Corfu 2011: Supersymmetry

Lecture 1

Literature

Weinberg vol.3
S.Martin
Bailin & Love
Drees et al

Lecture 1: Refreshing memory

Lecture 2: MSSM and extensions

Lecture 3: Supersymmetric Higgs sector

Lecture 4: Supersymmetric flavour problem

Crucial for the Brout-Englert-Higgs mechanism in electroweak theory:

spontaneous breaking of a global symmetry, at least

SU(2)xU(1) or better (for $\rho=1$)

$$SU(2)xSU(2) \rightarrow SU(2)$$

of some sector coupled to the weak gauge bosons as the origin of their masses

The BEH mechanism:
Goldstone bosons become the longitudinal modes of the gauge bosons *W, Z* which acquire masses.

The Scalar(s) are „by-product" of S (global) SB

Simplest dynamical sector with chiral symmetry (to be spontaneously broken) – self interacting scalar field

$$V = m_H^2 HH^+ + \frac{1}{2}\lambda(HH^+)^2$$

m_H^2, λ - free param

$v^2 \sim \frac{-m_H^2}{\lambda}$

Virtue - renormalizability; also easy description of fermion masses;

Prediction: ELEMENTARY scalar *h* as *The Higgs Scalar*;

hWW, hqq couplings are known;

Matching of the couplings $\Rightarrow$

- ***h*** **unitarizes the $W_L W_L$ amplitude ($W_L \equiv \pi$)**

$$A = \frac{1}{v^2}\left[s - \frac{s^2}{s-m_h^2} + (s \leftrightarrow t)\right]$$

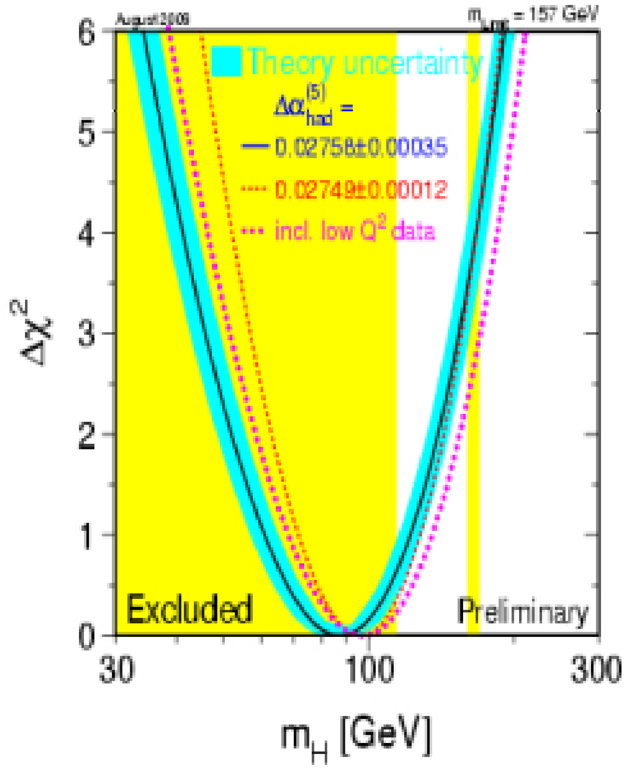
- **m_h serves as a cut-off**

$$\rho \sim \ln \frac{m_h^2}{M_V^2} + \ldots$$

We can predict the production cross sections and the decay rates as a function of The Scalar mass

***m_h*=?**

Precision data:



BUT…

so far, Nature does not seem to like elementary scalars

and the composite ones are never „unnaturally” light

PIONS:

Goldstone bosons of the spontaneously broken chiral (global) symmetry of QCD

New mass scale at $< 4\pi f_\pi$ (ρ mass) where $\pi\pi$ interactions become strong

The Scalar in QCD (accompanying SSB)
- σ meson (radial excitation)-
has the mass of the order of the ρ mass

Small pion mass and the σ meson mass are fully „natural" (in quantum field theory)

Well known drawbacks of the SM Higgs scalar:

- **highly unnatural, if the next mass scale (characterizing the embeding of the SM into a „bigger” theory) is much higher (right-handed neutrino mass? GUT scale?)**

- merely a parametrization of the EWSB, not a dynamical explanation

$$m_H^2 = m_H^2|_{tree} + \delta m_H^2$$ (loop correction)

At the scale *M*, embed the SM into some bigger theory and think in terms of the Appelquist-Carazzone decoupling

$$\delta m^2{}_H = \delta m^2_{SM} + \delta m^2_{NEW}$$

$$\delta m_H^2|_{SM} = c_2 M^2 + \ldots$$

$$\delta m_H^2|_{NEW} = c_2 \Lambda^2 + c_4 \ln \Lambda^2 + \ldots$$

M-cut-off to the Standard Model

Λ-cut-off to the extended theory

In the presence of a new scale *M*

$$\delta m^2_{SM} = (2m^2_W + m^2_Z + m^2_H - 4m^2_t)\frac{3M^2}{32\pi^2 v^2}$$

$$|\delta m^2_{SM}| \sim m^2_W \Rightarrow M < 0.6\ \text{TeV}$$

$$|\delta m^2_{SM}| < 10 M^2_W \Rightarrow M < 2\text{TeV}$$

$$|\delta m^2_{SM}| < 100 M^2_W \Rightarrow M < 6\text{TeV}$$

$$(m_H^2|_{tree},\ \delta m_H^2) \sim M_Z^2\ ?$$

We expect low scale *M*

We expect it to be built into a structure such that δm_{NEW}^2 is also small

A driving force for looking for extensions of the Standard Model

In summary, three questions about the electroweak symmetry breaking:

1) What is the dynamical origin of the electroweak scale?

2) What stabilises the electroweak scale ?
   (where the scale M comes from?)

3) What unitarizes the WW scattering amplitude?

massive W → *A ~ GF E2~ s/v2*

Related but not identical questions: in the SM WW is unitarized by an elementary scalar (Higgs boson) but we have no idea what is the origin of G_F and what stabilises the Fermi scale.

Are we going to face a new situation concerning scalars or, if new ones exist, are they similarly „natural” ?

Basic concepts to make EWSB „natural”:

- supersymmetry
- new strong interactions
- extra dimensions

SUPERSYMMETRY: RATIONALE FOR ELEMENTARY SCALARS; BETTER BEHAVIOUR OF QFT WITH SCALARS IN THE UV;

IT „ANSWERS” ALL 3 QUESTIONS ABOUT ELECTROWEAK BREAKING

Let's discuss a scalar field theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!}\varphi^4$$

We would like to calculate quantum corrections to the scalar mass.

Take $\quad -i\,\Sigma =$ 

$\sim \int^{\infty} dk^2$

Add a fermion

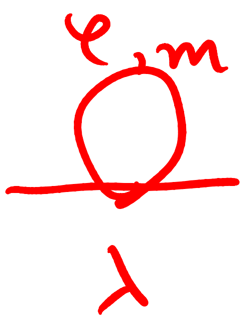
φ , ψ (fermion)

couplings $\lambda \varphi^4$, $y \varphi \bar{\psi} \psi$

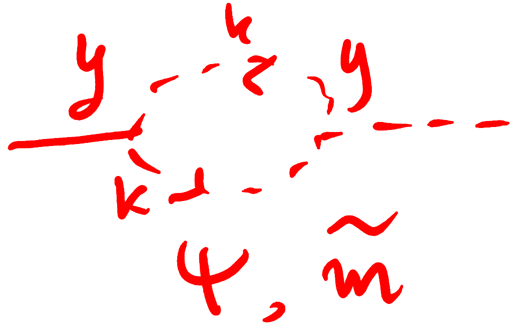
(Yukawa)

Correction to the mass of φ

(UV divergence)



,



$$\lambda \int^{\infty} dk^2\, k^2 \frac{1}{k^2+m^2}$$

$$-y^2 \int d^4k \, \frac{1}{\not{k}-\tilde{m}} \, \frac{1}{(\not{k}-\tilde{m})} =$$

$$= -y^2 \int d^4k \, \frac{(k^2-m^2)+2\tilde{m}^2}{(k^2-\tilde{m}^2)^2} \xrightarrow{\text{euclid}}$$

$$= -y^2 \int^{\infty} dk^2\, k^2 \frac{1}{k^2+\tilde{m}^2} - y^2\, 2\tilde{m}^2 \int^{\infty} \frac{dk^2}{k^2}$$

Cancellation of quadratic divergences if

1) $\lambda = y^2$, $m = \tilde{m}$

2) the same number of scalar and fermion degrees of freedom (proper Feynman rules)

Imagine now that $\lambda = y^2$ but

$m > \tilde{m}$ (softly broken supersymmetry)

$$\int dk^2 k^2 \left(\frac{1}{k^2+m^2} - \frac{1}{k^2+\tilde{m}^2} \right) \simeq -(m^2 - \tilde{m}^2) \int^{\infty} \frac{dk^2}{k^2}$$

logarithmically divergent

The emerging picture:

at low energy, we have non-supersymmetric theory of a fermion ψ and some scalar h (e.g. like in SM). There is a hierarchy problem for the scalar h.

It's solved by adding a scalar partner to ψ and a fermion partner to h with masses close to m_ψ, m_h.

The hierarchy problem of the non-supersymmetric model is then solved.
But we may still have a hierarchy problem of the full theory because of the terms

$$(m^2-\tilde{m}^2)\int^{\infty}\frac{dk^2}{k^2}\ ;$$

if there is one more physical scale, e.g. M_{GUT}, one gets $(m^2-\tilde{m}^2)\ln\frac{M^2_{GUT}}{m^2}$; it can still be large

SUPERSYMMETRY

Matter multiplets in some chosen representation of G

$$\hat{\Phi}_i \equiv (A_i \,,\ \psi_i \,,\ F_i)$$

$\hat{\Phi}$ → chiral

A_i → complex scalar

ψ_i → Weyl fermion

F_i → auxiliary field

complex scalar

Degrees of freedom: 2, 4, 2

$$\hat{\Phi}_i^{(x)} = A_i^{(x)} + 2\theta\psi_i^{(x)} + \theta\theta F_i^{(x)}$$

θ - Grassmann variables

chiral (depends only on θ)

	A	ψ	F	θ
dim :	1	3/2	2	-1/2

$$\theta\theta \equiv \epsilon_{ab}\theta_a\theta_b \equiv \theta_a\theta^b$$

$$\epsilon_{12} = 1$$

$$\epsilon^{12} = -1$$

To make the definition of a chiral superfield to be covariant with super-symmetric transformation:

$$x^\mu \to y^\mu = x + i\theta\sigma^\mu\bar{\theta}$$

and define

$$\bar{D}\Phi = 0 \qquad \text{chiral}$$

$$D\bar{\Phi} = 0 \qquad \text{anti-chiral}$$

$$\left.\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \\ \bar{D}_{\dot{\beta}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} + i\theta^\beta\sigma^\mu_{\beta\dot{\beta}}\partial_\mu \end{aligned}\right\} \text{covariant derivatives}$$

Expanding

$$\phi(x^\mu, \theta, \bar\theta) = A(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta F(x)$$

$$+ i\,\partial_\mu A\,\theta\sigma^\mu\bar\theta + i\sqrt{2}\,\theta\,\partial_\mu\psi\,\theta\sigma^\mu\bar\theta$$

$$- \frac{1}{2}\partial_\mu\partial_\nu A\,\theta\sigma^\mu\bar\theta\,\theta\sigma^\nu\bar\theta$$

Supersymmetry transformation $\theta \to \theta + \tilde{\theta}$

$\delta A = \tilde{\theta}\psi$, $\delta\psi = \tilde{\theta}F + \bar{\theta}\sigma^\mu \partial_\mu A$

$\delta F = \bar{\theta}\sigma^\mu \partial_\mu \psi$

↑ total derivative

WS

$\mathcal{L} = \partial_\mu A^* \partial^\mu A + \psi^+ \sigma^\mu \partial_\mu \psi + F^* F$

~~Summary~~

Gauge multiplets
in the adjoint represent.
of a gauge group G

degrees of freed. (for abelian)

$$\hat{V}^a = (\overset{3}{V_\mu^a}, \overset{4}{\lambda^a}, \overset{1}{D^a})$$

λ: left-handed Weyl spinor

Wess-Zumino gauge

$$\hat{V}^a = \theta\sigma^\mu\bar{\theta}V_\mu^a + i\theta\theta\bar{\theta}\bar{\lambda}^a - i\bar{\theta}\bar{\theta}\theta\lambda^a + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D^a$$

$\theta, \bar{\theta}$ - Grassmann variables

Susy Transf. $\theta \to \theta + \tilde{\theta}$

$$\delta D = \partial_\mu(-\tilde{\theta}\sigma^\mu\bar{\lambda} + \lambda\sigma^\mu\bar{\tilde{\theta}})$$

total derivative

Superspace $z = (x, \theta, \bar{\theta})$

Action is invariant under susy transform. up to total derivatives

Most general renormalizable susy gauge theory:

$$\mathcal{L} = \int d^4x\, d^2\theta\, d^2\bar\theta \left\{ \underbrace{\hat\phi^* e^{g\hat V} \hat\phi}_{\text{matter-gauge couplings}} + \xi \hat V \right\} + \int d^4x\, d^2\theta \left\{ \underbrace{\frac{1}{4}\hat W^\alpha \hat W_\alpha}_{\text{gauge kinetic terms}} + \underbrace{W(\hat\phi)}_{\text{superpotential}} \right\} + h.c$$

$\xi \hat V$: Fayet-Iliopoulos term (for $U(1)$)

$$\hat W_\alpha^{(x,\theta)} = 4i\lambda_\alpha^{(x)} + \theta^\beta \underbrace{\ell_{\alpha\beta} + \theta\theta f_\alpha(x)}_{\text{some combination of the original fields}}$$

$$\frac{1}{2}(W^\alpha W_\alpha)_F = -\frac{1}{4} V_{\mu\nu} V_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar\lambda + \underset{\text{dual}}{\ldots} + \frac{1}{2}D^2$$

In components (Weyl fermions)

$$\boxed{\not{D} = \sigma^\mu \nabla_\mu}$$

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \lambda^\dagger \not{D} \lambda + \frac{1}{2} D^2 \qquad \Big\{ (W_\alpha W^\alpha)_F$$

$$+ |D_\mu A|^2 + \psi^\dagger \not{D} \psi + i g \sqrt{2}\, A^* \psi^\dagger \lambda +$$

$$+ (g A^* A + \xi) D + \cancel{} + F_i F_i^* + \qquad \Big\} (\Phi^* e^{2gV} \Phi)_D$$

$$+ F_i \frac{\partial W(A)}{\partial A_i} + \psi_i \psi_j \frac{\partial^2 W(A)}{\partial A_i \partial A_j} + h.c. \qquad \Big\{ (W)_F$$

where

$$W = b_{ij} \hat{\Phi}_i \hat{\Phi}_j + c_{ijk} \hat{\Phi}_i \hat{\Phi}_j \hat{\Phi}_k$$

and $W(A)$ is obtained by $\hat{\Phi}_i \to A_i$

$\psi\psi = \epsilon^{ab} \psi_a \psi_b$

$\theta\theta = \epsilon^{ab} \theta_a \theta_b$

Equations of motion for auxiliary fields

$$\frac{\partial \mathcal{L}}{\partial D^a} = 0 \Rightarrow D^a = -(g A^* T^a A + \xi)$$

Hence

$$(\xi + g A^* T^a A) D^a + \frac{1}{2} D^a D_a = \quad -\frac{1}{2} D^a D_a =$$

4-scalar coupling proportional to g^2!

$$\frac{\partial \mathcal{L}}{\partial F_i} = 0 \Rightarrow F_i^* = -\frac{\partial W(A)}{\partial A_i}$$

Hence $\quad F_i F_i^* + F_i \frac{\partial W(A)}{\partial A_i} + F_i^* \frac{\partial W^*(A)}{\partial A_i^*} = -\left|\frac{\partial W(A)}{\partial A_j}\right|^2$

Scalar potential V

$$g\, A^* T^a A\, D^a \qquad + \xi^a D^a$$

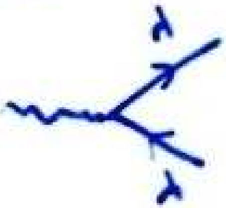
$$V = F_i F_i^* + \frac{1}{2} D^a D^a =$$

$$+\; a_i F_i + b_{ij} A_i F_j + c_{ijk} A_i A_j F_k \quad + h.c.$$

Couplings

1) $3 V_\mu$, $4 V_\mu$ $\quad \boxed{\text{from } W^\alpha W_\alpha}$

and

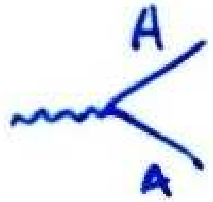


$$i g f_{abc} \lambda^a \sigma^\mu \bar{\lambda}^b V^c_\mu$$

2) from $\Phi_i^* e^{gV} \Phi$

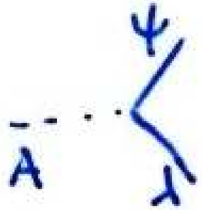
standard

$+$

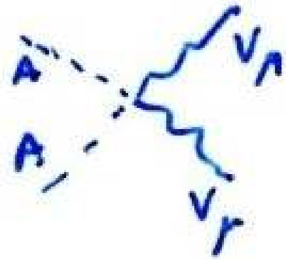


$$= -g T^a_{ij} V^a_\mu \left(\bar{\psi}_i \bar{\sigma}^\mu \psi_j + i A_i^* \partial^\mu A_j \right)$$

and



$$= i g \sqrt{2}\, T^a_{ij}\, \lambda^a \psi_j A_i^* + h.c.$$



$$= g^2 (T^a T^b)_{ij}\, V^a_\mu V^{\mu b} A_i^* A_j$$

$$(D_\mu A)(D_\mu A)^\dagger$$

and

$= g^a A_i^* T^a_{ij} A_j D^a$

where

$D^a = -g^a A_i^* T^a_{ik} A_k$

gives 4 A coupling $\sim g^2$

3) From $W(\phi_i)$

Yukawa

$\sim c_{ijk} A_i \psi_j \psi_k$ + h.c.

and

$= c_{ijk} A_i A_j F_k$ + h.c.

where $F_k = a_k + b_{ij} A_j + c_{ijk} A_j A_k$

(gives  + )

(No 4 Higgs coupling from this source)

APPENDIX: HIERARCHY PROBLEM

Let's discuss a scalar field theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!}\varphi^4$$

We would like to calculate quantum corrections to the scalar mass.

Take $-i\Sigma =$

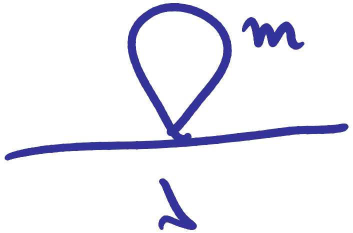
$\sim \int^{\Lambda} dk^2$

· Remember how the propagator is modified

$$G = \frac{i}{p^2-m^2} + \frac{i}{p^2-m^2}\left[-i\,\Sigma(p)\right]\frac{i}{p^2-m^2} + \ldots$$

$$= \frac{i}{p^2-m^2-\Sigma(p)}$$

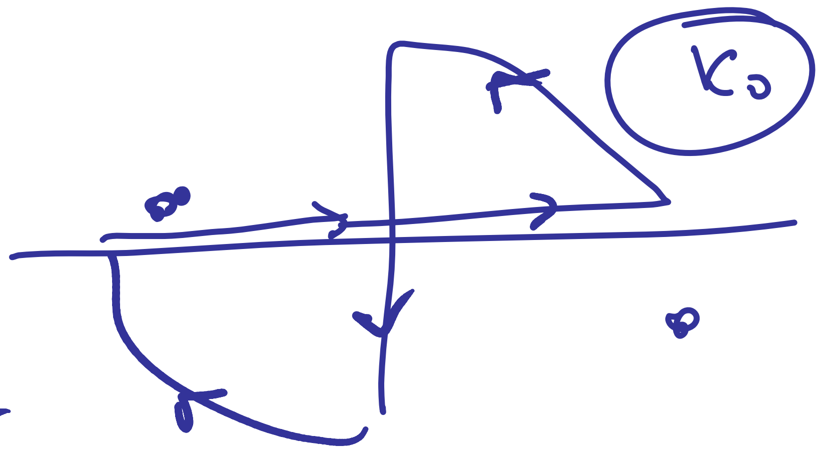
For the diagram

$-i\Sigma \equiv$ 

one gets

$$\frac{1}{2}(-i\lambda)\int \frac{d^4 k}{(2\pi)^n}\,\frac{i}{k^2-m^2+i\varepsilon}$$

$$\frac{1}{k^2 - m^2 + i\varepsilon} \Rightarrow$$



$$k_0 = \pm\left[\vec{k}^2 + m^2 - i\varepsilon\right]^{1/2}$$

Wick rotation: change the contour of integration to imaginary axis.

changing the variables

$ik_0 \to k_0$, $dk_0 \to -i\,dk_0$, $k_0^2 - \vec{k}^2 \to -k_0^2 - \vec{k}^2$,

$$\int_{+i\infty}^{-i\infty} dk_0 \to +i \int_{-\infty}^{\infty} dk_0$$

$$k^2 - m^2 \to -(k^2 + m^2)$$

In Euclidean space

$$d^4k = \frac{1}{2} k^2 \, dk^2 \, d\Omega_4 \,, \quad \Omega_4 = 2\pi^2$$

$$\Sigma \sim \int_0^\infty dk^2 \frac{k^2}{k^2+m^2} =$$

$$= \int_0^{\mu^2} dk^2 + \int_{\mu^2}^\infty dk^2 \qquad (\text{for } \mu^2 \gg m^2)$$

divergent

Let's impose a renormalization condition

$$\int_{\mu^2}^{\infty} dk^2 = 0$$

so that

$$\Sigma^R(\mu, p^2) = \int_0^{\mu^2} dk^2 \sim \mu^2$$

Then $m_F^2 = m_R^2(\mu) + \Sigma^R(\mu)$

Change the renormalization scheme

$$\mu_1^2 \to \mu_2^2$$

Then

$$m_R^2(\mu_2) \simeq m_R^2(\mu_1^2) + \int_{\mu_2^2}^{\mu_1^2} dk^2$$

$$= m_R^2(\mu_1^2) + (\mu_1^2 - \mu_2^2)$$

If $\mu_2^2 << \mu_1^2$, large fine tuning is necessary to keep $m_R^2(\mu_2) << \mu_1^2$

As long as there is only one physical scale (m) in the problem, the necessity of considering vastly different renormalization schemes is not so obvious.

However, let's consider a theory with two scalars, $m \ll M$.

As we shall see, renormalization at $\mu_1 = M$ and then its change to $\mu_2 = m$ is

convenient because it allows
for a decoupling of the heavy M
when calculating quantum corrections
to m

Let's discuss corrections to scalar masses in a theory with two scalars, with very different physical masses

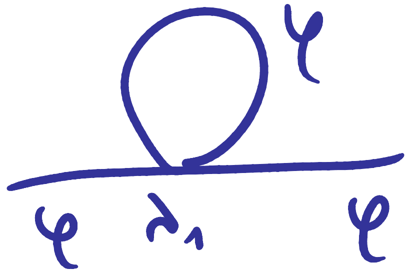
~~Let's see it using MS as the renormalisation scheme:~~

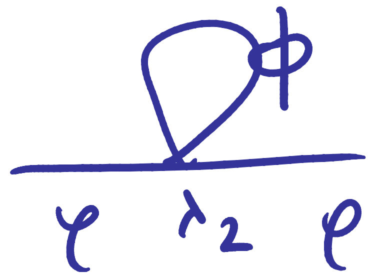
$$\mathcal{L} = \frac{1}{2}\left[(\partial\varphi)^2 + (\partial\phi)^2 - m^2\varphi^2 - M^2\phi^2\right]$$

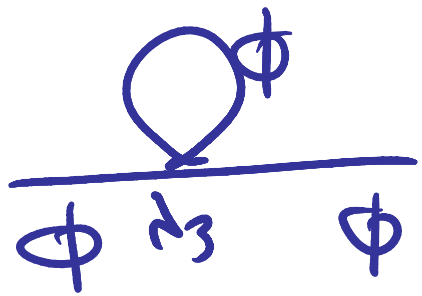
$$- \frac{\lambda_1}{4!}\varphi^4 - \frac{\lambda_2}{4!}\varphi^2\phi^2 - \frac{\lambda_3}{4!}\phi^4$$

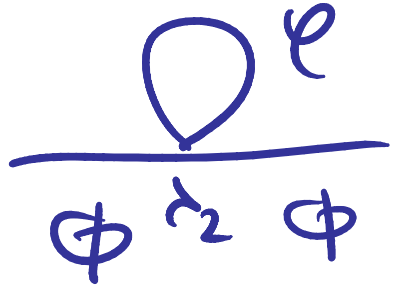
$$m \ll M$$

Among others, we get the diagrams

$-i\Sigma_\varphi \equiv$ 

$+$ 

$-i\Sigma_\Phi \equiv$ 

$+$ 

With two fields φ & Φ

$$\Sigma_\varphi \sim \lambda_1 \int_0^\infty dk^2 \frac{k^2}{k^2+m^2} + \lambda_2 \int_0^\infty dk^2 \frac{k^2}{k^2+M^2}$$

$$\simeq \lambda_1 \int_0^{M^2} dk^2 \frac{k^2}{k^2+m^2} + \lambda_2 \int_0^{M^2} dk^2 \frac{k^2}{k^2+M^2}$$

$$+ \int_{M^2}^\infty dk^2\, k^2 \left(\lambda_1 \frac{1}{k^2+m^2} + \lambda_2 \frac{1}{k^2+M^2} \right)$$

Let's impose a renormalization condition

$$\int_{M}^{\infty} dk^2 k^2 (\ldots) = 0$$

so that

$$\Sigma_{\varphi}^{R}(\mu = M) = \int_{0}^{M^2} dk^2 k^2 \left(\frac{1}{k^2+m^2} + \frac{1}{k^2+M^2} \right)$$

Then $m_F^2 = m_R^2(\mu = M) + \Sigma_{\varphi}^{R}(\mu = M)$.

$\approx m_R^2(M) + M^2$! (fine tuning)

It is useful to notice that I can organize this calculation also in another way:

Renormalizing at $\mu = m$ we put $\Sigma_\varphi(\mu = m) = \int_{m^2}^{\infty} dk^2 k^2 (\dots) = 0$

so that $m_F^2 \simeq m_R^2(\mu = m) =$

$$= m_R^2(\mu = M) + \int_0^{M^2} dk^2 \frac{k^2}{k^2+m^2} + \underbrace{\int_0^{M^2} dk^2 \frac{k^2}{k^2+M^2}}_{\approx M^2}$$

We can redefine

$$\tilde{m}_R^2(\mu=M) = m_R^2(\mu=M) + \int_0^{M^2} dk^2 \frac{k^2}{k^2+M^2}$$

and we get

$$m_R^2(\mu=m) = \tilde{m}_R^2(\mu=M) + \int_0^{M^2} dk^2 \frac{k^2}{k^2+m^2}$$

We get a theory without the particle M and with the physical cut-off at M ! (with some initial value $\tilde{m}_R^2(\mu=M)$)

If we choose the cut-off M close to m, we have no fine-tuning problem in the effective theory of the particle m.

But we may get another fine-tuning problem in the theory of $(m+M)$ if there is a third scale $\hat{M} \gg M$.

And so on ...

Another view at the hierarchy problem: take that model with two scalars, m & M, and suppose both of them develop vev's.

E.g. we want the first to break electroweak symmetry and the second to break the GUT group, so that we need $v \ll V$

Is this possible?

We look for solutions of two equations

$$\frac{\partial V}{\partial \varphi} = \left(-\frac{1}{2}m^2 + \frac{1}{3}\lambda_1|\varphi|^2 + \lambda_2|\phi|^2\right) = 0$$

$$\frac{\partial V}{\partial \phi} = \left(-\frac{1}{2}M^2 + \frac{1}{3}\lambda_3|\phi|^2 + \lambda_2|\varphi|^2\right) = 0$$

Replacing $\varphi \to v$, $\phi \to V$ and expanding in v/V we get (in the lowest order)

$$-3M^2 + 2\lambda_3 V^2 \cong 0$$

$$-3m^2 + 2\lambda_1 v^2 + 6\lambda_2 V^2 = 0$$

$$\Rightarrow \quad 2\lambda_1 v^2 = 3m^2 + 6\lambda_2 \frac{3}{2\lambda_3} M^2 \ll M^2$$

huge tuning between the tree level parameters m^2, M^2 is necessary

Large cancellations between parameters m and M are necessary to keep the physical scalar mass small

Two solutions are known :

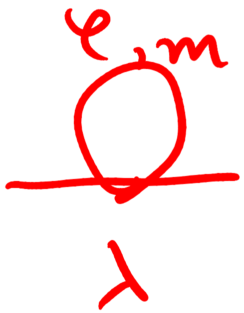
Solution $\underline{\mathrm{I}}$: supersymmetry

φ , ψ (fermion)

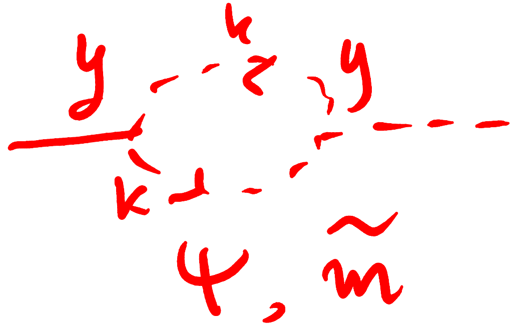
Couplings $\lambda \varphi^4$, $y \varphi \bar{\psi} \psi$

(Yukawa)

Correction to the mass of φ



,



(UV divergence)

$$\lambda \int_0^\infty dk^2 k^2 \frac{1}{k^2+m^2}$$

$$-y^2 \int d^4k \frac{1}{\not{k}-\tilde{m}} \frac{1}{(\not{k}-\tilde{m})} =$$

$$= -y^2 \int d^4k \frac{(k^2-m^2)+2\tilde{m}^2}{(k^2-\tilde{m}^2)^2} \xrightarrow{} \text{euclid}$$

$$= -y^2 \int dk^2 k^2 \frac{1}{k^2+\tilde{m}^2} - y^2 2\tilde{m}^2 \int \frac{dk^2}{k^2}$$

Cancellation of quadratic divergences if

1) $\lambda = y^2$, $m = \tilde{m}$

2) the same number of scalar and fermion degrees of freedom (proper Feynman rules)

Imagine now that $\lambda = y^2$ but $m > \tilde{m}$ (softly broken supersymmetry)

$$\int dk^2 k^2 \left(\frac{1}{k^2+m^2} - \frac{1}{k^2+\hat{m}^2} \right) \simeq -\left(m^2 - \tilde{m}^2\right) \underbrace{\int_0^\infty \frac{dk^2}{k^2}}_{\text{logarithmically divergent}}$$

The emerging picture:

at low energy, we have non-supersymmetric theory of a fermion ψ and some scalar h (e.g. like in SM). There is a hierarchy problem for the scalar h.

It's solved by adding a scalar partner to ψ and a fermion partner to h with masses close to m_ψ, m_h.

The hierarchy problem of the non-supersymmetric model is then solved.
But we may still have a hierarchy problem of the full theory because of the terms

$$(m^2 - \tilde{m}^2) \int_{\tilde{m}^2}^{\infty} \frac{dk^2}{k^2} \; ; \text{ if there is}$$

one more physical scale, e.g. M_{GUT},
one gets $(m^2 - \tilde{m}^2) \ln \frac{M_{GUT}^2}{\tilde{m}^2}$; it can still be large

Solution II:

~~Exception~~ Goldstone boson

Toy model - $U(1)$ global symmetry

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2$$
$$+ i \psi_1 \bar{\sigma}_\mu \partial^\mu \psi_1 + i \psi_2 \bar{\sigma}_\mu \partial^\mu \psi_2$$
$$- g(\phi \psi_1 \psi_2 + \phi^* \bar{\psi}_1 \bar{\psi}_2)$$

ψ_1, ψ_2 - Weyl fermions; $U(1)$ charges

$\phi : +1$

$\psi_1 : -1$, $\psi_2 : 0$

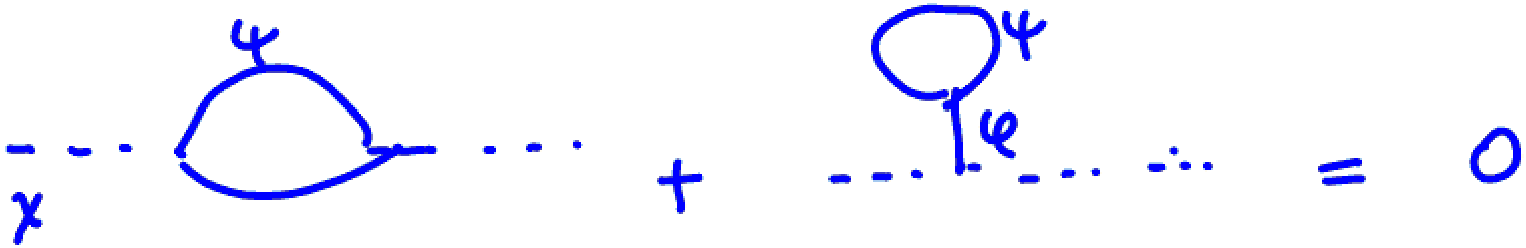
Spontaneous symmetry breaking

$$\phi = \frac{1}{\sqrt{2}}\left(\varphi + v + i\chi\right)$$

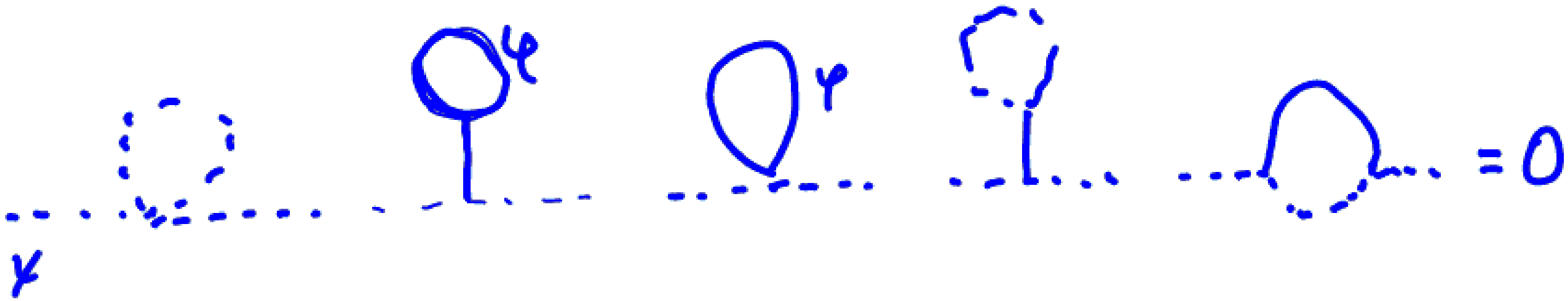
Dirac fermion $\psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}(\partial\chi)^2 + \bar{\psi}\not{\partial}\psi - \frac{1}{4}\underbrace{\lambda v^2}_{m_\varphi^2}\varphi^2 \\ & - \frac{\lambda v}{4}\varphi\left(\varphi^2 + \chi^2\right) - \frac{\lambda}{16}\left(\varphi^2 + \chi^2\right)^2 \\ & - \underbrace{\frac{gv}{\sqrt{2}}}_{m_\psi}\bar{\psi}\psi - \frac{1}{\sqrt{2}}g\varphi\bar{\psi}\psi - \frac{i}{\sqrt{2}}g\chi\bar{\psi}\gamma_5\psi\end{aligned}$$

Corrections to the χ mass (Goldstone)



(no dependence on cut-off Λ !)



fermion- fermion
Boson - boson

cancellations
(conspiracy of couplings)

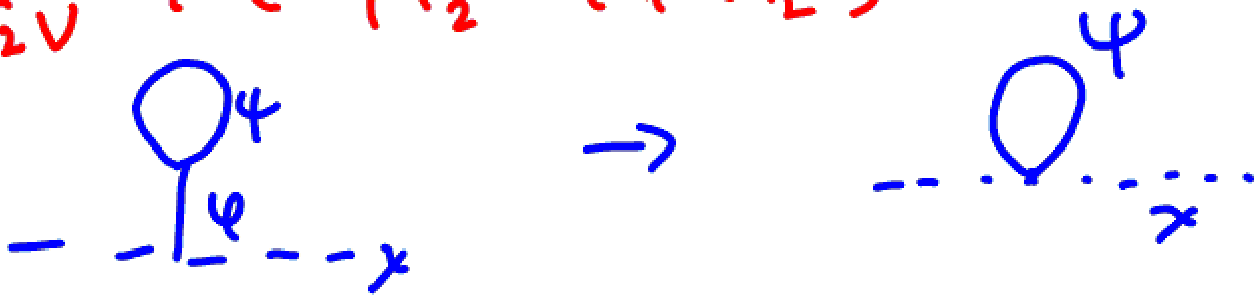
With non-linear parametrization

$$\phi = \frac{1}{\sqrt{2}}(\varphi + v)\, e^{i\chi/v}$$

$$-g(\phi\psi_1\psi_2 + \phi^*\bar{\psi}_1\bar{\psi}_2) = -\frac{gv}{\sqrt{2}}\Big(e^{i\frac{\chi}{v}}\psi_1\psi_2 +$$

$$+\, e^{-i\frac{\chi}{v}}\bar{\psi}_1\bar{\psi}_2\Big) = -\frac{ig}{\sqrt{2}}(\chi\psi_1\psi_2 - \chi\bar{\psi}_1\bar{\psi}_2)$$

$$+\frac{g}{\sqrt{2}v}\chi^2(\psi_1\psi_2 + \bar{\psi}_1\bar{\psi}_2) + \dots$$



Corrections to a Goldstone boson mass vanish independently of the scale of new physics.

Fermion-boson cancellations
of corrections to the Higgs boson
mass : supersymmetry

Fermion-fermion & boson-boson
cancellations : little Higgs models