Supersymmetry, Flavour
and Vacuum Selection

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Theory-Cosmology-Phenomenology
Corfu 2011

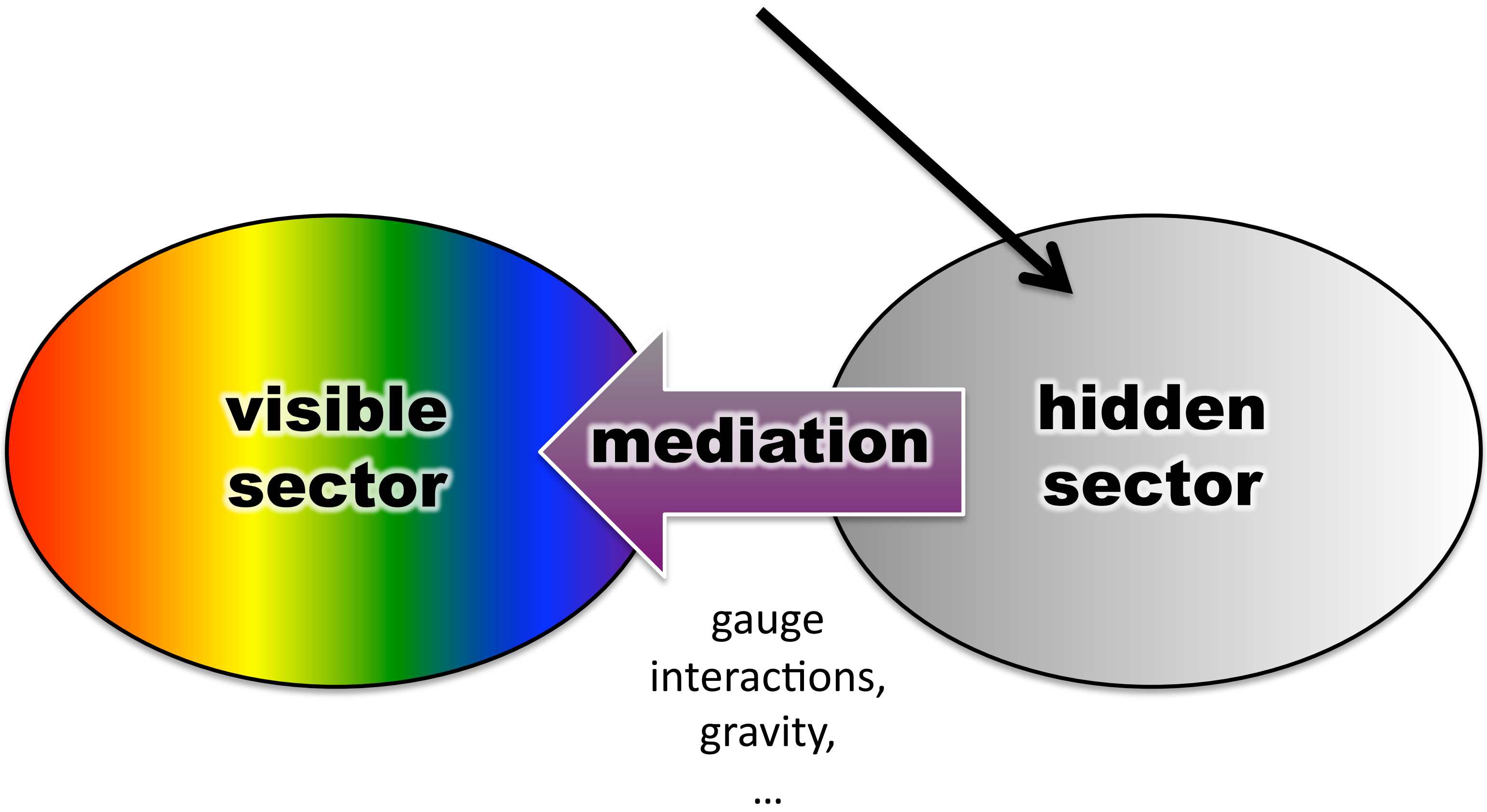
Z. Lalak, S. Pokorski, G. Ross, JHEP 1008:129, 2010
I. Dalianis, Z. Lalak, Phys. Lett. B697: 385, 2011
L. Calibbi, Z. Lalak, S. Pokorski, R. Ziegler, to appear

Outline:

- supersymmetry
- flavour problem and MFV
- horizontal symmetries
- horizontal symmetries and SUSY breaking
- flavour messenger sector
- SUSY breaking in the flavour sector and cosmology

SUPERSYMMETRY BSM

Supersymmetry breaking



Gauge vs gravity mediation

$$m_{3/2} = \frac{F^X}{\sqrt{3}M_P}$$

$$m_{gaugino} = \frac{\alpha(m)}{4\pi}\frac{F^X}{<X>}$$

$$m_{scalar} = \frac{\alpha(m)}{4\pi}\boxed{\frac{\sqrt{3}M_P}{<X>}}m_{3/2}$$

$$\Longrightarrow \quad m_{3/2,moduli} \ll m_{g,s}$$

- gauge mediation: natural suppression of flavour changing processes, works in the flat limit

FLAVOUR BBSM

Flavour group

$$G_F = SU(3)_q^3 \otimes SU(3)_l^2 \otimes U(1)^5,$$

where $SU(3)_q^3 = SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R}$, $SU(3)_l^2 = SU(3)_{L_L} \otimes SU(3)_{E_R}$

$$L_{q,e} = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R H_c + \bar{L}_L Y_E E_R H + h.c.$$

$$L_\nu = \bar{L}_L Y_\nu N_R H_c + \bar{N}_R M N_R + h.c.$$

$$m_{\nu\, eff} = -Y_\nu \frac{v^2}{M} Y_\nu^T = U_{11} m_d U_{11}^T$$

G. D'Ambrosio, G. Giudice, G. Isidori, A. Strumia

$$Y_U \sim (3, \bar{3}, 1)_{SU(3)_q^3},\ Y_D \sim (3, 1, \bar{3})_{SU(3)_q^3},\ Y_E \sim (3, \bar{3})_{SU(3)_l^2}$$

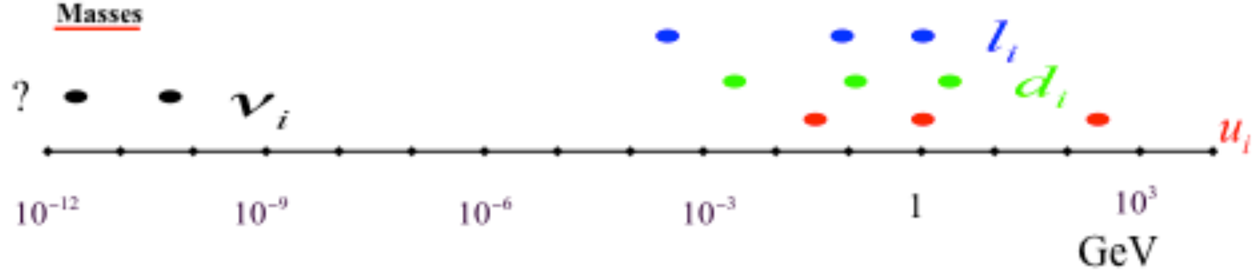
$$D_R^{'} = V_D^{\dagger} D_R,\ U_R^{'} = V_U^{\dagger} U_R,\ \bar{Q}_L = \bar{Q}_L^{'} S_d^{\dagger}.$$

$$L_W^{(+)} = \frac{g_2}{\sqrt{2}} \bar{U}_L \gamma^{\mu} V_{CKM} D_L W_{\mu}^{+} + \frac{g_2}{\sqrt{2}} \bar{\nu}_L \gamma^{\mu} V_{PMNS} E_L W_{\mu}^{+}$$

$$V_{CKM} = S_u^{\dagger} S_d \qquad\qquad V_{PMNS} = U_{11}^{T} S_e$$

data:

Masses



G.G. Ross

Mixing

$$V_{CKM} = \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix} \quad V_{MNS} = \begin{pmatrix} 0.79-0.88 & 0.48-0.61 & <0.2 \\ 0.27-0.49 & 0.45-0.71 & 0.52-0.82 \\ 0.28-0.5 & 0.51-0.65 & 0.57-0.81 \end{pmatrix}$$

MFV

$$Y_U \sim (3,\bar{3},1)_{SU(3)^3_q},\; Y_D \sim (3,1,\bar{3})_{SU(3)^3_q},\; Y_E \sim (3,\bar{3})_{SU(3)^2_l}$$

MFV:

- the only source of G_F breaking are the Yukawa spurions
- higher dimension flavour violating terms arise as the most general $SU(3)^5$ invariant higher dimension operators

The leading two-fermion operators from which one may determine the MFV predictions for the operators in the table are

G. D'Ambrosio et al; F. Gabbiani, A. Massiero; W. Altmannshoffer, A. Buras et al; G. Isidor, Y. Nir, G. Perez

$$\bar{Q}_L Y_u Y_u^\dagger Q_L,\; \bar{D}_R Y_d^\dagger Y_u Y_u^\dagger Q_L$$

$\gamma_{ij} \sim \alpha^2$

	Flavour violating dimension six operator	Λ_{MFV} (in TeV) $-$	$+$
$\mathcal{O}_0 =$	$\frac{1}{\Lambda^2}(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)^2$	6.4	5.0
$\mathcal{O}_{F1} =$	$\frac{1}{\Lambda^2} H^\dagger \left(\bar{D}_R \lambda_d \lambda_{\rm FC} \sigma_{\mu\nu} Q_L\right) F_{\mu\nu}$	9.3	12.4

EWDD: $\lambda_{FC} = (Y_u Y_u^\dagger)_{ij} = \lambda_t^2 U_{3i}^* U_{3j}$ where U is the CKM matrix.

Flavour vs hierarchy

- $\Lambda = M$ - where M is a mass of a heavy particle, but ...
- usually there exists a mechanizm easing the hierachy problem with its scale Λ_h
- possible suppressions are
  $1/\Lambda_h^2$ $1/(\Lambda_h M)$, $1/M^2$
- in SUSY luckily (or not) the first option is realized, with $\Lambda_h = M_{SUSY}$

in models of fermion masses based on SB family symmetries M is a mass of heavy fermions and hierarchy of masses and mixings is built from powers of a small parameter $< \theta > /M$ where θ is a familon field.

M_{SUSY}, F_{SUSY}, M, F_θ - are there any constraints?

Froggatt-Nielsen U(1)

M. Leurer, Y. Nir, N. Seiberg; L. Ibanez, G.G. Ross; E. Dudas, S. Pokorski, C. Savoy, Ch. Grojean; P. Binetruy, S. Lavignc, P. Ramond

$$\bar{Q}_L Y_U U_R H_c = \bar{Q}_L^i \left[a_i^j \left(\frac{\theta}{M} \right)^{|u_j + q_i|} \right] U_{Rj} H_c$$

Two familon fields, $X, \bar{X}$, with charges ± 1, $-w = q(H)$.

$$\begin{array}{rc} Q_{L\ 1,2,3}: & (-3+w, 2+w, w) \\ D^c\ 1,2,3: & (-5,0,0) \\ U^c\ 1,2,3: & (-5,0,0) \end{array}$$

$$Y_{U,D} = \begin{pmatrix} \epsilon_{u,d}^{|-8+w|} & \epsilon_{u,d}^{|-3+w|} & \epsilon_{u,d}^{|-3+w|} \\ \epsilon_{u,d}^{|-3+w|} & \epsilon_{u,d}^{|2+w|} & \epsilon_{u,d}^{|2+w|} \\ \epsilon_{u,d}^{|-5+w|} & \epsilon_{u,d}^{|w|} & \epsilon_{u,d}^{|w|} \end{pmatrix}$$

where $\epsilon_{u,d} = \frac{<X>}{M_{U,D}}$ - we allow for different messenger masses in the up and the down sectors.

For $w = 0$ $Y_t : Y_c : Y_u = 1 : \epsilon^2 : \epsilon^8$, $U_{12} = \epsilon$, $U_{13} = \epsilon^3$, $U_{23} = \epsilon^2$

Bounds on suppression scale

Flavour violating dimension six operator		Λ/Λ_{MFV} Ex. 1	Ex. 2	Ex. 3	$U(1)^2$	N-A	F
$\mathcal{O}_0 =$	$\frac{1}{2}(\bar{Q}_L X^Q_{LL} Q_L)^2$	ϵ^{-4}	ϵ^{-4}	1	1	ϵ^{-2}	1
$\mathcal{O}_{F1} =$	$H^\dagger \left(\bar{D}_R X^{D\dagger}_{LR} \sigma_{\mu\nu} Q_L\right) F_{\mu\nu}$	$x\epsilon^{-2}$	$x\epsilon^{-3/2}$	$x\epsilon^{-2}$	$x\epsilon$	$x\epsilon^{-2}$	$x\epsilon^{-2}$
$\mathcal{O}_{G1} =$	$H^\dagger \left(\bar{D}_R X^{D\dagger}_{LR} \sigma_{\mu\nu} T^a Q_L\right) G^a_{\mu\nu}$	$x\epsilon^{-2}$	$x\epsilon^{-3/2}$	$x\epsilon^{-2}$	$x\epsilon$	$x\epsilon^{-2}$	$x\epsilon^{-2}$
$\mathcal{O}_{\ell 1} =$	$(\bar{Q}_L X^Q_{LL} \gamma_\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$	ϵ^{-2}	ϵ^{-2}	1	1	ϵ^{-1}	1
$\mathcal{O}_{\ell 2} =$	$(\bar{Q}_L X^Q_{LL} \gamma_\mu \tau^a Q_L)(\bar{L}_L \gamma_\mu \tau^a L_L)$	ϵ^{-2}	ϵ^{-2}	1	1	ϵ^{-1}	1
$\mathcal{O}_{H1} =$	$(\bar{Q}_L X^Q_{LL} \gamma_\mu Q_L)(H^\dagger i D_\mu H)$	ϵ^{-2}	ϵ^{-2}	1	1	ϵ^{-1}	1
$\mathcal{O}_{q5} =$	$(\bar{Q}_L X^Q_{LL} \gamma_\mu Q_L)(\bar{D}_R \gamma_\mu D_R)$	ϵ^{-2}	ϵ^{-2}	1	1	ϵ^{-1}	1

Here $x = (m_t/m_b)^{1/2} \approx 6.2$.

M. Leurer, Y. Nir, N. Seiberg
S.King, G.G. Ross
I. de Medeiros-Varzielas, S. King, G.G. Ross
S. King, G. Leontaris, G.G. Ross
P. Chankowski, K. Kowalska, S. Lavignac, S. Pokorski

SUSY breaking

In the EW basis

$$m^2_{dLL\,ij} \sim m^2 \epsilon^{|q_{Li} - q_{Lj}|} + \Delta_i \delta_{ij}$$

where $\Delta_i = m^2_{ii} - m^2$

after rotation on the superfields to the EWDD basis

$$\left(S_d^\dagger m^2_{dLL} S_d\right)_{ij} \sim m^2 \epsilon^{|q_i - q_j|} + \Delta_i S_{dij} + \Delta_j S_{dji}$$

and $S_{ij} \geq \epsilon^{|q_i - q_j|}$

$\rightarrow$ the effect of initial diagonal splitting can be large

Non-degeneracy of squark masses

$$D^2 = g_f^2 \left(|\phi|^2 - |\bar{\phi}|^2 + c_{\tilde{d}L} |\tilde{d}_L|^2 + c_{\tilde{d}R} |\tilde{d}_R|^2 + ... \right)^2$$

$$\Delta m^2_{\tilde{f}L,R} = c_{\tilde{d}L,R} g_f < D >$$

$$V = \tfrac{1}{2} D^2 + m_\phi \phi^2 + m_{\bar{\phi}} \bar{\phi}^2 + |\phi\bar{\phi} - M^2|^2$$

with $\delta W = \psi(\phi\bar{\phi} - M^2)$

$$\delta^d_{12LL} \approx \frac{< D >}{\tilde{m}^2} \left(c_{\tilde{d}_L} S_{d11} S^*_{d21} + c_{\tilde{s}_L} S_{d12} S^*_{d22} + c_{\tilde{b}_L} S_{d13} S^*_{d23} \right)$$

this is $\frac{<D>}{m^2} \epsilon$!

while $\delta_{d12LL} < \epsilon^2$ and $\delta_{d12LR} < \epsilon^4$

RG running

FN predictions hold at $M \leq M_{GUT}$

At 1 TeV $m_g \approx 3m_{1/2}$ and $m_{\tilde{q}} \approx m_0^2 + 6m_{1/2}^2$

$x = m_g^2/m_{\tilde{q}}^2 = 1$ gives $x_0 = m_{1/2}^2/m_0^2 = 1/3$ and
$m_{\tilde{q}} = 350$ GeV implies $m_0 = 200$ GeV and $m_{1/2} = 120$ GeV

However, rising x_0 helps!
$m_{1/2}/m_0 = 7$ gives
$m_{\tilde{q}} = 350$ GeV implies $m_0 = 20$ GeV and $m_{1/2} = 140$ GeV
which gives additional suppression of $\frac{D}{m_{\tilde{q}}^2} \approx 1/300 \sim \epsilon^3$

Now, rising $m_{1/2}$:
$m_{1/2}/m_0 = 7$ with $m_{1/2} = 300$ GeV gives
$m_{\tilde{q}} = 800$ GeV and $m_g = 900$ GeV
consistent with low fine-tuning

For off-diagonal terms only a moderate suppression of 0.1 due to RG giving $x_0 \sim 1$ needed

Summary I

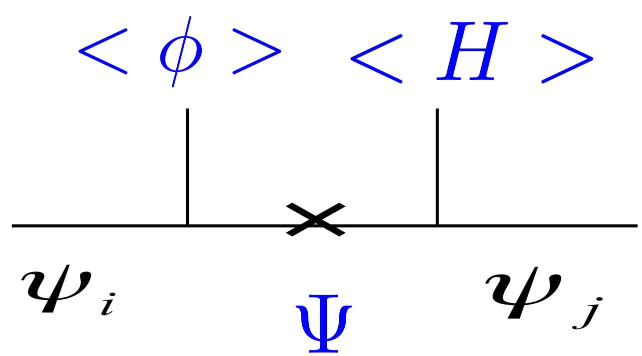
- Family symmetry models generally violate MFV

- However, they can be brought into agreement with data with certain, rather mild, constraints on susy breaking parameters, like $m_{1/2} \geq 7\, m_0$

Flavour sector

$$W_f = X_f \bar{\Psi}\Psi + a^{ij}\bar{\Psi}_i \psi_j \phi + H\Psi\bar{\psi}$$

where $X_f = M + \Theta^2 F_f$ is a gauge singlet, $\bar{\Psi}$, Ψ are heavy v-like quarks, $\psi, \bar{\psi}$ are light fermions and ϕ is the flavon (formerly θ)



$$\epsilon = \frac{< \phi >}{M}$$

Flavon F-term

$$W = HQ_iU_j^c a^{ij}\left(\frac{\Phi}{M}\right)^{|q_i+q_j|}$$

leads to sfermion masses of the form

$$m_{ij}^2 = \frac{vF_\Phi}{M}|q_i+q_j|\epsilon^{|q_i+q_j|-1}a^{ij}$$

This gives

$$\delta_{LRij} = \frac{vF_\phi}{M\epsilon m_{\tilde{q}}^2}\epsilon^{|q_i+q_j|}a'^{ij}$$

and requires

$$F_\phi \leq (350\,\mathrm{GeV})^2\,0.2\left(\frac{M}{v}\right)$$

S. Antusch, S.F. King, M. Malinsky, G.G. Ross

SUGRA: $F_\phi = m_{3/2} < \phi > c_\phi \rightarrow v m_{3/2} \leq m_{\tilde{q}}^2$

Stabilization of X_f

$$W_f = \lambda X \bar{\Psi}\Psi + a^{ij}\bar{\Psi}_i \psi_j \phi + H\Psi\bar{\psi} + \tilde{\lambda}\tilde{X}Q\bar{Q}$$

$$K = \bar{X}X + \sum_{i=1}^{n}\left(\bar{\tilde{\phi}}_i \tilde{\phi}_i + \bar{\phi}_i \phi_i\right)$$

$$-\frac{1}{16\pi^2} f_4 \frac{(\bar{X}X)^2}{\tilde{\Lambda}^2} - \frac{1}{16\pi^2} f_6 \frac{(\bar{X}X)^3}{\tilde{\Lambda}^4} + \ldots$$

$$W = \mu^2 X + \sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\phi}_i (m_{ij} + \lambda_{ij} X)\phi_j + c$$

Solutions

Z.L., S. Pokorski, K. Turzynski

$$X = \frac{1}{2\sqrt{3}} \frac{\tilde{\Lambda}^2}{f_4 M_P}$$ if $f_4 > 0$ and dominant

$$X^2 = \frac{8|f_4|}{9 f_6} \tilde{\Lambda}^2$$ if $f_4 < 0$ and $f_6 > 0$

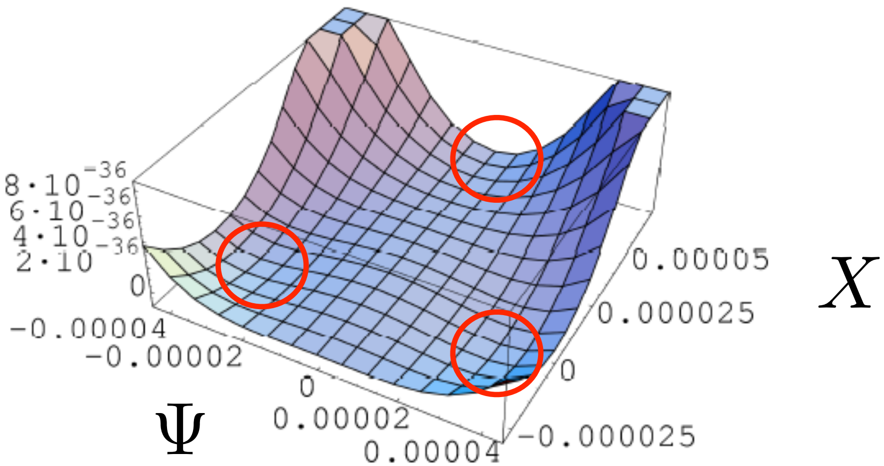
$$X^3 = \frac{16\pi^2}{9\sqrt{3} f_6} \frac{\tilde{\Lambda}^4}{M_P}$$ if $f_4 \approx 0$

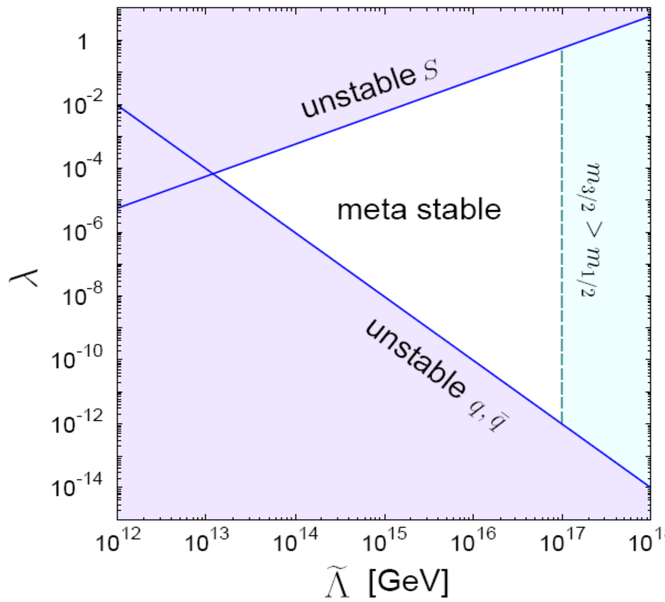
in all cases solutions exist if $\langle X \rangle \lesssim 10^{-3} M_P$

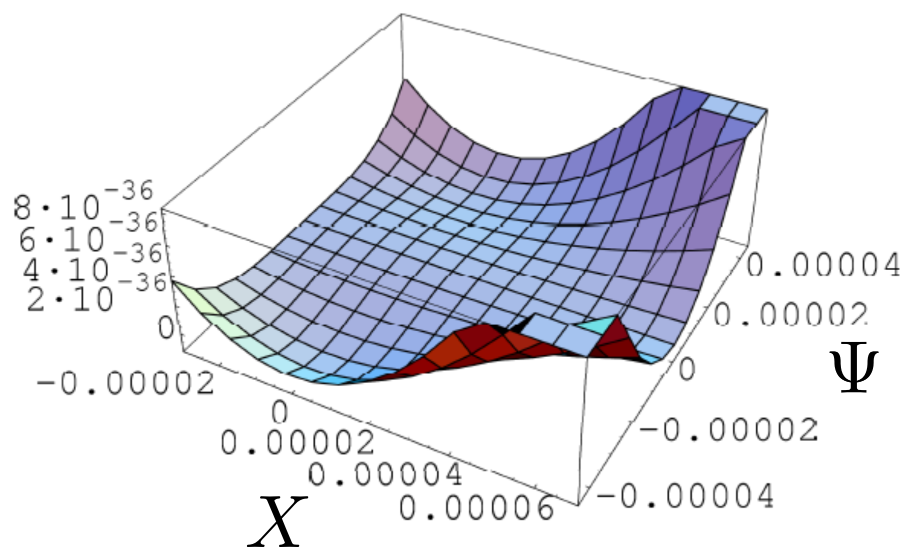
above this value of X the gravitational term gives to large negative slope

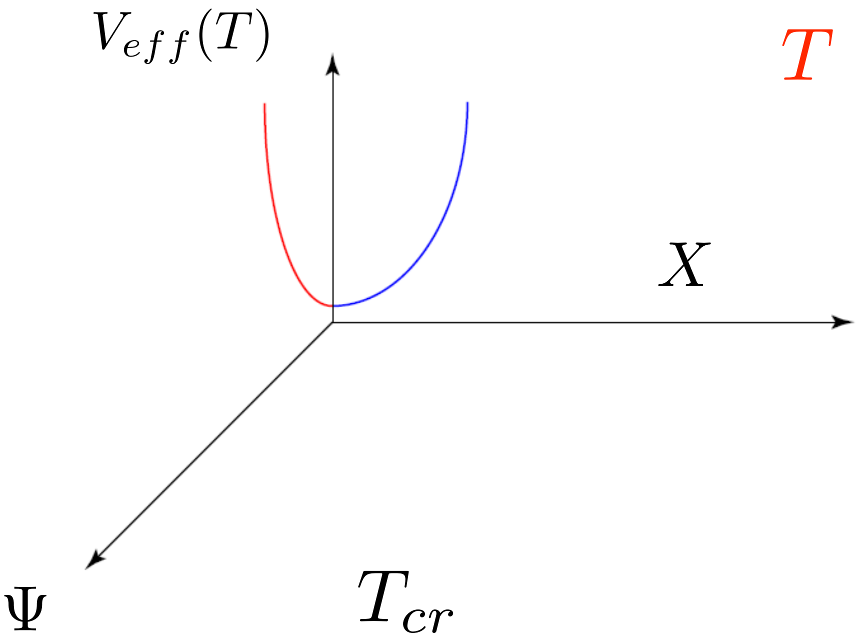
Cosmological vacuum selection

I. Dalianis, Z.L.







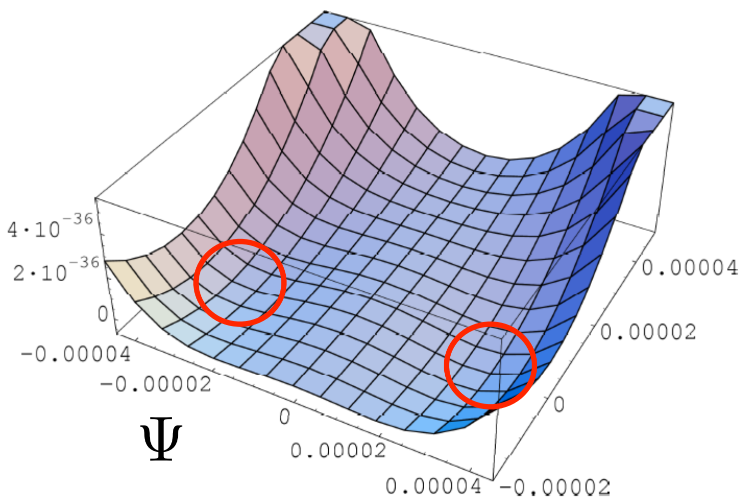


$T >> \tilde{\Lambda}$

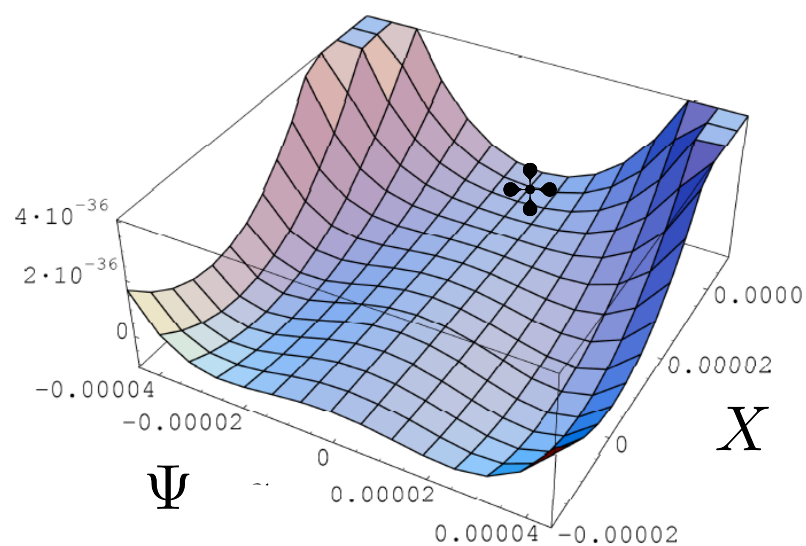
$$X \sim \frac{\mu^4}{\lambda^2 \tilde{\Lambda}^2}$$

$$\langle \Psi \rangle = 0$$

T_{cr}



T_S



Coupling to the SM

I. Dalianis, Z.L.

$$T_{cr}^{q} = \left(\frac{\lambda F_X}{8 g_s^2 + 4\lambda^2} \right)^{1/4}$$

$$T_S^2 = \frac{8}{5} \frac{F_X}{\lambda \sqrt{3} M_P} \left(\frac{F_X}{\lambda} \right)^{1/2}$$

$$T_S > T_{cr} \rightarrow \lambda < \left(\frac{10^{-3}}{2.5} \frac{\sqrt{F_X}}{\sqrt{3} M_P} \right)^{2/5}$$

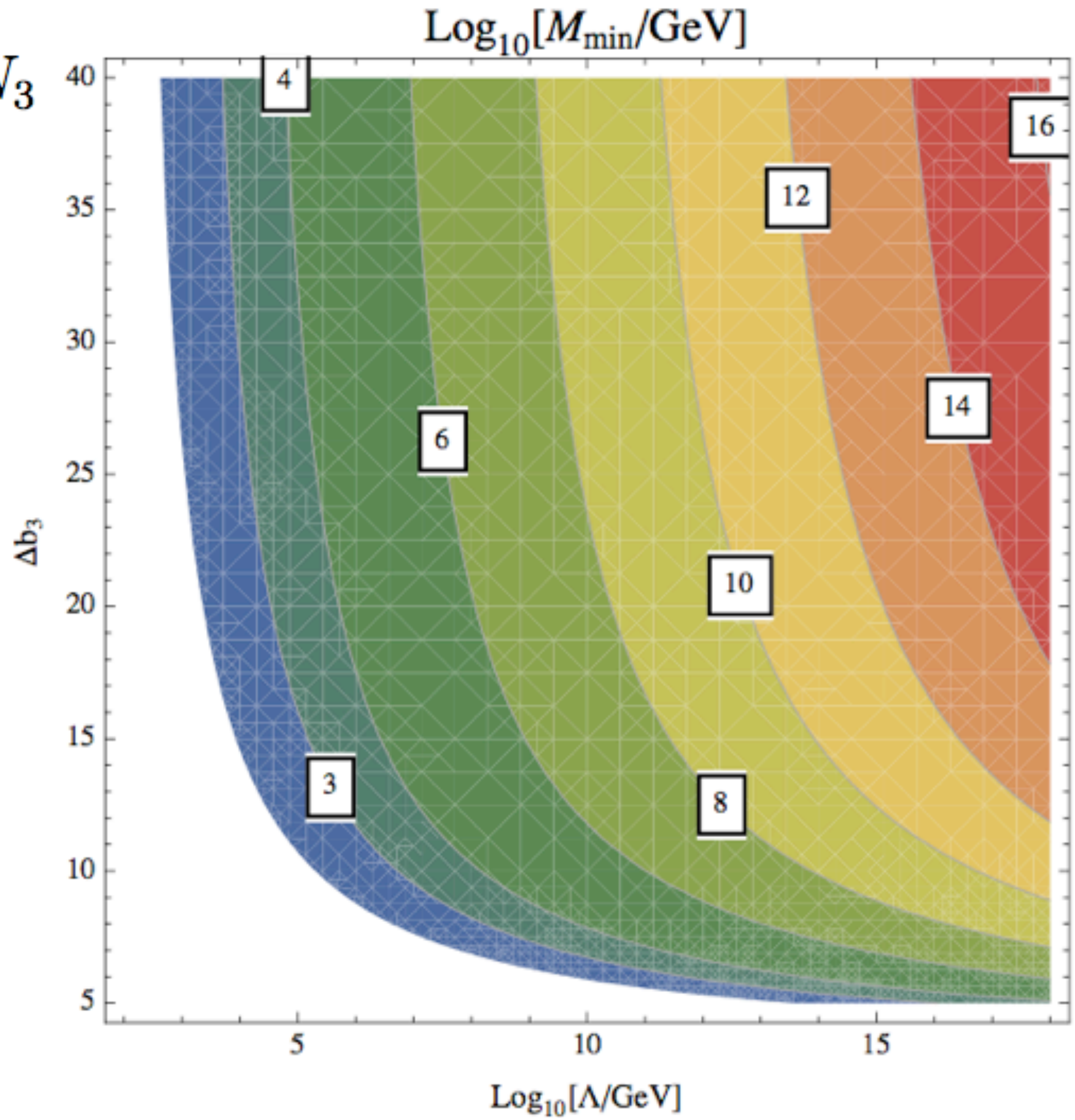
$$\sqrt{F_X} = 2.4\, 10^9 \,\mathrm{GeV} \;\; \text{gives} \;\; \lambda < 1.2\, 10^{-5}$$

$$\sqrt{F_X} = 2.4\, 10^8 \,\mathrm{GeV} \;\; \text{gives} \;\; \lambda < 4\, 10^{-6}$$

$$m_\Psi > 4\,\mathrm{TeV} \rightarrow \Lambda_{NP} = \langle X \rangle > \frac{4\,\mathrm{TeV}}{\lambda} > 4 \times 10^5\,\mathrm{TeV}$$

A. Buras, Ch. Grojean,
S.Pokorski, R. Ziegler

$$\Delta b_3 = -3 + N_3$$



L. Calibbi, Z.L., S. Pokorski, R. Ziegler

$$M \gtrsim M_{min} = \Lambda \exp\left[-\frac{2\pi}{\Delta b_3}\left(\frac{1}{\alpha_3(M_Z)} + \frac{3}{2\pi}\log\frac{\Lambda}{M_Z}\right)\right]$$

$$(\alpha_s \lesssim 4\pi)$$

$$\det Y \sim M^3 \Phi^N$$

$$N_3 \sim 10 - 20$$

$$m_\Psi > 10^{10} - 10^{12}\,\mathrm{GeV} \rightarrow$$

$$\Lambda_{NPF} = \langle X \rangle > \frac{10^{10} - 10^{12}\,\mathrm{GeV}}{\lambda} > 10^{15} - 10^{17}\,\mathrm{GeV}$$

Can one make X dominate supersymmetry breaking?

$$\mathcal{M} = M^2 \begin{pmatrix} \epsilon^2 & \epsilon & 0 \\ \epsilon & 1 & \frac{F_X^*}{M^2} \\ 0 & \frac{F_X}{M^2} & 1 \end{pmatrix}$$

M. Nardecchia, A. Romanino, R. Ziegler
L. Calibbi, Z.L., S. Pokorski, R. Ziegler

$$\tilde{m}^2 = -\epsilon^2 \frac{F_X^2}{M^2}$$

loops give $m_0^2 = \left(\frac{\alpha}{4\pi}\frac{F_X}{M}\right)^2$

$\alpha \sim \epsilon \sim 0.1 \rightarrow$ one needs $F_{\tilde{X}} \sim 10 \times F_X$

Non-minimal hidden sector

$$W = W_0 + f X e^{-T} + \tilde{f}\tilde{X} e^{+T} + \frac{1}{M}(X\tilde{X})^2$$

T. Jelinski, Z.L., J. Pawelczyk

$$K = X^\dagger X + \tilde{X}^\dagger \tilde{X} + \frac{m_V^2}{2}(T + T^\dagger)^2 + \delta K_R + \delta K_A$$

$$\delta K_R = -\frac{1}{\Lambda^2}(\bar{X}X)^2 - \frac{1}{\tilde{\Lambda}^2}(\bar{\tilde{X}}\tilde{X})^2$$

$$\delta K_A = -\frac{1}{m_V^2}(\bar{X}X - \bar{\tilde{X}}\tilde{X})^2$$

$$V_D = \frac{1}{2}D^2 = \frac{1}{2}\Big((|X|^2 - |\tilde{X}|^2) + m_V^2(T + T^\dagger)\Big)^2$$

Solutions:

- $X \sim \Lambda^2,\ \tilde{X} \sim \tilde{f}/f\ \tilde{\Lambda}^2$
- $F_X \sim F_{\tilde{X}} \sim f$

$\Lambda,\ \tilde{\Lambda} \ll (fM)^{1/4}$

- $X = 2\Lambda^2, \quad \tilde{X} = -\frac{\tilde{f}M}{8\Lambda^4}$
- $F_X = f,\ F_{\tilde{X}} = m_{3/2}\tilde{X} \ll f$

$\Lambda \gg (fM)^{1/8}$

- $\mathcal{O}_\mu = \eta \left(\frac{X^\dagger H_u H_d}{\Lambda} \right)_D,\ \mathcal{O}_{\mathcal{B}} = \left(\tilde{X} H_u H_d \right)_F$

e.g.

$\Lambda = 10^{-2},\ M = 10^{-1},\ f = 10^{-14},\ \tilde{f} = f$

gives

$X = 2\,10^{-4},\ \tilde{X} = -10^{-8},\ F_x = f,\ F_{\tilde{X}} = 6 \cdot 10^{-5} f$

Summary

- Family symmetry models generally violate MFV

- However, they can be brought into agreement with data with certain, rather mild, constraints on susy breaking parameters

- They need a separate flavour messenger sector which may be intertwined with the supersymmetry breaking sector

- Cosmological stability implies

$$\Lambda_{NP\ F} > 4 \times 10^5 \,\text{TeV}$$

- + perturbativity

$$\Lambda_{NP\ F} > 10^{15} \,\text{GeV}$$

Beyond Summary ...