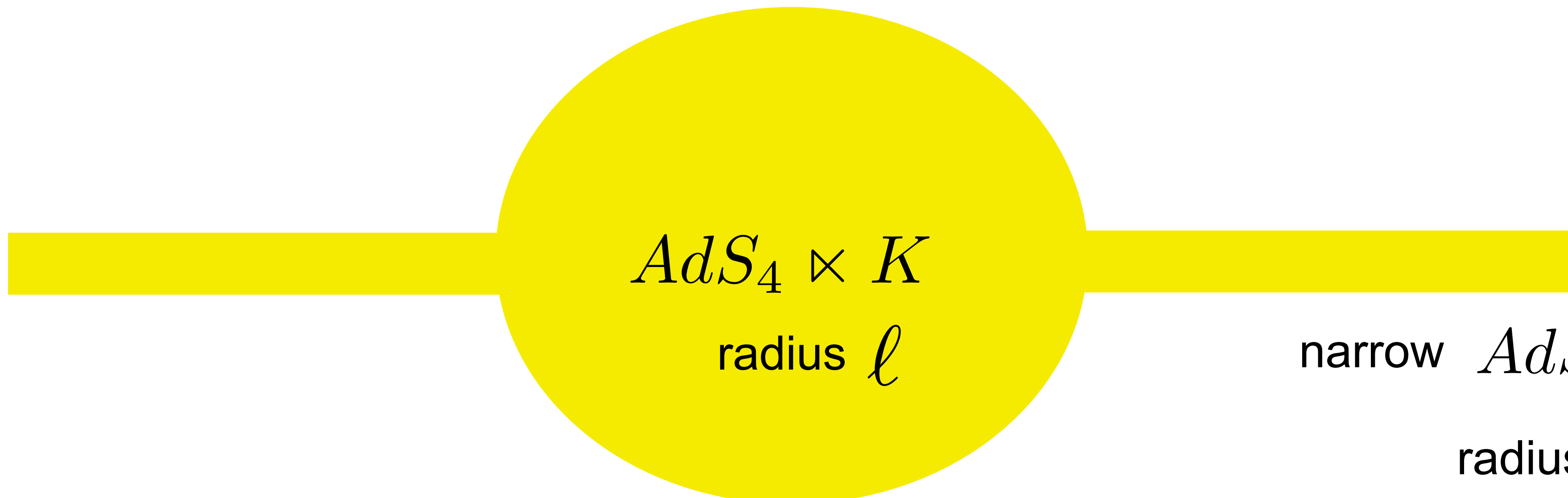


warp factor



sphere radii

The 10d geometry looks like this:



graviton mass $\sim \frac{L}{\ell} \ll 1$

Actually the limit $Q_3 \longrightarrow 0$ is smooth: transverse space compactifies,

the asymptotic regions $AdS_5 \times S^5$ go over to smooth $AdS_4 \times \mathcal{D}_6$ caps

These $AdS_4 \times_w \mathcal{M}_6$ solutions must be gravity duals to

3-dimensional (super)conformal field theories

Which ones ?

By studying the flat-space configurations, *Gaiotto and Witten* have proposed the existence of a class of interacting SCFTs in three dimensions that they called

$$T^{\hat{\rho}}_{\rho}(SU(N))$$

They are in 1-to-1 corrspondence with solutions of Nahm's equations:

$$\frac{dX^a}{dt} = i\epsilon_{abc}[X^b, X^c]$$

on the interval, with boundary conditions that are simple poles,

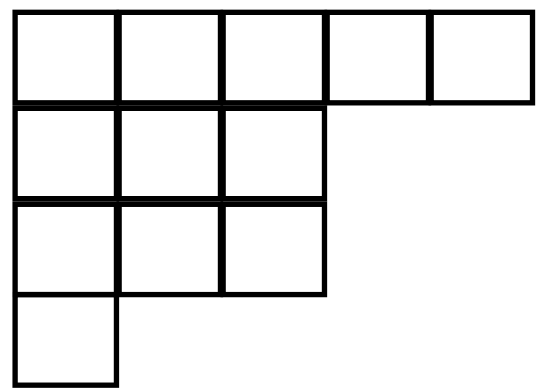
$$X^a \sim \frac{J^a}{t}$$

N-dimensional generators of SU(2)

This problem has been solved by *Kronheimer and Nakajima*

One can associate a partition of N with each choice of the J^a

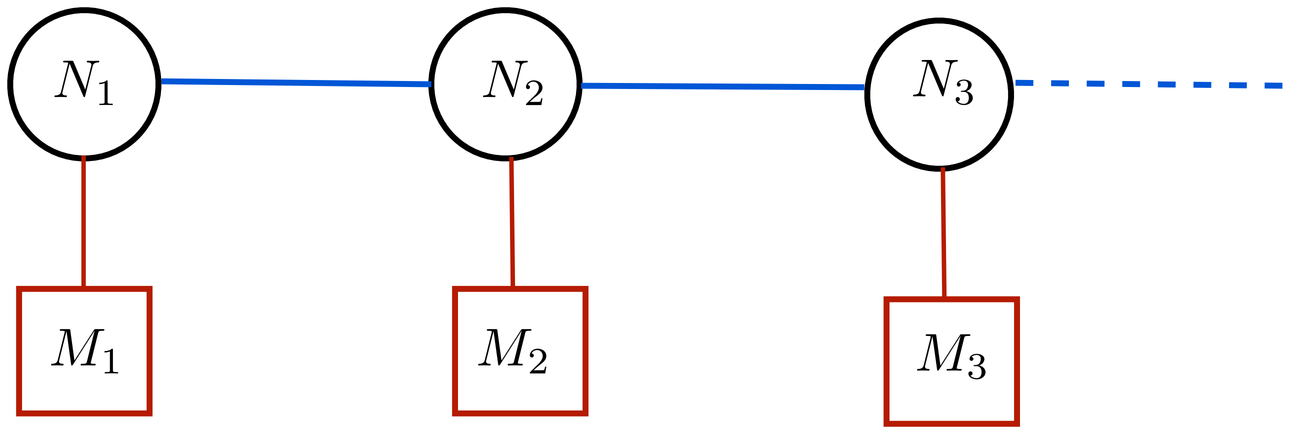
e.g. ρ: 12 = 5 + 3 + 3 + 1



K & N have shown that solutions exist iff $\rho^T > \hat{\rho}$

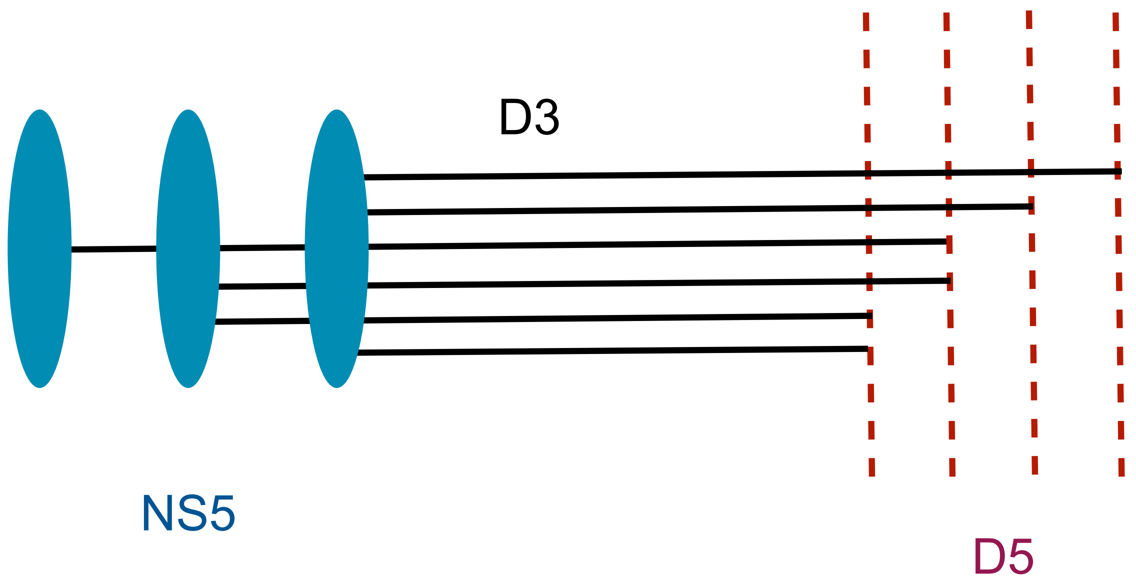
where these are the two partitions at the interval ends.

The underlying gauge theories are described by linear quivers

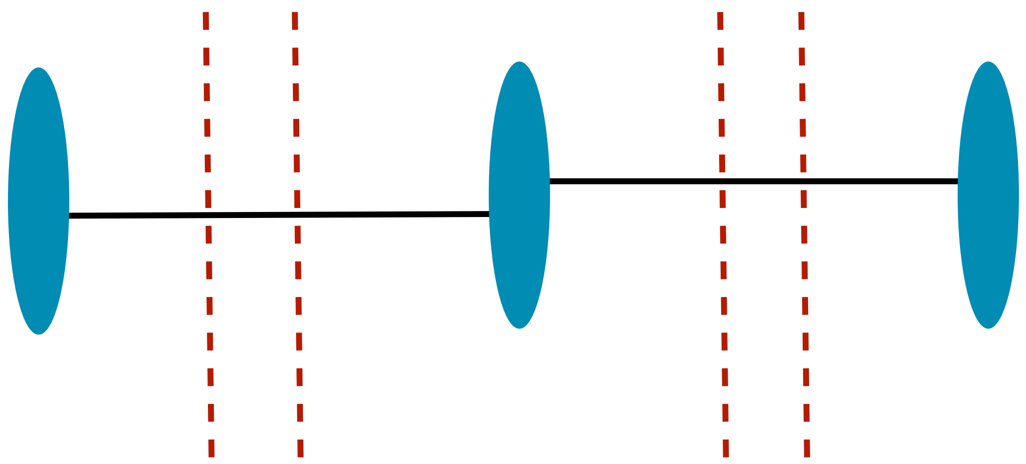


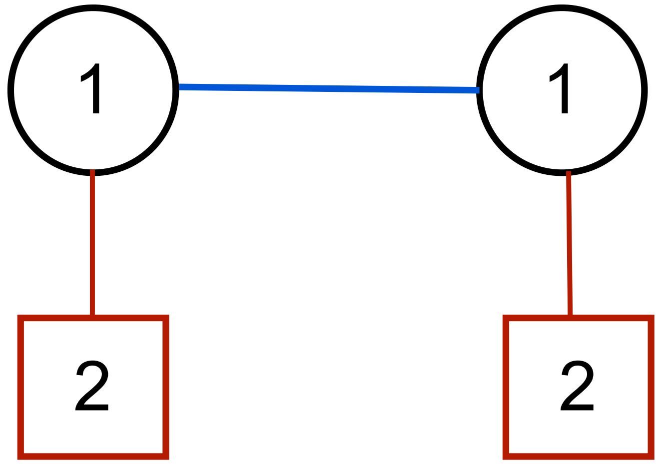
$$U(N_1) \times U(N_2) \times U(N3) \times \cdots$$

for example:



$$N = 6 \; ; \; \rho = (2,2,1,1) \; ; \; \hat{\rho} = (3,2,1)$$

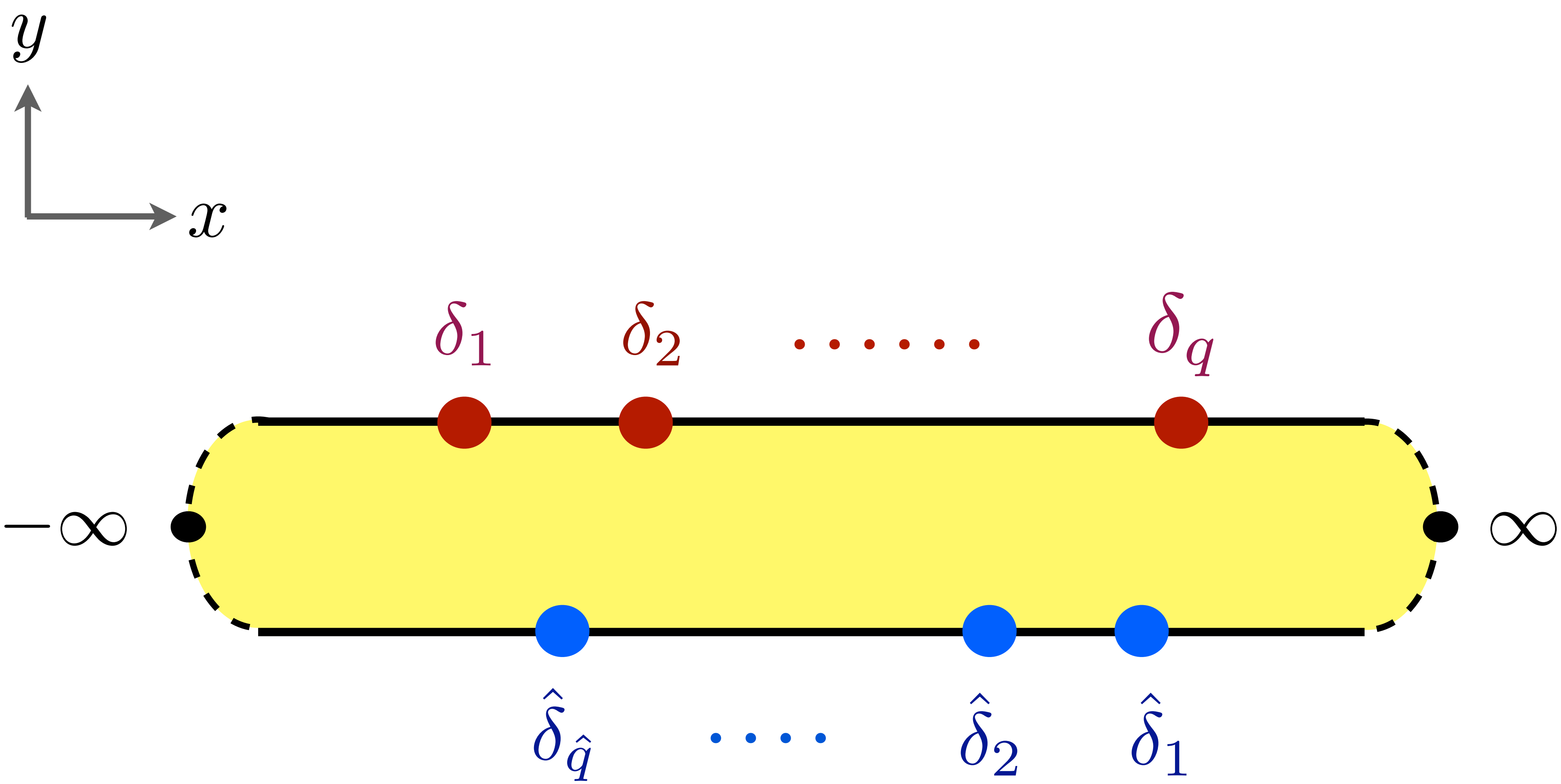




General result (by moving branes):

$$\text{supersymmetry} \iff \hat{\rho}^T \geq \rho\ , \quad \text{and} \quad \text{irreducibility} \iff \hat{\rho}^T > \rho\ .$$

When the inequality is saturated, the quiver breaks down to disjoint pieces.



$$h_1 = \left[-i\alpha \sinh(z-\beta) - \sum_{a=1}^{q} \gamma_a \ln\left(\tanh\left(\frac{i\pi}{4} - \frac{z-\delta_a}{2}\right)\right)\right] + c.c.$$

$$h_2 = \left[\hat{\alpha} \cosh(z-\hat{\beta}) - \sum_{b=1}^{\hat{q}} \hat{\gamma}_b \ln\left(\tanh\left(\frac{z-\hat{\delta}_b}{2}\right)\right)\right] + c.c.$$

D3-brane Page charges in fivebrane stacks:

$$Q_{D3}^{\mathrm{inv}(a)} = \int_{\mathcal{C}_a} F_5 - B_2 \wedge F_3 + \int_{\mathcal{C}_a} F_3 \wedge B_2 \Big|_{z=\infty}$$

$$= 2^8 \pi^3 \left(\hat{\alpha}\, \gamma_a \sinh(\delta_a - \hat{\beta}) - 2\, \gamma_a \sum_{b=1}^{\hat{q}} \hat{\gamma}_b \arctan(e^{\hat{\delta}_b - \delta_a}) \right)$$

$$\hat{Q}_{D3}^{\mathrm{inv}(b)} = \int_{\hat{\mathcal{C}}_b} F_5 + C_2 \wedge H_3 - \int_{\hat{\mathcal{C}}_b} H_3 \wedge C_2 \Big|_{z=-\infty}$$

$$= 2^8 \pi^3 \left(\alpha\, \hat{\gamma}_b \sinh(\hat{\delta}_b - \beta) + 2\, \hat{\gamma}_b \sum_{a=1}^{q} \gamma_a \arctan(e^{\hat{\delta}_b - \delta_a}) \right) \ .$$

$$N_{D3}^{(a)} = -N_{D5}^{(a)} \sum_{b=1}^{\hat{q}} \hat{N}_{NS5}^{(b)} \frac{2}{\pi} \arctan(e^{\hat{\delta}_b - \delta_a}) \ ,$$

$$\hat{N}_{D3}^{(b)} = \hat{N}_{NS5}^{(b)} \sum_{a=1}^{q} N_{D5}^{(a)} \frac{2}{\pi} \arctan(e^{\hat{\delta}_b - \delta_a})$$

Compute the linking numbers:

$$l^{(a)} \equiv -\frac{N_{D3}^{(a)}}{N_{D5}^{(a)}} \qquad \text{and} \qquad \hat{l}^{(b)} \equiv \frac{\hat{N}_{D3}^{(b)}}{\hat{N}_{NS5}^{(b)}} \ .$$

Prove $\rho^T > \hat{\rho}$ using the fact that $acrtan\theta \leq \pi/2$

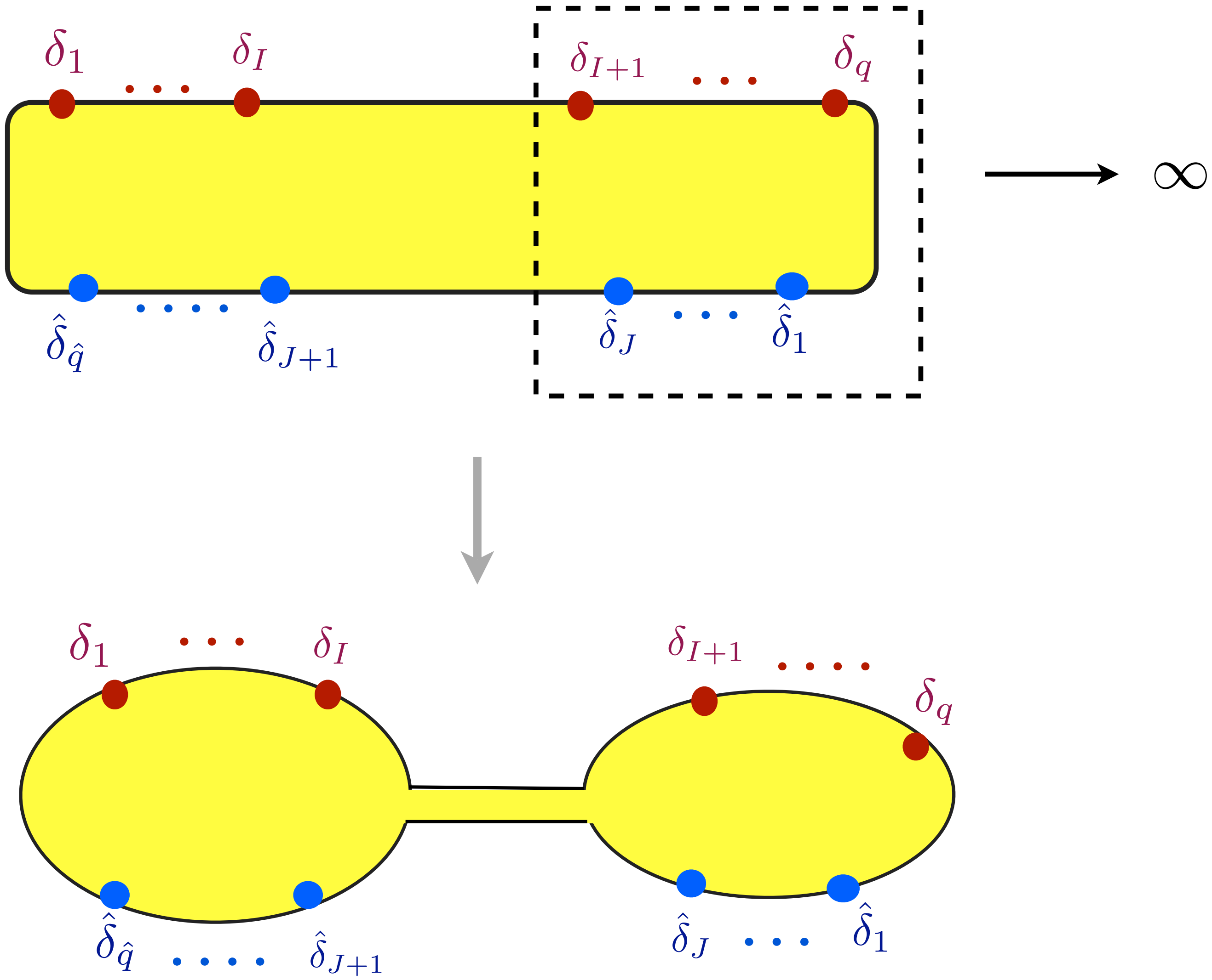
By counting parameters one can check that they are all quantized

These are examples of **IIB** AdS vacua with no moduli

Another interesting limit $\hat{\rho} \simeq \rho^T$ correspond to **severing**

one (or more) **link**, by taking $N_i \longrightarrow 0$

This corresponds to factorizing the 5-brane singularities on the strip.

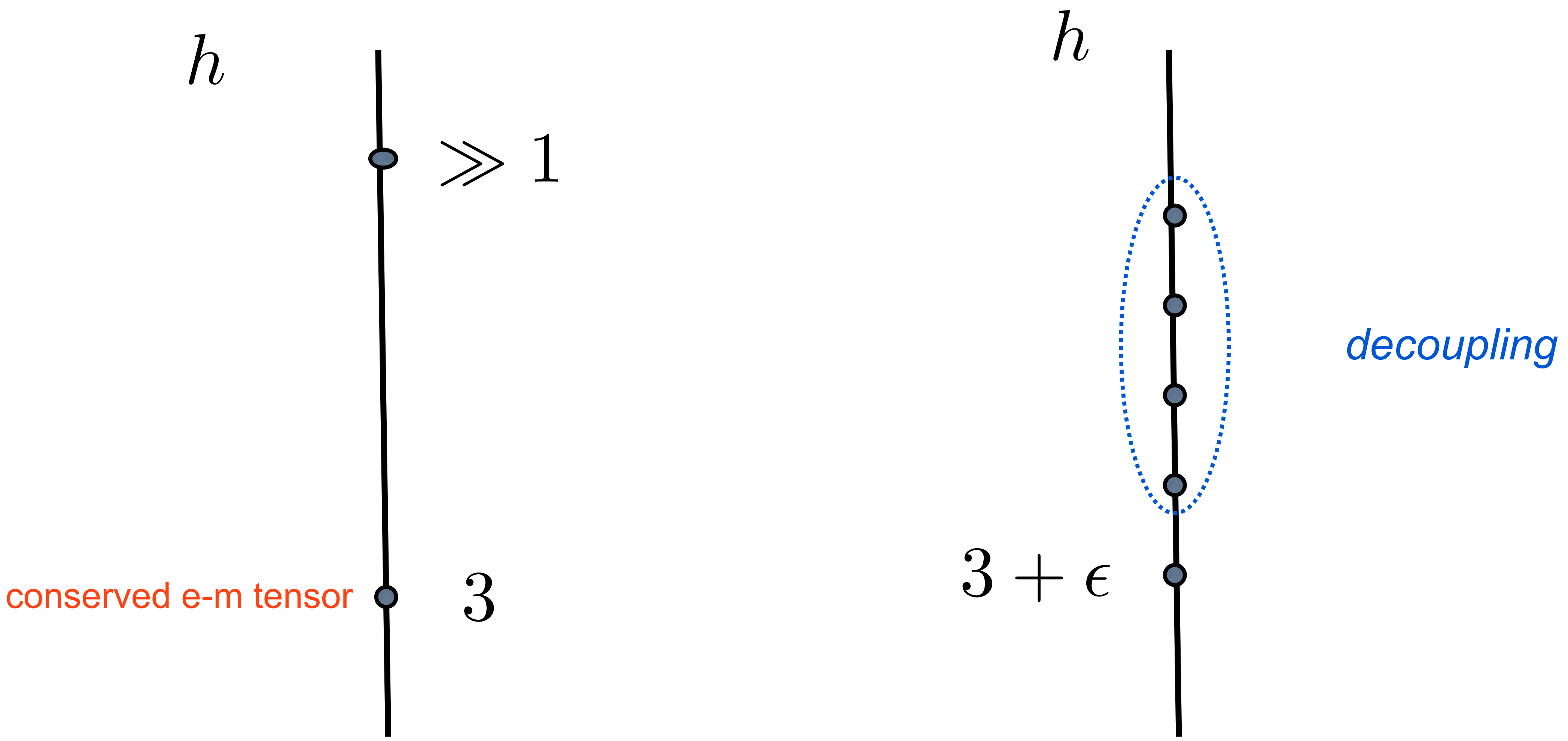


δ_1
δ_I
δ_{I+1}
δ_q
$\hat{\delta}_{\hat{q}}$
$\hat{\delta}_{J+1}$
$\hat{\delta}_J$
$\hat{\delta}_1$
∞
δ_1
δ_I
δ_{I+1}
δ_q
$\hat{\delta}_{\hat{q}}$
$\hat{\delta}_{J+1}$
$\hat{\delta}_J$
$\hat{\delta}_1$

this is a string-theory wormhole

Holographic comment

on massive AdS gravity theories:



CFT spectrum

defect CFT spectrum

CONCLUSIONS

(1) Embedding of Karch-Randall in string theory gives massive graviton in $AdS_4 \ltimes K$ backgrounds.

(2) New isolated vacua, and stringy wormholes

(3) Holographic duals for (large class of) N=4 d=3 SCFTs.

Thank you