RG flow for Yukawa matter and Einstein gravity

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ERG 2010s

Missed ERG 2010 previous events...

- February 7th , Canada, MONSTER ERG 2010 REGATTA
- June, Brussels, ERG 2010 (Ecole de Recherche Graphique)

Outline

- Target: $\boxed{\text{gravity}} \leftrightarrow \boxed{\text{SM of particle physics}}$
- Framework: RG flows and Asymptotic Safety paradigm.
- Study of a toy model with a real scalar field, Dirac Fermions and Einstein gravity.
- Also toy model $+$ additional spin 0, $1/2$ and 1 free massless fields.
- Possibility:
  completely non trivial UV structure in gravity+standard model.

Gravitational interaction

- General relativity is non renormalizable at perturbative level 't Hooft, Veltman, Nieuwenhuizen, Goroff-Sagnotti.
- Gravity certainly has an effect on (and is affected by) the dynamics of ordinary quantum fields of the SM and beyond.
  Even if quantum formulation of gravity is in general still not well established, it is very reasonable to apply an energy dependent effective theory approach Donoghue.
- Search for a fundamental theory: string theory, spin foams, lattice models...
- Old or recent proposals for induced Sakharov or emergent Jacobson, Padmanabhan, Verlinde nature of gravity from underline unknown dynamics.
  But in general we expect quantum gravitation fluctuations together with quantum matter fluctuations.
- A viable option: fundamental QFT in the sense of Asymptotic Safety Weinberg.

Standard model: Higgs sector

- Theory perturbatively renormalizable (Landau pole), but ...
- Triviality problem.
- Fine tuning.
- Due to these unsatisfactory features, this sector of the SM can be at least regarded as an effective QFT.
  Maybe the underline dynamics has different d.o.f.
- A viable option: fundamental QFT in the sense of Asymptotic Safety. Gies, Scherer, Rechenberger,...
  Several toy models studied.

Renormalization and RG flows

- Wilsonian renormalization (non perturbative) of a theory described up to some scale Λ_0: coarse-graining of d.o.f followed by a rescaling to restore the original cutoff value.
- At a scale k one obtains a description in terms of an Effective Action $\Gamma_k[\phi]$.
  RG flow: $\partial_t \Gamma_k[\phi] = \mathcal{F}[\Gamma_k[\phi], \Gamma'_k[\phi], \Gamma''_k[\phi]]$, $t = \log[k/\Lambda_0]$.
  Wilson, Wegner-Houghton, Polchinski, Wetterich
  Renormalizability when the limit $\Lambda_0 \to \infty$ is safe (in lattice the continuum limit).
- Fixed points: $\partial_t \Gamma^*[\phi] = 0$
  Linearizing the flow: $\Gamma_k[\phi] - \Gamma^*[\phi] = \sum_i a_i e^{-\lambda_i t} \mathcal{O}_i[\phi]$
  Positive λ_i associated to *relevant* (UV attractive) directions.
  Perturbing along a relevant direction $\to$ self-similar renormalized trajectory
  (k-dependence only in adimensional couplings and anomalous dimensions $g_i(k)$ and $\eta_j(k)$).

Type of flows:

- Gaussian fixed point ($g_i^* = 0$)
  - no interacting relevant directions, triviality, ($d \geq 4$ scalar QFT)
  - interacting relevant directions, asymptotic freedom (QCD)
- Non gaussian fixed point (some $g_i^* \neq 0$)
  - interacting relevant directions, asymptotic safety, Weinberg
    Predictivity requires a finite number of relevant directions.

Exact RG flow of the effective average action

- Define an IR scale ($k/\Lambda_0 = e^t$)-dependent generating functional of connected Green's functions

$$W_k[J] = \log\left[\int [d\Phi] e^{-S[\Phi]-\int J\Phi - \Delta S_k[\Phi]}\right]$$

via a cutoff term $\Delta S_k[\Phi] = \frac{1}{2}\int d^4x\, \Phi \mathcal{R}_k^{\Phi}(-\partial^2)\Phi$ which suppress the contributions to the functional integral from the infrared modes of the field Φ.

- Use a cutoff-corrected Legendre transform: $\Gamma_k = -W_k + \int d^4x\, J\phi - \Delta S_k(\phi)$
- Such average effective action satisfies the RG flow equation Wetterich:

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right]$$

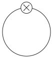
The r.h.s. takes contributions from a shell around the cutoff. It is finite and any UV cutoff dependence disappears (if not introduced in initial conditions)

- Note that Γ_k interpolates between Γ_{bare} ($k \to \infty$) and Γ(classical effective action) ($k \to 0$)
- Expand the Γ_k in a basis of operators to extract the running of the coefficients (couplings) $\Gamma_k[\phi^A, g_i] = \sum_i g_i(k)\mathcal{O}_i[\phi^A]$ A truncation is usually necessary!

Functional RG and anomalous dimensions

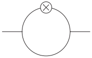
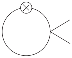
Consider $\Gamma_k[\Upsilon]$, functional of the field multiplet Υ.
Start from the flow of the effective average action:

- Erge: $\mathcal{G}_k \equiv \left(\Gamma_k^{(2)}[\Upsilon] + \mathcal{R}_k \right)^{-1}$

$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\, \mathcal{G}_k \dot{\mathcal{R}}_k = \frac{1}{2}$$ 

- Taking functional derivatives in the fields:

$$k\partial_k \frac{\delta^2 \Gamma_k}{\delta \Upsilon^A \delta \Upsilon^B} = \mathrm{STr}\, \mathcal{G}_k \frac{\delta \Gamma_k^{(2)}}{\delta \Upsilon^{(A}} \mathcal{G}_k \frac{\delta \Gamma_k^{(2)}}{\delta \Upsilon^{B)}} \mathcal{G}_k \dot{\mathcal{R}}_k - \frac{1}{2} \mathrm{STr}\, \mathcal{G}_k \frac{\delta^2 \Gamma_k^{(2)}}{\delta \Upsilon^A \delta \Upsilon^B} \mathcal{G}_k \dot{\mathcal{R}}_k$$

$=$  $- \frac{1}{2}$ 

- The β functions are obtained by substituting the expansion in the ERGE.
- The anomalous dimensions $\eta_i = -\partial_t \log Z_i$ are associated to the running of the field strengths of standard kinetic terms. Extract them by computing the flow of the two point functions.

Yukawa system & Einstein gravity

Euclidean effective action in a specific truncation:

$$\Gamma_k[g_{\mu\nu},\phi,\psi,\bar\psi] = \int d^4x (L_0 + L_{1/2} + L_g + L_{g.f.} + L_{gh})$$

- scalar field with potential $V(\phi)$

$$L_0 = \sqrt{g}\,(\frac{1}{2} Z_\phi \nabla^\mu \phi \nabla_\mu \phi + V(\phi))$$

- N_f fermion fields interacting with the scalar via $H(\phi) = y\phi$ ($U(N_f)$ symmetric)

$$L_{1/2} = \sqrt{g}\left[\frac{Z_\psi}{2}\left(\bar\psi\gamma^\mu iD_\mu\psi - iD_\mu\bar\psi\gamma^\mu\psi\right) + iH(\phi)\,\bar\psi\,\psi\right] .$$

Starting from local inertial reference frames the vierbein fields e^a_μ are introduced so that $D_\mu = \partial_\mu + \frac{1}{2}\omega_{\mu cd} J^{cd}$ is the covariant derivative, $\omega_{\mu cd} = e^\nu{}_c\left(e_{\nu d,\mu} - \Gamma^\rho{}_{\mu\nu} e_{\rho d}\right)$ is the metric compatible ($e_{\beta b;\mu} = 0$) spin connection and $J^{cd} = \frac{1}{4}[\gamma^c,\gamma^d]$ are the $O(4)$ generators.

- Einstein-Hilbert gravity

$$L_g = -\bar Z\,\sqrt{g}\,R\,[g_{\mu\nu}] \quad , \quad \bar Z = \frac{1}{16\pi G}$$

Quantization

Quantization is performed using the background field method.

Backgrounds: $\bar{g}_{\mu\nu}$, ϕ and ψ.
Quantum fluctuations: $h_{\mu\nu}$, φ and χ for metric, scalar and fermion fields.

- Invariance under reparameterization is fixed with the gauge fixing action term:

$$L_{g.f.} = \frac{\bar{Z}}{2\alpha} \delta^{\mu\nu} F_\mu F_\nu \quad ; \quad F_\mu = (\bar{g}_\mu^\beta \bar{\nabla}^\alpha - \frac{1+\beta}{4} \bar{g}^{\alpha\beta} \bar{\nabla}_\mu) h_{\alpha\beta}$$

  which leads to a corresponding ghost term.
- for fermions the vierbeins contain and extra $O(4)$ gauge symmetry. We employ a symmetric gauge ($e_{a\mu} = e_{\mu a}$) so that vierbein fluctuations can be written in terms of metric fluctuations and no $O(4)$ ghosts are present. (van Nieuwenhuizen, Woodard)
- for each field the effective action depends also on the average of the quantum fields (*classical* for $k \to 0$): for the metric $\langle g \rangle = \bar{g} + \bar{h}$
  Usually one employes the decomposition $\Gamma_k[\bar{g}, \bar{h}; \bar{c}, c; \cdots] = \bar{\Gamma}[\langle g \rangle; \cdots] + \hat{\Gamma}[h, \bar{c}, c, \bar{g}; \cdots]$ and assume a truncation for the effective action.
  The simpler choice: $\hat{\Gamma}[\bar{h}, \bar{c}, c, \bar{g}; \cdots] = S_{gf}[\bar{h}, \bar{g}] + S_{gh}[\bar{h}, \bar{c}, c, \bar{g}]$

Some details for the ERG equation

Running of $\bar{Z}$:
set the backgrounds for the metric as a sphere with curvature R, ϕ constant and $\psi = 0$.
Employ heat kernel techniques.

- spin decomposition of the metric fluctuations ($\nabla^\mu h^T_{\mu\nu} = h^{T\mu}_\mu = \nabla^\mu \xi_\mu = 0$) :

$$h_{\mu\nu} = h^T_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu + \nabla_\mu \nabla_\nu \sigma - \frac{1}{4} g_{\mu\nu} \nabla^2 \sigma + \frac{1}{4} g_{\mu\nu} h$$

  Jacobians cancelled after rescalings:

$$\hat{\sigma} = \sqrt{-\nabla^2} \sqrt{-\nabla^2 - \frac{R}{3}}\, \sigma \quad , \qquad \hat{\xi}_\mu = \sqrt{-\nabla^2 - \frac{R}{4}}\, \xi_\mu$$

- Second order expansion diagonal in the spin sectors on matter vacuum:

$$\mathcal{L}^{(2)} = \frac{1}{2} h^{T\mu\nu} \left(-\frac{\bar{Z}}{2} \nabla^2 + \frac{\bar{Z}}{3} R - \frac{1}{2} V \right) h^T_{\mu\nu} + \frac{1}{2} \hat{\xi}^\mu \left(-\frac{\bar{Z}}{\alpha} \nabla^2 - \frac{\bar{Z}\,(1-2\alpha)}{4\alpha} R - V \right) \hat{\xi}_\mu$$
$$+ \frac{1}{2} \begin{pmatrix} \hat{\sigma} & h & \varphi \end{pmatrix} S\left[\nabla^2, R\right] \begin{pmatrix} \hat{\sigma} \\ h \\ \varphi \end{pmatrix} + \begin{pmatrix} \chi & \bar{\chi}^T \end{pmatrix} Y\,[i\nabla\!\!\!/, R] \begin{pmatrix} \chi^T \\ \bar{\chi} \end{pmatrix}$$

- similarly for ghosts fluctuations.

Some details 2

For the running of the potential H we must keep $\psi \neq 0$.
Quadratic form less diagonal (fermion background)!

Cutoff scheme : type I
(vs type II for running of $\bar{Z}$, when curvature $R \neq 0$)

- remembering that $(i\nabla\!\!\!/)^2 = -\nabla^2 + \frac{R}{4}$
  the cutoff is chosen such that

$$\Gamma^{(2)} + R_k = \Gamma^{(2)}\Big|_{-\nabla^2 \to P_k[-\nabla^2],\, i\nabla\!\!\!/ \to i\nabla\!\!\!/ \frac{\sqrt{P_k[-\nabla^2]+\frac{R}{4}}}{\sqrt{-\nabla^2+\frac{R}{4}}}}$$

- profile: we use optimized cutoff function Litim, such that

$$P_k\,[z] = z\,\theta\Big(z - k^2\Big) + k^2\theta\Big(k^2 - z\Big)$$

- The scheme is called *diagonal*, since we do not include in R_k spin block off-diagonal terms proportional to the fermion background ψ.
  Neglecting anomalous dimensions we can compare some results with a non diagonal cutoff scheme O.Zanusso,L.Zambelli,G.P.V.,R.Percacci, PLB 2010. and find a very little dependence on this choice.

β functions

Define:

- renormalized fields: $\phi_R = Z_\phi^{1/2} k^{-1} \phi$ and $\psi_R = Z_\psi^{1/2} k^{-3/2} \psi$
- adimensional potentials:

$$v\left[\phi_R\right] = k^{-4} V\left[\phi\right]$$

$$h\left[\phi_R\right] = k^{-1} Z_\psi^{-1} H\left[\phi\right]$$

- and the dimensionless Planck mass square: $Z = \bar{Z}\, k^{-2}$, ($\eta_Z = -\dot{Z}/Z$).
- Inserting the truncation for the effective action into the flow equations one finds explicit partial differential equations for the potentials v and h:

$$\dot{v} = -4v + \phi\, v' \left(1 + \frac{\eta_\phi}{2}\right) + \frac{\mathrm{Nf}\left(\eta_\psi - 5\right)}{40\pi^2 \left(1 + h^2\right)} + \frac{5Z\left(\eta_Z - 8\right)}{192\pi^2 (v - Z)} - \frac{\eta_Z - 8}{64\pi^2} - \frac{1}{4\pi^2} + \frac{-(v - 2Z)\left(v'' + 1\right)\left(\eta_Z - 8\right) + 3\left(\left(v'\right)^2 \left(\eta_Z - 8\right) + 2(v - Z)\right) + (Z - v)\eta_\phi}{192\pi^2 \left((v - Z)\left(v'' + 1\right) - 3\left(v'\right)^2\right)}$$

$$
\begin{aligned}
\dot{h} \;=\;& h\left(\eta_\psi - 1\right) + \phi h'\left(1 + \frac{\eta_\phi}{2}\right) - \frac{5hZ\left(\eta_Z - 8\right)}{192\pi^2 (Z - v)^2} \\
&+ \frac{\left(\eta_\phi - 6\right)\left(h''(v - Z)^2 + 6h'(Z - v)v' + 3h\left(v'\right)^2\right) - Z\left(\eta_Z - 8\right)\left(3h''\left(v'\right)^2 - 6h'v'\left(1 + v''\right) + h\left(1 + v''\right)^2\right)}{192\pi^2\left((v - Z)\left(1 + v''\right) - 3\left(v'\right)^2\right)^2} \\
&+ \frac{h'(v - Z)v'\left(\eta_\phi - 6\right) - Zh'v'\left(1 + v''\right)\left(\eta_Z - 8\right)}{16\pi^2\left((v - Z)\left(1 + v''\right) - 3\left(v'\right)^2\right)^2} - \frac{h'v'\left((226 - 87\eta_Z)Z\left(1 + v''\right) + (29\eta_\phi - 168)(v - Z)\right)}{560\pi^2\left(1 + h^2\right)\left((v - Z)\left(1 + v''\right) - 3\left(v'\right)^2\right)^2} \\
&+ 3h\frac{\eta_Z Z\left(1 + v''\right)^2\left(\left(1120h^2 + 349\right) - 8960h^2 - 3106\right) + 3\left(v'\right)^2\left(-\left(1120h^2 + 349\right)\eta_\phi + 6720h^2 + 2408\right)}{35840\pi^2\left(1 + h^2\right)\left((v - Z)\left(1 + v''\right) - 3\left(v'\right)^2\right)^2} \\
&+ \frac{8h\left(h'\right)^2(v - Z)\left(5 - \eta_\psi\right) + h\left(1 + v''\right)\left(19 + 96h^2 - \left(20h^2 + \frac{23}{7}\right)\eta_\psi\right) + 4h'v'\left((1 + 11h^2)\eta_\psi - 6(1 + 9h^2)\right)}{640\pi^2\left(1 + h^2\right)^2\left((v - Z)\left(1 + v''\right) - 3\left(v'\right)^2\right)}
\end{aligned}
$$

- and an ODE for the running of Z ($v'(\phi_R) = 0$, since in the truncation Z is field independent):

$$
\begin{aligned}
\dot{Z} \;=\;& -2Z - 5Z\frac{9v - 25Z}{576\pi^2(Z - v)^2} - 5\dot{Z}\frac{7Z - 3v}{1152\pi^2(Z - v)^2} + \frac{\dot{Z}}{128\pi^2(Z)} - \frac{2Z\left(6 + 5v''\right) - v\left(9 + 7v''\right)}{192\pi^2(Z - v)\left(1 + v''\right)} \\
&+ \frac{\eta_\phi}{384\pi^2\left(1 + v''\right)} - \frac{\dot{Z}(2v - 3Z)}{384\pi^2(Z - v)} + N_f\frac{15 - 8\eta_\psi + (60 - 17\eta_\psi)h^2}{1440\pi^2\left(1 + h^2\right)^2} + \frac{5}{96\pi^2} + \frac{5}{64\pi^2} - \frac{7}{128\pi^2}
\end{aligned}
$$

Anomalous dimensions.

- Anomalous dimensions are extracted from the flow of the two point functions for scalar and fermion fields looking for contributions in momentum space of the form $\Delta\dot{Z}_\phi\, p^2$ and $\Delta\dot{Z}_\psi \not{p}$.
  Again $Z_{\phi,\psi}$ are field independent so we set also $v'(\phi_R) = 0$.
  We use <u>three and four point vertices</u> of the truncated action for all d.o.f.

  Results not shown here. System of coupled equations (η_ϕ, η_ψ).

  Leading gravitational corrections at GFP in DeDonder gauge:
  $\eta_\phi = 0$, $\eta_\psi = -Gk^2 123/80\pi$.

  So we have a perturbative leading gravitational correction:

  $$\dot{h}^{(grav)} = +\frac{673}{1120\pi} Gh + \cdots$$

- We choose $h_R[\phi_R] = y\phi_R$ and approximate the scalar potential $v_R[\phi_R]$ with a polynomial:
  - Symmetric phase , $v_R[\phi_R] = \lambda_0 + \lambda_2\phi_R^2 + \lambda_4\phi_R^4 + \lambda_6\phi_R^6 + \cdots$
  - SSB phase, $v_R[\phi_R] = \theta_0 + \theta_4(\phi_R^2 - \kappa)^2 + \theta_6(\phi_R^2 - \kappa)^3 + \cdots$

Asymptotic safety and previous RG flow analysis

What has been studied in the literature till now?

$D_c =$ dimension of the UV critical surface

- Pure gravity theories. All studies with several truncations have shown a NGFP
  - Einstein-Hilbert gravity with cosmological constant $\rightarrow D_c = 2$
  - Modified gravity theories quadratic in the curvature $\rightarrow D_c = 3$
  - $f(R)$ theories (polinomial up to R^8) $\rightarrow D_c = 3$

  (Reurer, Lauscher, Saueressig, Percacci, Codello, Rahmede, Niedermaier, Litim, Benedetti, Machado, . . .)
- Pure matter theories
  - Scalar theories in $d = 3$, Wilson-Fisher fixed point $\rightarrow D_c = 2$
  - 1 real scalar+N_f Dirac fermions ($d = 4$): NGFP only in SSB phase for $N_f \lesssim 0.3 \rightarrow D_c = 2$
    N_L (complex scalar+left handed fermion)+1 right handed fermion. NGFP only in SSB phase $\rightarrow D_c = 1, 2$
  - Thirring model in $d = 3$ and N_f fermions, NGFP $\rightarrow D_c = 2$

  (Gies, Scherer, Rechenberger, Janssen)
- Matter-gravity theories (Gaussian Matter FP)
  - Non minimally coupled scalar (no η_ϕ) and Einstein gravity $\rightarrow D_c = 2$ (4)
  - Spin 1 gauge fields and Einstein gravity $\rightarrow D_c = 3$

  (Percacci, Perini, Narain, Rahmede, Reuter, Daum, Harst, Litim, Pawlowski,... . . .)

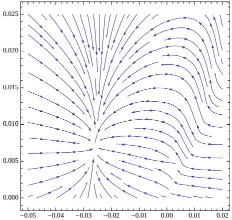
Fixed points: symmetric phase

- Beyond the fully gaussian fixed point (GFP), there is for any N_f a gaussian matter fixed point (GMFP) with $-1 < \eta_\psi < 0$ and $\eta_\phi = 0$ in DeDonder gauge. UV critical surface with dimension 4 (3 for GFP and $N_f < 3$)

- NGFP: In DeDonder gauge and for a truncation up to λ_4 it has physical values for $N_f > 3.6$ The critical surface has dimension $D_c = 3$

N_f	λ_0	λ_2	λ_4	y	Z	η_ψ	η_ϕ
4	-0.0129	0.243	0.832	2.70	0.00562	-0.215	0.400
8	-0.0260	0.0872	0.437	1.25	0.00700	-0.102	0.225
20	-0.0642	0.0289	0.0543	0.467	0.0130	-0.0407	0.0958
$N_f \to \infty$	$\lambda_{0,\infty} - \frac{N_f}{32\pi^2}$	$\frac{\lambda_{2,\infty}}{N_f}$	$\frac{\lambda_{4,\infty}}{N_f^2}$	$\frac{y_\infty}{N_f}$	$Z_\infty + \frac{N_f}{192\pi^2}$	$\frac{\eta_{\psi,\infty}}{N_f}$	$\frac{\eta_{\phi,\infty}}{N_f}$

N_f	c_1	c_2	c_3	c_4	c_5
4	-1.02	-0.566	1.30	1.38	3.88
8	-0.613	-0.240	1.66	1.68	3.96
20	-0.235	-0.0937	1.86	1.87	3.99
$N_f \to \infty$	$\frac{c_{1\infty}}{N_f}$	$\frac{c_{2\infty}}{N_f}$	$2+\frac{c_{3\infty}}{N_f}$	$2+\frac{c_{4\infty}}{N_f}$	$4+\frac{c_{5\infty}}{N_f}$



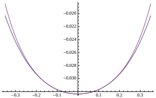
Therefore for $N_f \to \infty$ the fixed point becomes GMFP.

NGFP in the symmetric phase

We find:

- stability of the FP on varying the gauge fixing parameters α and β. Values change mildly.

  stability under change of truncation up to λ_6 or to λ_8.
- Indeed shape of the potential is stable within the radius of convergence of the expansion



  Introducing λ_6 the maximum relative change in y^* is fractions of %, in λ_2^* is of %, completely negligible for λ_0^* and Z^*.
- we expect it to be stable under change of cutoff function.

What if anomalous dimensions of matter fields are neglected?

- The NGFP found exists, with couplings of the same order of magnitude, for $5 < N_f \leq 12$ But another one with much larger values and a completely different asymptotic dependence in N_f appears.

Ergo: Considering matter anomalous dimensions might be relevant to understand the FP structure.

Broken phase

- Let us start by looking at the flow of the VEV itself in a general case. Differentiating $v'(\sqrt{\kappa}) = 0$ one immediately gets

$$\partial_t \kappa = -(2+\eta_\phi)\kappa + \frac{N_f(\eta_\psi - 5)h\,h'\sqrt{\kappa}}{10\pi^2(1+h^2)^2 v''} + \frac{(6-\eta_\phi)v'''\sqrt{\kappa}}{96\pi^2(1+v'')^2 v''}\bigg|_{\phi=\sqrt{\kappa}}$$

  We note that the dependence on the gravitation coupling is only through η_ϕ and η_ψ.

- Results: 5 branches of FPs.
  - Branch 1: exist for any N_f, η_ϕ increase with N_f
  - Branch 2: $N_f \gtrsim 3$, large ηs
  - Branch 3: $y = 0$, $\theta_4 \neq 0$, only $N_f = 1, 2$, acceptable ηs, UV repulsive
  - Branch 4: GMFP $y = \theta_4 = 0$, $\kappa = 3/32\pi^2$, any N_f, $\eta_\phi = 0$ and $|\eta_\psi| \ll 1$, $D_c = 4$
  - Branch 5: any N_f, large ηs

Running of Z: type II cutoff scheme

We note that with a type II cutoff there is a change in the leading contribution of matter fields to the running of Z.

Example of type II: Dirac field

$$\Gamma^{(2)} + R_k = \Gamma^{(2)}\Big|_{i\nabla\!\!\!/ \to i\nabla\!\!\!/ \frac{\sqrt{P_k[-\nabla^2+\frac{R}{4}]}}{\sqrt{-\nabla^2+\frac{R}{4}}}}$$

Computation of the trace in the ERGE may give (and it does!) a different term linear in the curvature R.

For free massless n_s scalar, n_D Dirac fermions and n_M vector fields, same vacuum energy contribution, but the gravitational β function acquires a term proportional to

- $(-n_s + n_D + \frac{7}{4}n_M)$ (type I)
- $(-\frac{1}{2}n_s - n_D + 2n_M)$ (type II)

Applying type II cutoff scheme to the toy model kills the NGFP in the symmetric phase!!!

So we consider a slightly extended model: toy model + free massless fields.
We set $N_f = 3$ (for top quark) and n_s, n_D, n_M to match qualitatively the content of the standard model.

- The NGFP in the symmetric phase now exists for both type I and II schemes.
- It is stable under variations in a wide range of the number of the different fields.
- at NGFP the yukawa coupling is of order unity. Generally $\lambda_0 < 0$.

Summary and Outlook

- We have considered gravity coupled to scalar and fermion matter taking into account anomalous dimensions.
- Eventually we added other free massless matter fields of various spin.
- Asymptotic safety appears in these toy models involving gravity and matter. Situation more clear in the Z_2 symmetric phase
  In such a case values of Yukawa coupling at fixed point about unity.
- As for pure gravity, more general ”truncations” should be considered.
- The interesting fact is that this could happen in the standard model coupled to gravity. One may consider two possible ways:
  - a fixed point in a symmetric phase and mechanism for SSB at lower energies
  - a fixed point directly in a broken phase which might appear in an analysis of a more realistic model.