ASYMPTOTIC W-SYMMETRIES IN 3D HIGHER-SPIN GAUGE THEORIES

Andrea Campoleoni
Albert Einstein Institute, Potsdam

A.C., S. Fredenhagen, S. Pfenninger, S. Theisen, 1008.4744

A.C., S. Fredenhagen, S. Pfenninger, 1107.0290

Workshop on Fields and Strings, Corfu Summer Institute, 15/9/2011

HIGHER SPINS IN D=2+1

- D=2+1 ⇒ no irreps of arbitrary helicity for the little group of massless particles ⇒ no spin in the usual sense

- Still... look at Fronsdal equation:

No dependence on D!

$$\mathcal{F}_{\mu_1\dots\mu_s} \equiv \Box\varphi_{\mu_1\dots\mu_s} - \partial_{(\mu_1}\partial\cdot\varphi_{\mu_2\dots\mu_s)} + \partial_{(\mu_1}\partial_{\mu_2}\varphi_{\mu_3\dots\mu_s)\lambda}{}^{\lambda} = 0$$

- We can consider Fronsdal equations in D=2+1
  - No local d.o.f for $s > 1$ (i.e. no wave solutions)
  - Nothing new with respect to gravity: no graviton in D=2+1 but the Einstein-Hilbert action is non-trivial

- Non-linear theory? Asymptotic symmetries for Λ<0?

OUTLINE

- Higher-spin gauge theories in D=2+1
  - Chern-Simons formulation of the non-linear dynamics
  - The Lie algebras hs[λ]
- Asymptotic symmetries
  - Drinfeld-Sokolov reduction in highest-weight gauge
  - The W-algebras $W_\infty[\lambda]$
- Higher-spin geometry?
  - Metric-like fields & invariant tensors of hs[λ]

CHERN-SIMONS ACTIONS & HIGHER SPINS IN D=2+1

CHERN-SIMONS GRAVITY IN D=2+1

- $\Lambda < 0 \;\Rightarrow$ "gauge algebra" so(2,2) = sl(2,R) ⊕ sl(2,R)
  - Algebra: $[J_a, J_b] = \epsilon_{abc} J^c$ $\quad [\tilde{J}_a, \tilde{J}_b] = \epsilon_{abc} \tilde{J}^c$
  - Gauge potentials: $A = A_\mu{}^a J_a \, dx^\mu$ $\quad \widetilde{A} = \widetilde{A}_\mu{}^a J_a \, dx^\mu$
  - Dreibein and spin connection:

$$e = \frac{l}{2}\left(A - \widetilde{A}\right) \qquad \omega = \frac{1}{2}\left(A + \widetilde{A}\right)$$

- Einstein-Hilbert action

Achúcarro and Townsend (1986); Witten (1988)

$$\boxed{S \;=\; S_{CS}[A] - S_{CS}[\widetilde{A}]} \qquad S_{CS}[A] \;=\; \frac{k}{4\pi}\int \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

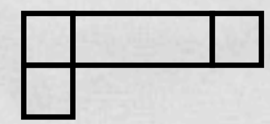
$$k = \frac{l}{4G}$$

HS INTERACTIONS IN D=2+1?

- Frame-like formulation of the dynamics

Vasiliev (1980)

  - HS “vielbeins” and “spin connections”

$e_\mu{}^{a_1 \dots a_{s-1}} \Rightarrow$  $\qquad \omega_\mu{}^{b, a_1 \dots a_{s-1}} \Rightarrow$ 

- Everything is traceless, then in D=2+1...

  -  $\approx$  $\qquad$ (e.g. $\omega_\mu{}^a = \frac{1}{2} \epsilon^a{}_{bc}\, \omega_\mu{}^{b,c}$)

  - Vielbeins and spin connections have the same structure!

- Consider the 1-forms

Blencowe (1989)

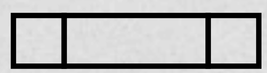
$$A_{(s)} = \left(\omega_\mu{}^{a_1 \dots a_{s-1}} + \frac{1}{l}\, e_\mu{}^{a_1 \dots a_{s-1}} \right) dx^\mu$$

$$\widetilde{A}_{(s)} = \left(\omega_\mu{}^{a_1 \dots a_{s-1}} - \frac{1}{l}\, e_\mu{}^{a_1 \dots a_{s-1}} \right) dx^\mu$$

HS INTERACTIONS IN D=2+1?

- Frame-like formulation of the dynamics — Vasiliev (1980)
    - HS “vielbeins” and “spin connections”

$$e_\mu{}^{a_1\dots a_{s-1}} \Rightarrow$$  $$\omega_\mu{}^{b,a_1\dots a_{s-1}} \Rightarrow$$ 

- Everything is traceless, then in D=2+1...
    -  $\approx$  (e.g. $\omega_\mu{}^a = \frac{1}{2}\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$)
    - Vielbeins and spin connections have the same structure!

- Consider the 1-forms (Lie-algebra valued) — Blencowe (1989)

$$A_{(s)} = \left(\omega_\mu{}^{a_1\dots a_{s-1}} + \frac{1}{l}\,e_\mu{}^{a_1\dots a_{s-1}}\right) dx^\mu\ T_{a_1\dots a_{s-1}}$$

$$\widetilde{A}_{(s)} = \left(\omega_\mu{}^{a_1\dots a_{s-1}} - \frac{1}{l}\,e_\mu{}^{a_1\dots a_{s-1}}\right) dx^\mu\ T_{a_1\dots a_{s-1}}$$

G×G
Chern-Simons?

CHERN-SIMONS ACTION!

- Natural guess: extend CS reformulation of gravity

Blencowe (1989)

$$A = A_{grav} + \sum_s A_{(s)}$$

$$\widetilde{A} = \widetilde{A}_{grav} + \sum_s \widetilde{A}_{(s)}$$

$$\boxed{S \; = \; S_{CS}[A] - S_{CS}[\widetilde{A}] \qquad k = \frac{l}{4G}}$$

  - No local d.o.f. as in the linearised theory

- Correct linearised dynamics $\Leftrightarrow$ $[\, J_a, T_{b_1 \ldots b_\ell} \,] = \epsilon^c{}_{a(b_1} T_{b_2 \ldots b_\ell)c}$

THE LIE ALGEBRAS hs[λ]

- Natural guess: extend CS reformulation of gravity

Blencowe (1989)

$$A = A_{grav} + \sum_s A_{(s)}$$

$$\widetilde{A} = \widetilde{A}_{grav} + \sum_s \widetilde{A}_{(s)}$$

$$S = S_{CS}[A] - S_{CS}[\widetilde{A}] \qquad k = \frac{l}{4G}$$

  - No local d.o.f. as in the linearised theory

- Correct linearised dynamics $\Leftrightarrow$ $[\, J_a, T_{b_1 \ldots b_\ell} \,] = \epsilon^c{}_{a(b_1} T_{b_2 \ldots b_\ell)c}$

  - $T_{a_1 \ldots a_\ell} \sim \Pi_{\rm tr}\, J_{(a_1} \ldots J_{a_\ell)}$ satisfies this property!

  - Choose a sl(2) irrep:

$$C_2 := J_0^2 - \frac{1}{2}\,(\, J_+ J_- + J_- J_+ \,) \equiv \frac{1}{4}\,(\lambda^2 - 1)\mathbb{1}$$

- One-parameter family of non-isomorphic Lie algebras

Bergshoeff, Blencowe and Stelle; Pope, Romans and Shen; Vasiliev; Fradkin and Linetsky; Bordemann, Hoppe and Schaller (1989)

★−PRODUCT & KILLING METRIC

$$\boxed{T_{a_1\dots a_\ell} \sim W^\ell_m := (-1)^{\ell-m}\frac{(\ell+m)!}{(2\ell)!}L_-^{\ell-m}J_+^\ell}$$

$$L_i\,x := [\,J_i, x\,]$$

- Abstract characterisation of hs[λ]: $\dfrac{\mathcal{U}(sl(2,\mathbb{R}))}{\langle\, C_2 - \mu\mathbb{1}\,\rangle} = hs[\lambda] \oplus \mathbb{C}$

- ★−product on the quotient → Killing form

Vasiliev (1989)

$$\mathrm{tr}\left(W^k_m W^\ell_n\right) \sim W^k_m \star W^\ell_n\Big|_{W^i_p=0 \text{ for } i>0}$$

- For $\lambda \in \mathbb{N}$ the Killing form degenerates!

★−PRODUCT & KILLING METRIC

$$\boxed{T_{a_1\ldots a_\ell} \sim W^\ell_m := (-1)^{\ell-m}\frac{(\ell+m)!}{(2\ell)!}L_-^{\ell-m}J_+^\ell}$$

$$L_i\, x := [\, J_i, x\,]$$

- Abstract characterisation of hs[λ]: $\dfrac{\mathcal{U}(sl(2,\mathbb{R}))}{\langle\, C_2 - \mu\mathbb{1}\,\rangle} = hs[\lambda] \oplus \mathbb{C}$

- ★−product on the quotient → Killing form

Vasiliev (1989)

$$\mathrm{tr}\left(W^k_m W^\ell_n\right) \sim W^k_m \star W^\ell_n \Big|_{W^i_p = 0 \text{ for } i>0}$$

- For λ ∈ N the Killing form degenerates!
  - λ = N → hs[λ]⊕hs[λ] ≅ sl(N,R)⊕sl(N,R)
  - λ = 1/2 → hs[λ]⊕hs[λ] ≡ Fradkin-Vasiliev algebra

A.C., Fredenhagen, Pfenninger and Theisen (2010)

Henneaux and Rey (2010)

ASYMPTOTIC SYMMETRIES & DRINFELD-SOKOLOV REDUCTION

ASYMPTOTICALLY ADS SOLUTIONS

- What are the correct boundary conditions i.e. how to select asymptotically AdS solutions?
  - First fix the gauge: $A_\rho = b^{-1}(\rho)\,\partial_\rho\, b(\rho)$ with $b(\rho) = e^{\rho J_0}$
  - Then impose boundary conditions: $\left(\frac{A_t}{l} - A_\theta\right)\Big|_{\partial\mathcal{M}} = 0$

compatible with BTZ

- The space of solutions of the e.o.m. is parameterised by

$$\boxed{\begin{aligned} A &= b^{-1}(\rho)\, a(x^+)\, b(\rho) dx^+ + b^{-1}\partial_\rho b\, d\rho \\ \widetilde{A} &= b(\rho)\, \tilde{a}(x^-)\, b^{-1}(\rho) dx^- + b\,\partial_\rho b^{-1} d\rho \end{aligned}}$$

$$x^\pm = \frac{t}{l} \pm \theta$$

$$a(x^+),\, \tilde{a}(x^-) \in hs[\lambda]$$

ASYMPTOTICALLY ADS SOLUTIONS

- What are the correct boundary conditions i.e. how to select asymptotically AdS solutions?
  - First fix the gauge: $A_\rho = b^{-1}(\rho)\,\partial_\rho\, b(\rho)$ with $b(\rho) = e^{\rho J_0}$
  - Then impose boundary conditions: $\left(\frac{A_t}{l} - A_\theta\right)\Big|_{\partial\mathcal{M}} = 0$ compatible with BTZ
- The space of solutions of the e.o.m. is parameterised by

$$\boxed{\begin{aligned} A &= b^{-1}(\rho)\, a(x^+)\, b(\rho) dx^+ + b^{-1}\partial_\rho b\, d\rho \\ \widetilde{A} &= b(\rho)\, \tilde{a}(x^-)\, b^{-1}(\rho) dx^- + b\,\partial_\rho b^{-1} d\rho \end{aligned}}$$

$$x^\pm = \frac{t}{l} \pm \theta$$

$$a(x^+),\, \tilde{a}(x^-) \in hs[\lambda]$$

  - No longer an arbitrary time dependence: the gauge is completely fix
  - Still, there are formal residual gauge symmetries → “global” symmetries

Regge and Teitelboim (1974)

ASYMPTOTICALLY ADS SOLUTIONS

- Is that enough? NO, extra conditions are needed
  - $A_- = 0 \Rightarrow$ algebra of boundary charges $\approx$ Kac-Moody
  - Pure gravity: extra boundary conditions $\Rightarrow$ conformal symmetry
  - Extra conditions fixed by comparison with Brown-Henneaux

For a review see Banados, 9901148

Brown and Henneaux (1986)

- And here?

ASYMPTOTICALLY ADS SOLUTIONS

- Is that enough? NO, extra conditions are needed
  - $A_- = 0 \Rightarrow$ algebra of boundary charges $\approx$ Kac-Moody
  - Pure gravity: extra boundary conditions $\Rightarrow$ conformal symmetry
  - Extra conditions fixed by comparison with Brown-Henneaux
- And here? Asymptotically AdS $\Leftrightarrow$ $A - A_{\mathrm{AdS}}\Big|_{\mathrm{boundary}} = \mathcal{O}(1)$

For a review see Banados, 9901148

Brown and Henneaux (1986)

ASYMPTOTICALLY ADS SOLUTIONS

- Is that enough? NO, extra conditions are needed
  - $A_- = 0 \Rightarrow$ algebra of boundary charges ≈ Kac-Moody
  - Pure gravity: extra boundary conditions ⇒ conformal symmetry
  - Extra conditions fixed by comparison with Brown-Henneaux

For a review see Banados, 9901148

Brown and Henneaux (1986)

- And here? Asymptotically AdS ⇔ $A - A_{\rm AdS}\Big|_{\rm boundary} = \mathcal{O}(1)$
- The extra condition put a constraint on $a(x^+)$

$$a(\theta) = J_+ + a_-(\theta) \quad \text{with} \quad L_- a_- = 0$$

Drinfeld-Sokolov constraint in highest-weight gauge

- DS constraint → Dirac-bracket algebra ≡ W-algebra

ASYMPTOTIC SYMMETRIES

- Look for "gauge transf." that preserve the HW gauge
  - $\delta_\lambda a(\theta) \, = \, \lambda'(\theta) \, + \, [\, a(\theta)\,,\, \lambda(\theta)\,]$ so that $\quad L_-(\delta_\lambda a) \, = \, 0$
- Equation for the gauge parameter:
  - $(\, L_- D_\theta + L_- L_+ \,)\, \lambda(\theta) = 0$ with $\quad D_\theta := \, \partial_\theta \, + \, [\, a_-(\theta), \, \cdot \,]$
  - The second operator acts diagonally $\quad L_- L_+ \, = \, -\, \Delta \, + \, L_0(L_0 - 1)$
  - It is invertible on the complement of lowest-weight states

$$\boxed{R := \, - \, \frac{1}{\Delta - L_0(L_0 - 1)} \, (\, 1 - P_+ \,)} \qquad\qquad R L_- L_+ = 1 - P_+$$

  - Act with R on the equation: $\quad \lambda(\theta) \, = \, \dfrac{1}{1 + R L_- D_\theta} \, \lambda_+(\theta)$ with $\; \lambda_+ = P_+ \lambda$

THE W-ALGEBRAS $W_\infty[\lambda]$

- Symmetries generated by the lowest-weight part of λ

$$\delta_\lambda a(\theta) = P_- \sum_{n=0}^{\infty} \big(- D_\theta R L_- \big)^n D_\theta \lambda_+(\theta)$$

A.C., Fredenhagen and Pfenninger (2011)

see also de Boer and Goeree (1993)

THE W-ALGEBRAS $W_\infty[\lambda]$

- Symmetries generated by the lowest-weight part of λ

$$\delta_\lambda a(\theta) = P_- \sum_{n=0}^{\infty} \big(- D_\theta R L_- \big)^n D_\theta \lambda_+(\theta)$$

A.C., Fredenhagen and Pfenninger (2011)

see also de Boer and Goeree (1993)

- Apply the previous construction to hs[λ]
  - Different λ → non-isomorphic W-algebras
  - λ = N → classical W_N algebras

Gaberdiel and Hartmann (2011)

THE W-ALGEBRAS W∞[λ]

- Symmetries generated by the lowest-weight part of λ

$$\delta_\lambda a(\theta) = P_- \sum_{n=0}^{\infty} \big(- D_\theta R L_- \big)^n D_\theta \lambda_+(\theta)$$

A.C., Fredenhagen and Pfenninger (2011)

see also de Boer and Goeree (1993)

- Apply the previous construction to hs[λ]
  - Different λ → non-isomorphic W-algebras
  - λ = N → classical W_N algebras

Gaberdiel and Hartmann (2011)

$$a_-(\theta) = \frac{2\pi}{k} \sum_{j=1}^{\infty} \mathcal{W}_j(\theta)\, W^j_{-j}$$

$$\boxed{\begin{aligned}\delta_i \mathcal{W}_j &= \frac{k}{2\pi(2i)!}\, \epsilon^{(2i+1)}\, \delta_{i,j} \\ &+ \sum_{r=1}^{i} \sum_{\substack{L=|i-j|+r \\ i+j+L+r \ \text{even}}}^{i+j-r} \sum_{\{a_t\}} \sum_{\{p_t\}} C[i,j]_{a_1 \dots a_r ; p_1 \dots p_r}\, \mathcal{W}^{(p_1)}_{a_1} \dots \mathcal{W}^{(p_r)}_{a_r}\, \epsilon^{(\hat{n} - \sum_1^r p_t)}\end{aligned}}$$

THE W-ALGEBRAS $W_\infty[\lambda]$

- Symmetries generated by the lowest-weight part of λ

$$\delta_\lambda a(\theta) = P_- \sum_{n=0}^{\infty} \big(- D_\theta R L_- \big)^n D_\theta \lambda_+(\theta)$$

A.C., Fredenhagen and Pfenninger (2011)

see also de Boer and Goeree (1993)

- Apply the previous construction to hs[λ]
  - Different λ → non-isomorphic W-algebras

    Gaberdiel and Hartmann (2011)
  - λ = N → classical W_N algebras

$$c = \frac{3\,l}{2\,G}$$

$$\delta_i \mathcal{W}_j = \frac{k}{2\pi (2i)!}\,\epsilon^{(2i+1)}\,\delta_{i,j} + \sum_{r=1}^{i} \sum_{\substack{L=|i-j|+r \\ i+j+L+r \ \text{even}}}^{i+j-r} \sum_{\{a_t\}} \sum_{\{p_t\}} C[i,j]_{a_1\dots a_r;\,p_1\dots p_r}\, \mathcal{W}^{(p_1)}_{a_1} \dots \mathcal{W}^{(p_r)}_{a_r}\, \epsilon^{(\hat{n} - \sum_1^r p_t)}$$

METRIC-LIKE FORMULATION?

LORENTZ-LIKE SYMMETRIES

- $\mathfrak{g} \oplus \mathfrak{g}$ Chern-Simons $\approx$ Cartan-Weyl approach to gravity
- How to extract information on the metric-like theory?
  - How to express Frondal's fields in terms of vielbeins?
  - How to generalise $g_{\mu\nu} = \eta_{ab}\, e_\mu{}^a e_\nu{}^b$?
- Look at gauge transformations

$$\boxed{\begin{aligned} \delta A &= dA + [\,A\,,\,\lambda\,]\,, \\ \delta \tilde{A} &= d\tilde{A} + [\,\tilde{A}\,,\,\tilde{\lambda}\,] \end{aligned} \quad \Rightarrow \quad \begin{aligned} \delta e &= d\xi + [\,\omega\,,\,\xi\,] + [\,e\,,\,\Lambda\,]\,, \\ \delta\omega &= d\Lambda + [\,\omega\,,\,\Lambda\,] + \frac{1}{l^2}\,[\,e\,,\,\xi\,] \end{aligned}}$$

$$e = \frac{l}{2}\left(A - \tilde{A}\right) \qquad \xi = \frac{l}{2}\left(\lambda - \tilde{\lambda}\right)$$

$$\omega = \frac{1}{2}\left(A + \tilde{A}\right) \qquad \Lambda = \frac{1}{2}\left(\lambda + \tilde{\lambda}\right)$$

LORENTZ-LIKE SYMMETRIES

- $\mathfrak{g} \oplus \mathfrak{g}$ Chern-Simons ≈ Cartan-Weyl approach to gravity
- How to extract information on the metric-like theory?
  - How to express Frondal's fields in terms of vielbeins?
  - How to generalise $g_{\mu\nu} = \eta_{ab}\, e_\mu{}^a e_\nu{}^b$?
- Look at gauge transformations

$$\begin{aligned} \delta A &= dA + [\,A\,,\,\lambda\,]\,, \\ \delta \tilde{A} &= d\tilde{A} + [\,\tilde{A}\,,\,\tilde{\lambda}\,] \end{aligned} \quad \Rightarrow \quad \begin{aligned} \delta e &= d\xi + [\,\omega\,,\,\xi\,] + [\,e\,,\,\Lambda\,]\,, \\ \delta\omega &= d\Lambda + [\,\omega\,,\,\Lambda\,] + \frac{1}{l^2}[\,e\,,\,\xi\,] \end{aligned}$$

$$e = \frac{l}{2}\left(A - \tilde{A}\right) \qquad \xi = \frac{l}{2}\left(\lambda - \tilde{\lambda}\right)$$

$$\omega = \frac{1}{2}\left(A + \tilde{A}\right) \qquad \Lambda = \frac{1}{2}\left(\lambda + \tilde{\lambda}\right)$$

Lorentz-like symmetries!

METRIC-LIKE FIELDS

- The Lorentz algebra is extended to $\mathfrak{g} \supseteq sl(2,\mathbb{R}) \sim so(1,2)$
- Metric-like fields must be invariant under generalised Lorentz transformations
  - This fixes the non-linear structure of fields in the SL(3)×SL(3) theory

$$g \,=\, e_a\, e^a \,-\, 2\,\sigma\; e_{ab}\, e^{ab}$$

$$\varphi \,=\, e_a\, e_b\, e^{ab} \,+\, \frac{4}{3}\,\sigma\; e_{ac}\, e_b{}^c e^{ab}$$

METRIC-LIKE FIELDS

- The Lorentz algebra is extended to $\mathfrak{g} \supseteq sl(2,\mathbb{R}) \sim so(1,2)$
- Metric-like fields must be invariant under generalised Lorentz transformations
  - This fixes the non-linear structure of fields in the SL(3)×SL(3) theory

$$g = e_a\, e^a - 2\,\sigma\; e_{ab}\, e^{ab} \qquad \sim \mathrm{tr}\,(e\cdot e) \qquad\qquad e = e^A\, T_A$$

$$\varphi = e_a\, e_b\, e^{ab} + \frac{4}{3}\sigma\; e_{ac}\, e_b{}^c e^{ab} \sim \mathrm{tr}\,(e\cdot e\cdot e)$$

A.C., Fredenhagen, Pfenninger and Theisen (2010)

METRIC-LIKE FIELDS

- The Lorentz algebra is extended to $\mathfrak{g} \supseteq sl(2,\mathbb{R}) \sim so(1,2)$
- Metric-like fields must be invariant under generalised Lorentz transformations
  - This fixes the non-linear structure of fields in the SL(3)×SL(3) theory

$$g \;=\; e_a\, e^a \;-\; 2\,\sigma\; e_{ab}\, e^{ab} \qquad \sim \mathrm{tr}\,(e\cdot e) \qquad\qquad e = e^A\, T_A$$

$$\varphi \;=\; e_a\, e_b\, e^{ab} \;+\; \frac{4}{3}\,\sigma\; e_{ac}\, e_b{}^c e^{ab} \;\sim \mathrm{tr}\,(e\cdot e\cdot e)$$

A.C., Fredenhagen, Pfenninger and Theisen (2010)

- Lorentz-like invariance guaranteed by the trace

$$\delta_\Lambda e \;=\; [\,e\,,\,\Lambda\,] \quad\Rightarrow\quad \delta_\Lambda \mathrm{tr}\,(e^n) \;=\; n\,\mathrm{tr}\,(e^{n-1}[\,e\,,\,\Lambda\,]) \;=\; 0$$

- For rank >3 multiple invariants in SL(N)×SL(N) theories!

INVARIANT TENSORS OF hs[λ]

- How to identify metric-like fields?

A.C., Fredenhagen and Pfenninger (2011)

  - First look for possible candidates...
  - Invariant tensors of hs[λ] can be built with the help of the ★−product

$$k_{A_1 \dots A_s} \equiv \frac{1}{s!} \operatorname{tr}\left(T_{(A_1} \dots T_{A_s)} \right) := \frac{6}{(\lambda^2-1)\, s!} \, T_{(A_1} \star \dots \star T_{A_s)} \Big|_{T_A = 0}$$

  - Most general combination of rank s

$$\boxed{\varphi_s = a \operatorname{tr}(e^s) + b \operatorname{tr}(e^2) \operatorname{tr}(e^{s-2}) + \dots} \qquad \operatorname{tr}\left(e^n\right) := e^{A_1} \dots e^{A_n} k_{A_1 \dots A_n}$$

- Conditions that φₛ must satisfy
  - Double tracelessness
  - Correct linearised dynamics i.e. $\varphi_{\mu_1 \dots \mu_s} \sim \bar{e}_{(\mu_1}{}^{a_1} \dots \bar{e}_{\mu_{s-1}}{}^{a_{s-1}} h_{\mu_s)\, a_1 \dots a_{s-1}}$

INVARIANT TENSORS OF hs[λ]

- How to identify metric-like fields?

A.C., Fredenhagen and Pfenninger (2011)

  - First look for possible candidates...
  - Invariant tensors of hs[λ] can be built with the help of the ★−product

$$k_{A_1 \dots A_s} \equiv \frac{1}{s!} \operatorname{tr}\left(T_{(A_1} \dots T_{A_s)} \right) := \frac{6}{(\lambda^2 - 1)\, s!} \, T_{(A_1} \star \dots \star T_{A_s)} \Big|_{T_A = 0}$$

  - Most general combination of rank s

$$\boxed{\varphi_s = a \operatorname{tr}(e^s) + b \operatorname{tr}(e^2) \operatorname{tr}(e^{s-2}) + \dots} \qquad \operatorname{tr}\left(e^n\right) := e^{A_1} \dots e^{A_n} k_{A_1 \dots A_n}$$

- Conditions that φ_s must satisfy
  - Double tracelessness
  - Correct linearised dynamics i.e. $\varphi_{\mu_1 \dots \mu_s} \sim \bar{e}_{(\mu_1}{}^{a_1} \dots \bar{e}_{\mu_{s-1}}{}^{a_{s-1}} h_{\mu_s)\, a_1 \dots a_{s-1}}$

FIRST EXAMPLES & PROBLEMS...

- Match the standard linearised expression for φ_s
  - No background for s > 2
  - $\varphi_{\mu_1 \dots \mu_s} \sim \bar{e}_{(\mu_1}{}^{a_1} \dots \bar{e}_{\mu_{s-1}}{}^{a_{s-1}} h_{\mu_s)\, a_1 \dots a_{s-1}}$
- First positive results...

$$\varphi_4 \sim \operatorname{tr} e^4 - \frac{1}{10}(3\lambda^2 - 7)\left(\operatorname{tr} e^2\right)^2 ,$$

$$\varphi_5 \sim \operatorname{tr} e^5 - \frac{5}{21}(3\lambda^2 - 13)\operatorname{tr} e^2 \operatorname{tr} e^3 .$$

FIRST EXAMPLES & PROBLEMS...

- Match the standard linearised expression for φ_s
  - No background for s > 2
  - $\varphi_{\mu_1 \dots \mu_s} \sim \bar{e}_{(\mu_1}{}^{a_1} \dots \bar{e}_{\mu_{s-1}}{}^{a_{s-1}} h_{\mu_s)\, a_1 \dots a_{s-1}}$
- First positive results...

$$\varphi_4 \sim \operatorname{tr} e^4 - \frac{1}{10}(3\lambda^2 - 7)\left(\operatorname{tr} e^2\right)^2 ,$$

$$\varphi_5 \sim \operatorname{tr} e^5 - \frac{5}{21}(3\lambda^2 - 13)\operatorname{tr} e^2 \operatorname{tr} e^3 .$$

- ... and first problems
  - $\varphi_6 \sim \operatorname{tr} e^6 + \alpha(\lambda)\operatorname{tr} e^2 \operatorname{tr} e^4 + \beta(\lambda)\left(\operatorname{tr} e^2\right)^3 + \gamma(\lambda)\left(\operatorname{tr} e^3\right)^2$
  - Imposing the matching at linearised level

$$\alpha(\lambda) = -\frac{5}{6}(\lambda^2 - 7)\, ; \qquad \beta(\lambda) = \frac{1}{42}(6\lambda^4 - 71\lambda^2 + 125)\, ; \qquad \gamma(\lambda) = \, ?$$

CONCLUSIONS & OUTLOOK

- Higher spins in D=2+1: a relatively simple setup to address questions that are still unaccessible for D≥4
  - Asymptotic symmetries ⇔ non-linear W-algebras
  - Metric-like formulation ⇔ proper Lorentz-like invariants
- First applications
  - HS extensions of the AdS_3/CFT_2 correspondence — Gaberdiel, Gopakumar and many others...
  - Study of BTZ-like solutions with HS hairs — Gutperle, Kraus and collaborators

CONCLUSIONS & OUTLOOK

- Higher spins in D=2+1: a relatively simple setup to address questions that are still unaccessible for D≥4
  - Asymptotic symmetries ⇔ non-linear W-algebras
  - Metric-like formulation ⇔ proper Lorentz-like invariants
- First applications
  - HS extensions of the AdS_3/CFT_2 correspondence — Gaberdiel, Gopakumar and many others...
  - Study of BTZ-like solutions with HS hairs — Gutperle, Kraus and collaborators

- **Asymptotic symmetries in presence of matter couplings?**
- **HS geometry? Aka complete the recovering of Fronsdal**