CORFU2 2011:MSSM

Lectures 2 and 3

Gauge bosons of the SM are part of gauge multiplets described (in the Wess-Zumino gauge) by chiral vector multiplets W :

$$g \to \tilde{g} \text{ (Weyl)}$$
$$W \to \tilde{W}$$
$$B \to \tilde{B}$$
$$\vdots$$

Equal number of <u>physical</u> degrees of freedom

Every quark and lepton of the SM is viewed as the spinor component of a Wess-Zumino chiral superfield denoted by Φ :

$$L \to \Phi_L \, , \quad Q \to \Phi_Q \, , \quad \bar{u} \to \Phi_{\bar{u}} \, , \quad \bar{d} \to \Phi_{\bar{d}} \, ,$$
$$\bar{e} \to \Phi_{\bar{e}}$$

and

$$\Phi_L : \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \text{ and } \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$$

$\phi_{\bar{e}}$: $\bar{e}_L$ and $\hat{e}^*_R$ (right anti slepton)

ϕ_Q : $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ and $\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$

$\phi_{\bar{u}}$, $\phi_{\bar{d}}$

Two Higgs doublets (for anomaly cancellation)

ϕ_{H_d} : $\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}_{Y=-\frac{1}{2}}$ and $\begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}_{-\frac{1}{2}}$ (higgsino)

$$\Phi_{H_u} : \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}_1 \quad \text{and} \quad \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}_1$$

<u>Yukawa interactions of MSSM</u>

$$W_{MSSM} = Y^u_{ij} \Phi_Q^i \Phi_{\bar{u}}^j \Phi_{H_u} + Y^d_{ij} \Phi_Q^i \Phi_{\bar{d}}^j \Phi_{H_d}$$

$$+ Y^e_{ij} \Phi_L^i \Phi_{\bar{e}}^j \Phi_{H_d}$$

i, j - family indices

This superpotential cannot by itself reproduce the real world because it is invariant under too many global symmetries and does not give a potential with proper ground state.

Global transformations on the superfields are of the form

$$\Phi_s \to e^{i\eta n_s} \Phi_s$$

where s denotes $L, \bar{e}, \bar{u}, \bar{d}$ or Q

The global transformations that preserve supersymmetry obey the relations

$$n_{L_i} + n_{\bar{e}_i} + n_{H_d} = 0 \, , \qquad i = 1,2,3$$

$$n_Q + n_{\bar{u}} + n_{H_u} = 0 \qquad \text{any flavour}$$

$$n_Q + n_{\bar{d}} + n_{H_d} = 0 \qquad \text{any flavour}$$

$\rightarrow$ because, without loss of generality, we can bring the lepton Yukawa to diagonal form

Suppose for a moment there is only one chiral family ∋ 7 superfields and 3 couplings in the superpotential.

We get 4 independent symmetries, identified with

	n_L	$n_{\bar{e}}$	n_Q	$n_{\bar{u}}$	$n_{\bar{d}}$	n_{H_u}	n_{H_d}
L	1	-1	0	0	0	0	0
B	0	0	$1/3$	$-1/3$	$-1/3$	0	0
Y	-1	2	$1/3$	$-4/3$	$2/3$	1	-1
PQ	$-1/2$	$-1/2$	$-1/2$	$-1/2$	$-1/2$	1	1

These are Lepton number (L), the vectorial Baryon number (B), the chiral Peccei-Quinn (PQ) and the local hyperchange symmetry (Y).

With 3 families, we get in addition $L_e - L_\mu$ and $L_\mu - L_\tau$ symmetries

Except for PQ, all of them appear in the SM

In addition, we have a special global $U(1)$ symmetry called R-symmetry.

Invariance under R is achieved if, under a phase change of the Grassmann variables $\theta \to e^{i\beta}\theta$, the superpotential transforms as

$$W \to e^{2i\beta} W$$

So, our superpotential is R invariant with $\Phi_s \to e^{i2\beta/3}\Phi_s$ and the for the chiral spinor superfields that contain the gauge bosons we have

$$W^\alpha(x, \theta) \to e^{i\beta} W^\alpha(x, e^{i\beta}\theta)$$

and for gauginos

$$\lambda^a(x) \to e^{i\beta}\lambda^a(x)$$

Important : invariance under R- symmetry requires the gauginos to be massless, since their Majorana masses $\lambda^a \lambda_a$ transform under $R \Rightarrow R$ symmetry must be broken in the real world

PQ symmetry must also be broken: it involves fields that transform as weak isospinors, so it is necessarily broken at the electroweak scale.

If broken only spontaneously, by the electroweak vevs, we get a weakly coupled axion (Goldstone boson) with a mass in the keV range (ruled out by experiment)

μ - term

$$\mu \, H^T_{H_u} \, \tau_2 \, \Phi_{H_d}$$

breaks PQ and R symmetries leaving unbroken one linear combination

$$R' = R + \frac{1}{3} PQ$$

$$\mu \, \Phi_{H_u} \Phi_{H_d} \quad : \quad \Delta PQ = 2$$
$$\Delta R = -2/3$$

Supersymmetry must be broken to describe the real world.

Gaugino masses $m\,\lambda^a\lambda_a = m\,\epsilon^{ab}\lambda_b\lambda_a$

break R - symmetry but leave unbroken R - parity

$\lambda \to -\lambda \qquad (p = \bar{\lambda})$

quarks, leptons and Higgs bosons are even under R parity; all superparticles are odd
(LSP is stable)

Construction of MSSM (summary):

supersymmetrize all interactions of the SM; avoid as much as possible unwanted symmetries (PQ, R). Finally, we need supersymmetry breaking

Susy breaking

Hard breaking – affects UV

Soft breaking – OK, but we don't like explicit breaking

Spontaneous breaking: generated by the vacuum

Can MSSM break supersymmetry by itself?

Spontaneous supersymmetry breaking

In general: SSB = "vacuum" is not invariant under symmetry transformations.

For internal symmetries, e.g. $U(1)$ global symmetry and a complex scalar field

$$\varphi \to e^{i\beta} \varphi(x)$$

The Lagrangian

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - \lambda\left(\varphi^*\varphi - \frac{v^2}{2}\right)^2$$

Noether current corresponding to $U(1)$

$$j_\mu = i\,\varphi^* \overleftrightarrow{\partial_\mu}\, \varphi \quad , \quad \partial_\mu j^\mu = 0$$

Minimum of the potential

$$\varphi^*\varphi = \frac{v^2}{2}$$

$$\phi_0 = \frac{1}{\sqrt{2}}\, v\, e^{i\beta}$$

$$\delta\phi_0 \sim \beta v$$

The lowest energy configuration corresponds to a set of degenerate minima

Choosing one minimum v_0 (spontaneous breaking)

and writing $\varphi(x) = e^{i\eta(x)}\left(v_0 + \rho(x)\right)$

one sees that under $U(1)$ transformation

$$\eta(x) \to \eta(x) + \beta$$

Nambu-Goldstone boson $\xi(x) = v\,\eta(x)$

Indeed, rewriting the Lagrangian in terms of ρ, ξ we get

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho + \frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi\left(1+\frac{\rho}{v}\right)^2$$

$$- \frac{1}{2}m^2\rho^2 - \frac{\lambda}{4}\rho^4 - v\lambda\rho^3$$

$m_\xi = 0$, no potential for ξ

$m^2 = 2v^2\lambda$

N-G boson couples to the rest of the system

$$\mathcal{L}_{NG} = \frac{1}{v}\xi(x)\partial_\mu J^\mu$$

where $J_\mu(x)$ is the Noether current of the broken symmetry. It is obvious that a constant shift in $\xi(x)$ generates a surface term and leaves the action invariant $\left[J_\mu = -v\left(1+\frac{\rho(x)}{v}\right)^2\partial_\mu\xi(x)\right]$

Supersymmetric case, with a chiral superfield. We expect now to see a massless fermion (Goldstino), since the supersymmetry parameter is fermionic, that shifts by a constant under supersymmetry. In a constant field configuration, the supersymmetric algebra (under $\theta \to \theta + \alpha$, $\bar\theta \to \bar\theta + \bar\alpha$) gives

$$\delta \varphi_0 = \bar\alpha\, \psi_0 \,, \quad \delta \psi_0 = \alpha F_0 \,, \quad \delta F_0 = 0$$

For Lorentz invariance, we need $\psi_0 = 0$,

so $\delta\varphi_0 = 0$, $\delta\psi_0 = \alpha F_0$, $\delta F_0 = 0$,

with $\varphi_0 \neq 0$, $F_0 \neq 0$

This is the so-called F-term breaking (also $V = FF^* > 0$)

The chiral fermion then shifts under supersymmetry (Goldstino)

Let's now take a vector multiplet:

A_μ and λ must vanish in the vacuum (for Lorentz invariance), so only D can have non-zero vacuum expectation value D_0. Under $\theta \to \theta + \alpha$

$$\delta A_0^\mu = 0, \quad \delta \lambda_0 = \frac{1}{\sqrt{2}} \alpha D_0, \quad \delta D_0 = 0$$

In this case the gaugino λ is the Goldstino (D-term breaking)

For spontaneous supersymmetry breaking

$$V = F^* F + D^2 > 0$$

(one can also prove quite generally that for spont susy breaking $V > 0$)

MSSM

$$V = \sum_{\substack{i = H_u, H_d, L, \\ Q, \bar{e}, \bar{u}, \bar{d}}} F_i^+ F_i + \frac{1}{2}\left(D^2 + D^\alpha D^\alpha + D^A D^A\right)$$

$$U(1): \; D = \frac{1}{2} g_1 \Big[-\tilde{L}^+_{L_i} \tilde{L}_{L_i} + 2 \tilde{e}^*_{R_i} \tilde{e}_{R_i} + \frac{1}{3} \tilde{Q}^+_{L_i} \tilde{Q}_{L_i} + \frac{4}{3} \tilde{u}^+_{R_i} \tilde{u}_{R_i} - \frac{2}{3} \tilde{d}^+_{R_i} \tilde{d}_{R_i} + H_u^+ H_u - H_d^+ H_d \Big]$$

$$SU(2): \quad D^{\alpha} = g_2 \Big[\tilde{L}^{+}_{Li} \frac{\tau^{\alpha}}{2} \tilde{L}_{Li} + \tilde{Q}^{+}_{Li} \frac{\tau^{\alpha}}{2} \tilde{Q}_{Li}$$

$$+ H_u^{+} \frac{\tau^{\alpha}}{2} H_u + H_d^{+} \frac{\tau^{\alpha}}{2} H_d \Big]$$

$$SU(3): \quad D^{A} = g_3 \Big[\tilde{Q}^{+}_{Li} \frac{\lambda^{A}}{2} \tilde{Q}_{Li} + \tilde{u}^{+}_{Ri} \cdots + \tilde{d}^{+}_{Ri} \Big]$$

<u>The F - terms</u>

$$F_{H_u} = \mu H_d - \tilde{\mu}^{*}_{R} M_u \, u \tilde{Q}_L$$

$$F_{H_d} = -\mu H_u - \tilde{e}^*_R M_e \tilde{L}_2 - \tilde{d}^*_R M_d \tilde{Q}_u$$

$$F_Q = M_d \tilde{d}_R H_d + \tilde{u}^T M_u \tilde{u}^*_R H_u$$

$$F_L = M_e \tilde{e}^*_R H_d \qquad F_{\bar{e}} = \tilde{L}^T M_e H_d$$

$$F_{\bar{u}} = \tilde{Q}^T_L \ldots H_u \,, \qquad F_{\bar{d}} = \tilde{Q}^T \ldots H_d$$

We see that, except for the μ term, the MSSM potential is quartic in the fields

The μ term gives an equal mass to the Higgs and Higgsinos, and also generates cubic couplings among the Higgs and sleptons & squarks.

Does this potential break susy and/or electroweak symmetry?

Its minimum occurs for $F_i = D_i = 0$ and there are many field configurations for which this is true

(e.g. all fields equal to zero).

Hence the MSSM potential does not by itself break susy.

<u>Electroweak breaking in the supersymmetric limit</u>

Higgs potential with unbroken supersymmetry

$$V = |\mu|^2 \left(|H_d|^2 + |H_u|^2 \right) +$$

$$+ \frac{g_1^2 + g_2^2}{8} \left(|H_d|^2 - |H_u|^2 \right)^2$$

$$+ \frac{1}{2} g_2^2 \, |H_d H_u|^2$$

To preserve susy we need $V = 0$.

This is consistent with $\langle H_{u,d} \rangle \neq 0$

only if

$$\mu = 0 \quad , \quad tg\,\beta = 1$$

The potential has then a flat direction (the scale of gauge symmetry breaking is not fixed)

Supersymmetric limit

$\mu = 0$, $\operatorname{tg}\beta = 1$ $(v_1 = v_2 = v/\sqrt{2})$

- Chargino mass eigenvalues

$$m_{\chi^\pm} = gv/\sqrt{2} = \pm M_W$$

- Neutralino mass eigenvalues

  2 massless Majorana states $(\tilde{\gamma}, \tilde{h})$

  2 massive states $(\pm M_Z)$ $(\tilde{Z}, \tilde{h})$

- Higgs boson masses

$$m_h = m_A = 0 \quad , \quad m_H = M_Z$$

$$m_{H^\pm} = M_W$$

- $m_f = m_{\tilde{f}}$

So, we need

Spontaneous symmetry breaking in a "hidden" sector, its transmission to the MSSM and soft super-symmetry breaking in MSSM

We add soft terms that are $SU(2) \times U(1)$ invariant (we don't want to break electroweak symmetry)

"Hidden" $\longrightarrow$ MSSM

gravity mediation
gauge mediation

e.g. gravity mediation

$$K = \phi_i^\dagger \phi_i + \frac{X^\dagger X}{M_{Pl}^2} a_{ij} \phi_i^\dagger \phi_j + \ldots$$

$$F_X \neq 0 \Rightarrow \tilde{m}_{ij}^2 = a_{ij} \frac{|F_X|^2}{M_{Pl}^2}$$

Models of supersymmetry breaking

O'Raifeartaigh model (F-term breaking)

3 chiral superfields (at least)

$\phi_j \quad (j = A, B, C)$

$$W = m\phi_A\phi_B + \lambda(\phi_A^2 - M^2)\phi_C$$

The theory is invariant under global $R' = R - \frac{2}{3} X$

where the X is an Abelian symmetry with values $x_j = (1, -2, -2)$:

$$X : \phi_j(x,\theta) \to e^{i x_j \alpha} \phi_j(x,\theta)$$

$$R : \phi_j(x,\theta) \to e^{i\frac{2\beta}{3}} \phi_j(x, e^{i\beta}\theta)$$

Supersymmetry and R' are spontaneously broken

Sum rule

$$\mathrm{sTr}\, M^2 \equiv \sum_s (-1)^s (2s+1) m_s^2 = 0$$

but $m_\psi \neq m_\varphi$

D - term breaking (Fayet & Iliopoulos)

It necessarily involves an $U(1)$ gauge supermultiplet.

Basic mechanism: two chiral superfields Φ_1, Φ_2 of opposite $U(1)$ charge ($\Phi_i = (A_i, \psi_i, F_i)$)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} + \underbrace{\lambda^{+} \sigma^{\mu} \partial_{\mu} \lambda}_{\text{gaugino}} + \frac{1}{2} D^2 +$$

$$+ \sum_{i=1}^{2} \left[(D_\mu A_i)^* (D^\mu A_i) + \psi_i^+ \sigma^\mu D_\mu \psi_i + F_i^* F_i \right]$$

$$+ g D (A_1^* A_1 - A_2^* A_2) - (\sqrt{2} g \lambda (\psi_1 A_1^* - \psi_2 A_2^*) + cc)$$

$$+ m \underbrace{(A_1 F_2 + A_2 F_1}_{\text{from superpotential } m \Phi_1 \Phi_2} - \psi_1 \psi_2 + c.c) \mp \xi D$$

$$\left.\begin{aligned} D_\mu A_i &= (\partial_\mu + q_i g V_\mu) A_i \\ D_\mu \psi_i &= (\partial_\mu + i q_i g V_\mu) \psi_i \end{aligned}\right\} \text{covariant derivatives}$$

It is R-symmetry invariant under

$$\Phi_j(x,\theta) \to e^{i2\beta}\Phi_j(x, e^{i\beta}\theta)$$

Equations of motion for F_i & D give

$$D = \pm\xi - g(|A_1|^2 - |A_2|^2)$$

$$F_1 = -mA_2^* \qquad F_2 = -mA_1^*$$

All three cannot vanish and supersymmetry is broken

The potential is given by

$$V = \frac{1}{2}\left\{\pm\xi - g\left(|A_1|^2 - |A_2|^2\right)\right\}^2 +$$

$$+ m^2\left(|A_1|^2 + |A_2|^2\right)$$

Minimizing, we get

$$A_1^*\left\{m^2 - g\left(\pm\xi - g|A_1|^2 + g|A_2|^2\right)\right\} = 0$$

$$A_2^*\left\{m^2 + g\left(\pm\xi - g|A_1|^2 + g|A_2|^2\right\} = 0$$

Two solutions:

1) $A_1 = A_2 = 0$

$$D = \pm \xi \,, \quad F_1 = F_2 = 0 \,, \quad V_0 = \frac{\xi^2}{2}$$

λ is the massless goldstino

$U(1)$ and R remain unbroken

2 scalars with masses $m^2 \pm \xi g$

— " — $m^2 \mp \xi g$

A Dirac fermion — " — m^2

$$\mathrm{STr}\, M^2 = 2(m^2 \pm \xi g) + 2(m^2 \mp \xi g) - 4 m^2 = 0$$

We notice that the scalar masses are shifted by $q_i g D$ ()

So in fact $\mathrm{STr}\, M^2 = 2g(q_1+q_2)D$

2) The second solution: $A_2 = 0$, $A_1 = v$

It breaks both gauge symmetry and R-symmetry, but leaves a linear combination invariant, hence only one Goldstone boson, to be eaten by U(1) gauge boson

Minimization of the potential gives

$$v^2 = \frac{1}{g^2}\left(\pm g\xi - m^2\right)$$

so, for $v^2 > 0$ we need $\oplus g$ and $g\xi > m^2$

(we assumed v to be real, one can always redefine the field)

We get in the vacuum

$$\Phi = \frac{m^2}{g} \qquad F_1 = 0, \quad F_2 = -\frac{m}{g}\sqrt{\xi - m^2}$$

$$A_1 = \frac{1}{g}\sqrt{\xi - m^2} \qquad A_2 = 0$$

The Goldstino is a linear combination of the gaugino λ and the chiral ψ_2. This can be seen from the fermion mass matrix.

The gauge boson gets a mass $m_v^2 = 2g^2v^2$

Expanding the potential around its vacuum value one gets scalar masses

$$2m^2 , \quad 2m^2 , \quad 0 , \quad 2g^2v^2$$

appears here because we are not in the unitary gauge

Sum rule

$$\mathrm{STr}\, M^2 = g\,(q_1 + q_2)\,D$$

The D term susy breaking is not necessarily so "hidden". Suppose, e.g. the matter (MSSM) chiral multiplets are charged under U(1). Then the squark masses get contribution

$$\tilde{m}_D^2 = g\, q_i\, D$$

F_2 contributes to the squark masses, too, via gravity mediation

$$\tilde{m}^2_F = \frac{F_2^2}{M_{Pl}^2} \qquad \text{vs} \qquad \tilde{m}^2_D \simeq g\, q_i\, D$$

$$\frac{m^2 \xi}{M_P^2} \qquad \text{vs} \qquad m^2$$

determined by $\dfrac{\xi}{M_P^2}$

Explicit supersymmetry breaking needed.

3rd ASSUMPTION:

The effective theory has no quadratic divergences

$\hookrightarrow$ NO DIM 4 SUSY BREAKING TERMS

(dim 4 $\Rightarrow$ $\delta m^2 = g^2 \Lambda^2$

g new coupling multiplying dim 4 term)

IN ADDITION

Soft terms must not break explicitly the $SU(2) \times U(1)$

The most general SUSY breaking terms

$$\Delta \mathcal{L} = -\frac{1}{2} m_{\tilde{g}} \tilde{g}\tilde{g} - \frac{1}{2} m_{\tilde{W}} \tilde{W}\tilde{W} - \frac{1}{2} m_{\tilde{b}} \tilde{b}\tilde{b} + h.c.$$

$$- m^2_{H_1} H_1 H_1^* - m^2_{H_2} H_2 H_2^* + m^2_3 H_1 H_2 + h.c.$$

$$+ \Sigma (m^2_Q)^{ij} \tilde{Q}^i \tilde{Q}^{j*} + \Sigma (m^2_u)^{ij} \tilde{u}^i \tilde{u}^{c*}$$

$$+ \Sigma (m^2_D)^{ij} \ldots \quad + \Sigma (m^2_L)^{ij} \ldots$$

$$+ \Sigma (m^2_E)^{ij} \ldots$$

$$+ A_u \tilde{u}^c \tilde{Q} H_2 + \ldots\ldots$$

ELECTROWEAK SYMMETRY BREAKING IN MSSM

The Higgs sector of MSSM

$\mathrm{Re}\, H^0_{u,d}\,,\ \mathrm{Im}\, H^0_{u,d}$

$H_u\,,\ H_d \to$ 2 charged, 4 neutral fields; spontaneous breaking of global chiral $SU(2)\times SU(2) \to SU(2)_V$ gives 3 Goldstones (to be eaten by W, Z) and physical scalars

$$H^\pm\,,\quad h\,,\quad H\,,\quad \underset{CP-\text{odd}}{A}$$

With the soft breaking terms, the scalar potential reads

$$
\begin{aligned}
V = {} & B\mu \left(H_u^+ H_d^- - H_u^0 H_d^0 \right) + c.c. + \\
& + \sum_{i=u,d} \left(|\mu|^2 + m_{H_i}^2 \right) H_i^\dagger H_i + \\
& \left.\begin{aligned}
& + \frac{1}{8} g_1^2 \left[H_u^\dagger H_u - H_d^\dagger H_d \right]^2 + \\
& + \frac{1}{8} g_2^2 \left[H_u^\dagger \tau^a H_u - H_d^\dagger \tau^a H_d \right]^2
\end{aligned}\right\} \text{D-terms}
\end{aligned}
$$

The potential contains the charged fields $H^{\pm}_{u,d}$, e.g.

$$H^{+}_{u} H_{u} = |H^{0}_{u}|^{2} + |H^{+}_{u}|^{2}$$

They do not acquire vevs (fortunately). Note that by a weak isospin rotation we can set $\langle H^{+}_{u} \rangle = 0$. Then one can see that the minimum condition also requires $\langle H^{+}_{d} \rangle = 0$.

Hence charge is preserved and we can set charged fields to zero at the minimum. We get

$$V = \sum_{i=u,d} \left(|\mu|^2 + m^2_{H_i} \right) |H^0_i|^2 - B\mu H^0_u H^0_d$$

$$+ c.c. + \frac{1}{8}(g_1^2 + g_2^2)\left(|H^0_u|^2 - |H^0_d|^2 \right)^2$$

The quartic terms vanish along the flat direction $|H^0_u| = |H^0_d|$

A positive quadratic term requires then

$$2|\mu|^2 + m^2_{H_u} + m^2_{H_d} > 2B_\mu$$

for the potential to be bounded from below. For this potential to break electroweak symmetry, the determinant of the mass matrix must be negative, so that one linear combination of the fields has a negative $(\text{mass})^2$

So

$$B^2\mu^2 > \left(|\mu|^2 + m^2_{H_u}\right)\left(|\mu|^2 + m^2_{H_d}\right)$$

The two conditions exclude $m^2_{H_u} = m^2_{H_d}$

Let's set at the minimum

$$\langle H^0_{u,d} \rangle = v_{u,d}$$

We need $v_u^2 + v_d^2 = v^2 = (240)^2\ GeV^2$

but $\quad tg\,\beta = \dfrac{v_u}{v_d}\quad$ is a free parameter

Interesting possibility: breaking of the electroweak symmetry appears as a consequence of the quantum corrections of the theory.

(Hierarchy M_Z / M_X related to presence in nature of large Yukawa coupling(s))

$$t = \ln \frac{M_X}{M_Z}$$

$$(4\pi)^2 \frac{d}{dt} m^2_{H_d} = \left\{ 6 g_2^2 M_2^2 + \frac{6}{5} g_1^2 M_1^2 \right\}$$

$$- \underline{6 Y_b^2} \left(m^2_{\tilde{Q}_3} + m^2_{\tilde{D}_3} + m_1^2 + A_b^2 \right)$$

$$- 2 Y_\tau^2 \left(m^2_{\tilde{L}_3} + m^2_{\tilde{E}_3} + m_1^2 + A_\tau^2 \right)$$

$$(4\pi)^2 \frac{d}{dt} m^2_{H_u} = \left\{ 6 g_2^2 M_2^2 + \frac{6}{5} g_1^2 M_1^2 \right\}$$

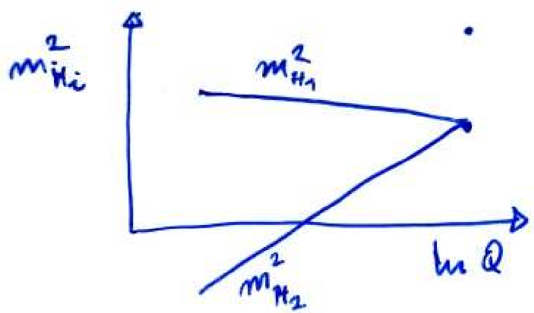
$$- \underline{6 Y_t^2} \left(m^2_{\tilde{Q}_3} + m^2_{\tilde{u}_3} + m_2^2 + A_t^2 \right)$$

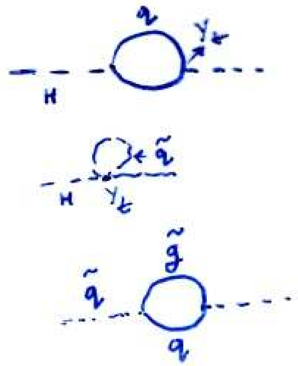
$$(4\pi)^2 \frac{d}{dt} m^2_{\tilde{u}_3} = \underline{\frac{32}{3} g_3^2 M_3^2} + \frac{32}{15} g_1^2 M_1^2 - 4 Y_t^2 \left(m^2_{\tilde{Q}_3} + m^2_{u_3} + \ldots \right.$$

$$(4\pi)^2 \frac{d}{dt} m^2_{\tilde{D}_3} = \underline{\frac{32}{3} g_3^2 M_3^2} + \frac{8}{15} g_1^2 M_1^2 - 4 Y_b^2 \left(\right.$$

Large logs $\ln \frac{M_X}{M_Z}$ summed up $\Rightarrow m_2^2 \ll m_1^2$

Works for $Y_t = Y_b \tan$!! $(m_2^2 \leq 0)$





$m^2_{\tilde q}$ remains positive

$m^2_{H_2} < 0 \Rightarrow SU(2) \times U(1)$ broken

M_t plays the crucial role (Y_t)

~~For large Y_t the RGE have the following approximate solutions~~

$$\cancel{m^2_{H_1} = m_0^2 + [illegible] M^2_{1/2}}$$

$$\cancel{m^2_{H_2}(M_Z) = -[illegible] m_0^2 - [illegible] M^2_{1/2}}$$

$$\cancel{m^2_{\tilde Q}(M_Z) = [illegible] m_0^2 + 6 M^2_{1/2}}$$

$$\cancel{m^2_{\tilde e}(M_Z) = m_0^2 + 0.15 M^2_{1/2}}$$

Then

2

$$\frac{1}{2} M_Z^2 = \sum_{i,j} c_{ij}\, \tilde{m}_i \tilde{m}_j \qquad (*)$$

•

$$c_{ij} = c_{ij}\left(g, Y, t_{g\beta}, \ln \frac{\Lambda}{M_Z}\right)$$

~~Two constraints on acceptable solutions to (*) :~~

- ~~not too large cancellations in (*)~~
- ~~no~~ large FCNC ~~transitions~~

No large cancellations (in presence of large log)

what is "large" ?

are $\tilde{m}_i$'s correlated by the mechanism of supersymmetry breaking?

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Nevertheless, some qualitative sense in this requirement ?

How large are c_{ij}'s ?

E.g. cancellations 1 : 100

$c_{ij} \sim O(1)$ $\qquad \tilde{m}_i \sim O(500-1000)\,\mathrm{GeV}$

$c_{ij} \sim O(0.1)$ $\qquad \tilde{m}_i \sim O(2-3)\,\mathrm{TeV}$

$c_{ij} \sim O(0.01)$ $\qquad \tilde{m}_i \sim O(5-7)\,\mathrm{TeV}$

~~RG evolution and c_{ij}'s - well understood~~

~~see, for instance,~~

~~Barbieri, Giudice (1988)~~

~~[illegible] (1993)~~

~~[illegible] (1994)~~

~~Carena, Olechowski, Pokorski, Wagner (1994)~~

~~[illegible], Giudice (1995)~~

~~Kane, King (1998)~~

Dominant effects come from α_s, Y_t

$c_{ii} \gtrsim O(1)$ for $\tilde{\mu}$ and $\tilde{m}_{\text{gluino}}$ (M_3)

$c_{ij} \sim O(0.01)$ for $\tilde{m}_{sq}$ of the first two generations

weak dependence on M_1, M_2, A terms

Dependence on $\underbrace{\tilde{m}_{H_d},\ \tilde{m}_{H_u},\ \tilde{m}_{Q_3},\ \tilde{m}_{U_3},\ \tilde{m}_{D_3}}_{\tilde{m}_0}$:

low $\tan\beta$ ($\lesssim 5$) intermediate large ($Y_t \approx Y_b$)

$C_{\tilde{m}_0} \sim O(1)$

$C_{\tilde{m}_0} \lesssim O(0.1)$

$$\boxed{\frac{1}{2} M_Z^2 \approx -\check{\mu}^2 + 3\tilde{M}_3^2}$$

Weak dependence on all scalar masses has been more recently called focus point (1999)

Conclusion: absence of large cancellations $\Rightarrow$ small μ, M_3 ; unconstrained $M_{1,2}$ and scalar masses ?

Crucial constraint: $m_{Higgs} > 115$ GeV

(1998)

Feng, Matchev, Moroi (2000)

End of Lecture 3