noncommutative spectral geometry
algebra doubling
and the seeds of quantisation

m.s., stabile, vitiello, PRD 84 (2011) 045026

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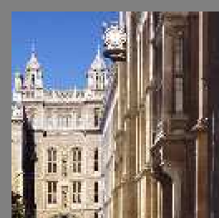

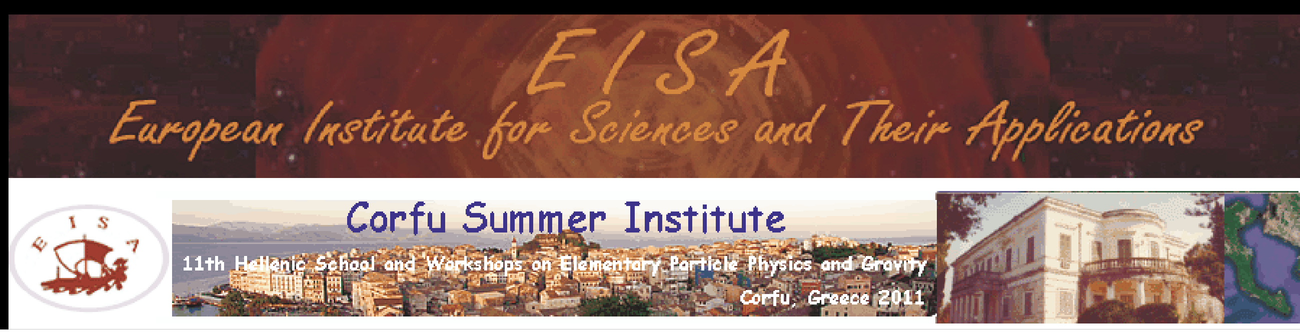

noncommutative spectral geometry (NCG with spectral triples):
purely geometric explanation for the SM of EW & strong interactions

chamseddine, connes, marcolli (2007)

cosmological consequences:

nelson, **m.s.**, PLB 680 (2009) 263

nelson, **m.s.**, PRD 81 (2010) 085038

buck, fairbairn, **m.s.**, PRD 82 (2010) 043509

nelson, ochoa, **m.s.**, PRD 82 (2010) 085021

nelson, ochoa, **m.s**, PRL 105 (2010) 101602

- simple almost commutative space

  extend to less trivial noncommutative geometries

- purely classical model

  it cannot be used within EU when QC cannot be neglected

- action functional obtained through perturbative approach in inverse powers of cut-off scale

  it ceases to be valid at lower energy scales (astrophysics)

- model developed in euclidean signature

  physical studies must be done in lorentzian signature

noncommutative spectral geometry (NCG with spectral triples):
purely geometric explanation for the SM of EW & strong interactions

chamseddine, connes, marcolli (2007)

algebra doubling

dissipation

gauge structure

dissipation (information loss) plays a key role
in the quantisation ('t hooft's conjecture)

dissipation leads to the notion of temperature
SM: zero-temperature QFT describing a closed system
thermal field theory is often unavoidable (EU physics)

the gauge field acting as a
reservoir for the matter field

noncommutative spectral geometry:

a two-sheeted space $\mathcal{M} \times \mathcal{F}$

4dim smooth compact riemannian manifold

discrete noncommutative space composed by 2 points

$$(\mathcal{A}, \mathcal{H}, \mathcal{D})$$

involution of operators on hilbert space $\mathcal{H}$ of euclidean fermions

self-adjoint unbounded operator in $\mathcal{H}$

noncommutative spectral geometry:

a two-sheeted space $\mathcal{M} \times \mathcal{F}$

4dim smooth compact riemannian manifold

discrete noncommutative space composed by 2 points

$$(\mathcal{A}, \mathcal{H}, \mathcal{D})$$

$\mathcal{H} = L^2(\mathcal{M}, S)$ the hilbert space of square integrable sections of the spinor bundle

$\mathcal{A} = C^\infty(\mathcal{M})$ the algebra of smooth functions on $\mathcal{M}$ acting on $\mathcal{H}$ as simple multiplication operators $(f\xi)(x) = f(x)\xi(x), \quad \forall f \in C^\infty(\mathcal{M})$ and $\forall \xi \in L^2(\mathcal{M}, S)$.

$$\mathcal{D} = \sqrt{-1}\,\gamma^\mu \nabla_\mu^{\mathrm{s}}$$

spin connection

noncommutative spectral geometry:

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$$(\mathcal{A}, \mathcal{H}, \mathcal{D})$$

$\mathcal{H} = L^2(\mathcal{M}, S)$ the hilbert space of square integrable sections of the spinor bundle

$\mathcal{A} = \mathcal{C}^\infty(\mathcal{M})$ algebra of coordinates, related to the gauge group of local gauge transformations

$\mathcal{D} = \sqrt{-1}\,\gamma^\mu \nabla^{\mathrm{s}}_\mu$ plays the role of the inverse of the line element ds

spin connection

for $\mathcal{M} \times \mathcal{F}$:

$$\begin{aligned} \mathcal{A} &= C^\infty(\mathcal{M}) \otimes \mathcal{A}_\mathcal{F} = C^\infty(\mathcal{M}, \mathcal{A}_\mathcal{F}) \,, \\ \mathcal{H} &= L^2(\mathcal{M}, S) \otimes \mathcal{H}_\mathcal{F} = L^2(\mathcal{M}, S \otimes \mathcal{H}_\mathcal{F}) \,, \\ D &= \partial\!\!\!/_\mathcal{M} \otimes 1 + \gamma_5 \otimes D_\mathcal{F} \,; \end{aligned}$$

$$\mathcal{A} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$$

$$k = 2a$$

$k = 4$ leading to the correct number of 16 fermions in each of the 3 generations spectral action principle:

the bare bosonic euclidean action is given by the trace of the heat kernel associated with the square of the noncommutative dirac operator

$$\mathrm{Tr}(f(\mathcal{D}/\Lambda))$$

the bare action at mass scale Λ *(a la wilson)*

4dim riemannian geometry: perturbatively in terms of geometric seeley- de witt coefficients a_n

$$\begin{aligned}\mathrm{Tr}(f(D/\Lambda)) \sim\ & 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4 + \cdots \\ & + \Lambda^{-2k} f_{-2k} a_{4+2k} + \cdots\ ,\end{aligned}$$

f: smooth even cut-off function which decays fast at infinity

asymptotic expansion:

$$\mathrm{Tr}(f(D/\Lambda)) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4$$

$$f_0 \equiv f(0)$$
$$f_k \equiv \int_0^\infty f(u) u^{k-1} \mathrm{d}u \quad , \quad \text{for} \quad k > 0$$

f_0, f_2, f_4 : related to coupling constants at unification, gravitational constant, cosmological constant

asymptotic expansion:

$$\mathrm{Tr}(f(D/\Lambda)) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4$$

cosmological term

einstein-hilbert action functional

yang-mills action for the gauge fields

asymptotic expansion:

$$\mathrm{Tr}(f(D/\Lambda)) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4$$

it leads to the full lagrangian for the SM minimally coupled to gravity, with neutrino mixing and majorana mass terms

↓

geometric explanation for the SM

VEV of higgs is related to noncommutative distance between 2 sheets connes discusses the relation between matrix mechanics

(physical quantities are governed by noncommutative algebra)

to

- experiments
- discretisation of energy of atomic levels and angular momentum

the doubling of the algebra $\mathcal{A} \to \mathcal{A}_1 \otimes \mathcal{A}_2$ acting on the doubled space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is also present in the standard QM formalism of the density matrix and the wigner function

$$W(p,x,t) = \frac{1}{2\pi\hbar} \int \psi^*\left(x - \frac{1}{2}y, t\right) \psi\left(x + \frac{1}{2}y, t\right) e^{-i\frac{py}{\hbar}} dy$$

$$x_\pm = x \pm \frac{1}{2}y$$

density matrix: $W(x_+, x_-, t) \equiv \langle x_+ | \rho(t) | x_- \rangle = \psi^*(x_-, t)\psi(x_+, t)$

the coordinate $x(t)$ of a quantum particle is split into two coordinates $x_+(t)$ (forward in time) and $x_-(t)$ (backward)

the forward and backward in time evolution of the density matrix is described by 2 copies of the schroedinger equation

$$i\hbar\frac{\partial\psi(x_+,t)}{\partial t} = H_+\psi(x_+,t)$$

$$-i\hbar\frac{\partial\psi^*(x_-,t)}{\partial t} = H_-\psi^*(x_-,t)$$

$$i\hbar\frac{\partial\langle x_+|\rho(t)|x_-\rangle}{\partial t} = H\langle x_+|\rho(t)|x_-\rangle$$

$$H = H_+ - H_-$$

the connection between spectroscopic experiments, noncommutative matrix algebra, energy level discretisation and algebra doubling:

the density matrix and the wigner function require the introduction of a doubled set of coordinates $(x_\pm, p_\pm)$ and of their respective algebras

$$i\hbar \frac{\partial \langle x_+|\rho(t)|x_-\rangle}{\partial t} = H \langle x_+|\rho(t)|x_-\rangle$$

$$H = H_+ - H_-$$

eigenvalues of H are the bohr transition frequencies $h\nu_{nm} = E_n - E_m$

i.e., spectroscopic structure

the need to double the degrees of freedom is implicit even in the classical theory when considering the brownian motion

e.o.m. $$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

derived from a delta functional classical constraint representation as a functional integral

$$\delta[m\ddot{x} + \gamma\dot{x} - f] = -\int \mathcal{D}y \, \exp\left[\frac{i}{\hbar}\int dt \; y \; [m\ddot{x} + \gamma\dot{x} - f]\right]$$

the need to double the degrees of freedom is implicit even in the classical theory when considering the brownian motion

e.o.m. $m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$

$$\langle\delta[m\ddot{x} + \gamma\dot{x} - f]\rangle = \int \mathcal{D}y < \exp[\frac{i}{\hbar}\int dt\, L_f(\dot{x}, \dot{y}, x, y)] >$$

$$L_f(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

the constraint condition at the classical level introduced a new coordinate y and the standard euler-lagrange eqs are obtained:

$$\frac{d}{dt}\frac{\partial L_f}{\partial \dot{y}} = \frac{\partial L_f}{\partial y} \; ; \qquad \frac{d}{dt}\frac{\partial L_f}{\partial \dot{x}} = \frac{\partial L_f}{\partial x}$$

i.e.

$$m\ddot{x} + \gamma\dot{x} = f \; , \quad m\ddot{y} - \gamma\dot{y} = 0$$

$$L_f(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

the constraint condition at the classical level introduced a new coordinate y and the standard euler-lagrange eqs are obtained:

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i.e.

$$m\ddot{x} \ldots \; ; \qquad m\ddot{y} - \gamma\dot{y} = 0$$

canonical formalism for dissipative systems

$$L_f(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

canonical formalism for dissipative systems

the x-system: open (dissipating) system
to set up the canonical formalism, one must close the system

this is the role of the y-system, which is the time reversed copy of the x-system; $\{x - y\}$ is a closed system

the two-sheeted space of NCSG is related to the gauge structure

classical 1dim damped harmonic oscillator (open system):

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

to set up a canonical formalism:

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

time-reversed image

$$\gamma \to -\gamma$$

closed system

$$L_f(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

$$f = kx$$

canonical transformation:

$$x_1(t) = \frac{x(t)+y(t)}{\sqrt{2}} \,, \qquad x_2(t) = \frac{x(t)-y(t)}{\sqrt{2}}$$

e.o.m.

$$\begin{aligned} m\ddot{x}_1 + \gamma\dot{x}_2 + kx_1 &= 0 \\ m\ddot{x}_2 + \gamma\dot{x}_1 + kx_2 &= 0 \end{aligned}$$

$$p_1 = m\dot{x}_1 + (1/2)\gamma x_2 \; ; \; p_2 = -m\dot{x}_2 - (1/2)\gamma x_1$$

hamiltonian:

$$H = H_1 - H_2 = \frac{1}{2m}(p_1 - \frac{\gamma}{2}x_2)^2 + \frac{k}{2}x_1^2 - \frac{1}{2m}(p_2 + \frac{\gamma}{2}x_1)^2 - \frac{k}{2}x_2^2$$

$$H = H_1 - H_2 = \frac{1}{2m}(p_1 - \frac{\gamma}{2}x_2)^2 + \frac{k}{2}x_1^2 - \frac{1}{2m}(p_2 + \frac{\gamma}{2}x_1)^2 - \frac{k}{2}x_2^2$$

introduce vector potential $A_i = \frac{B}{2}\epsilon_{ij}x_j \quad (i,j = 1,2)$

with $B \equiv \frac{c}{e}\gamma \ , \ \epsilon_{ii} = 0 \ , \ \epsilon_{12} = -\epsilon_{21} = 1$

H_1, H_2 describe 2 particles with opposite charges $e_1 = -e_2 = e$
in the oscilllator potential $\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2) \equiv \Phi_1 - \Phi_2$
and constant magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$

$$L = \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{c}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi$$

$$A_i = \frac{B}{2}\epsilon_{ij}x_j \quad (i, j = 1, 2)$$

- doubled coordinate, e.g. x_2 acts as gauge field component A_1 to which x_1 coordinate is coupled
- energy dissipated by one system is gained by the other one
- gauge field as *bath/reservoir* in which the system is embedded

in agreement with 't hooft's conjecture, loss of information (dissipation) in a regime of completely deterministic dynamics appears to be responsible of the system's QM evolution

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

the system's hamiltonian:

$$H = \sum_{i=1}^{2} p_i f_i(q)$$

nonsingular functions of q's

eqs. for q's

$$\dot{q}_i = \{q_i, H\} = f_i(q)$$

are decoupled of p's

class of hamiltonians considered by 't hooft a complete set of observables, *beables*, exists which poisson commutes

the system admits deterministic description even when expressed in terms of operators acting on a functional space of states (hilbert space)

quantisation as a consequence of dissipation (loss of information)

the system's hamiltonian:

$$H = \sum_{i=1}^{2} p_i f_i(q)$$

nonsingular functions of q's

eqs. for q's

$$\dot{q}_i = \{q_i, H\} = f_i(q)$$

are decoupled of p's

class of hamiltonians considered by 't hooft

$$H = H_{\mathrm{I}} - H_{\mathrm{II}}$$

impose constraint $H_{\mathrm{II}}|\psi\rangle = 0$

it defines physical states and guaranties that H is bounded from below

this constraint introduces information loss

$$H = H_{\mathrm{I}} - H_{\mathrm{II}}$$

impose constraint $H_{\mathrm{II}}|\psi\rangle = 0$

it defines physical states and guaranties that H is bounded from below

$$H|\psi\rangle = H_{\mathrm{I}}|\psi\rangle = \left(\frac{1}{2m} p_r^2 + \frac{K}{2} r^2\right)|\psi\rangle$$

$$K \equiv m\Omega^2$$

$$\Omega \equiv \sqrt{\frac{1}{m}\left(\kappa - \frac{\gamma^2}{4m}\right)}$$

H_{I} describes a 2dim isotropic (radial) harmonic oscillator

$$\ddot{r} + \Omega^2 r = 0$$

physical states are invariant under time reversal

$$|\psi(t)\rangle = |\psi(-t)\rangle$$

and periodical with period $\tau = 2\pi/\Omega$

$$\begin{aligned}
&{}_H\langle\psi(\tau)|\psi(0)\rangle_H \\
&= {}_{H_I}\langle\psi(0)|\exp\left(i\int_{C_{0\tau}} A(t')dt'\right)|\psi(0)\rangle_{H_I} \\
&\equiv e^{i\phi},
\end{aligned}$$

$$A(t) \equiv \frac{\Gamma m}{\hbar}(\dot{x}_1 x_2 - \dot{x}_2 x_1)$$

$$
{}_{H}\langle\psi(\tau)|\psi(0)\rangle_{H} \\
= {}_{H_I}\langle\psi(0)|\exp\left(i\int_{C_{0\tau}} A(t')dt'\right)|\psi(0)\rangle_{H_I} \\
\equiv e^{i\phi},
$$

calculate integral by rewriting as a contour integral in a complex plane

$$\phi = \alpha\pi$$

dimensionless

$$\mathcal{H}^n_{1,\mathrm{eff}} \equiv \langle \psi_n(\tau) | H | \psi_n(\tau) \rangle = \hbar\Omega \left(n + \frac{\alpha}{2} \right)$$

effective **n**th energy level of the physical system,
energy $\hbar\Omega n$ corrected by interaction with environment

the dissipation term of the hamiltonian is responsible for the zero point (n=0) energy: $E_0 = (\hbar/2)\Omega\alpha$

zero point energy is the signature of quantisation

dissipation term in H of a couple of classical damped-amplified oscillators manifests itself as a geometric phase and is responsible for the appearance for the zero point energy

$$\frac{\partial S}{\partial U} = \frac{1}{T}$$

$$H = H_{\rm I} - H_{\rm II} = U - TS = \mathcal{F}$$

it controls the dissipative
(irreversible) part of the dynamics

constraint to define physical states: condition for an adiabatic system

the thermodynamical picture is consistent with canonical
quantisation of dissipative systems in QFT

conclusions

the doubling of the algebra is related to dissipation and the gauge field structure

⇒ *the two-sheeted geometry is the construction that can lead to the gauge fields required to explain the SM*

dissipation, implied by the algebra doubling, may lead to quantum features (loss of information within completely deterministic dynamics may lead to a quantum evolution)

⇒ *the NCSG classical construction carries in the doubling of the algebra the seeds of quantisation* *remark*

in NCSG, the doubled degrees of freedom are associated with unlikely processes in the classical limit

⟹ perturbative approach: drop higher order terms (unlikely processes in the classical limit)

BUT

higher order terms are responsible for quantum corrections, so the second sheet cannot be neglected at the classical level

the second sheet (gauge fields) cannot be neglected in RDE ;
at GUTS gauge fields (discrete space of 2 points) plays no role